QCD ANALYSIS OF POLARIZED SCATTERING DATA AND NEW POLARIZED PARTON DISTRIBUTIONS*

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In this talk results from a new QCD analysis in Leading (LO) and Next-to-Leading (NLO) Order are presented. New parametrizations of the polarized quark and gluon densities are derived together with parametrizations of their fully correlated 1σ error bands. Furthermore the value of $\alpha_{\rm s}(M_Z^2)$ is determined. Finally a number of low moments of the polarized parton densities are compared with results from lattice simulations. All details of the analysis are given in J. Blümlein, H. Böttcher, *Nucl. Phys.* **B636**, 225 (2002).

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1. Formalism

The twist-2 contributions to the structure function $g_1(x,Q^2)$ can be represented in terms of a MELLIN convolution of the polarized singlet density $\Delta\Sigma$, the polarized gluon density ΔG, the polarized non–singlet density $\Delta q_j^{\rm NS}$, and the corresponding polarized Wilson coefficient functions ΔC_i^A by

$$g_1(x,Q^2) = \frac{1}{2}\sum_{j=1}^{N_f} e_j^2 \int_x^1 \frac{dz}{z}\left[\frac{1}{N_f}\Delta\Sigma\left(\frac{x}{z},\mu_{\rm f}^2\right)\Delta C_q^{\rm S}\left(z,\frac{Q^2}{\mu_{\rm f}^2}\right) + \Delta G\left(\frac{x}{z},\mu_{\rm f}^2\right) \times\Delta C^G\left(z,\frac{Q^2}{\mu_{\rm f}^2}\right) + \Delta q_j{}^{\rm NS}\left(\frac{x}{z},\mu_{\rm f}^2\right)\Delta C_q^{\rm NS}\left(z,\frac{Q^2}{\mu_{\rm f}^2}\right)\right], \quad (1)$$

where e_j denotes the charge of the jth quark flavor, N_f the number of flavors, and $\mu_{\rm f}$ is the factorization scale. In addition, the above quantities depend on the renormalization scale $\mu_{\rm r}$ of the strong coupling constant $a_{\rm s}(\mu_{\rm r}^2) = g_{\rm s}^2(\mu_{\rm r}^2)/(16\pi^2)$.

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The change of the parton distributions w.r.t. the factorization scale $\mu_f^2 = Q^2$ is described by the evolution equations which contain the polarized splitting functions ΔP_{ij}. Both the polarized Wilson coefficient [2] and the polarized splitting functions [3] are known in the $\overline{\rm MS}$ scheme up to NLO.

The evolution equations are solved in MELLIN–N space. A MELLIN–transformation turns the MELLIN convolution into an ordinary product. The solutions for the evolved parton distributions are structured such that the input and the evolution part factorize [4], which is of key importance for the error calculation. An inverse MELLIN–transformation to x–space is then performed numerically by a contour integral in the complex plane around all singularities.

2. Parametrization and error calculation

The shape chosen for the parameterization of the polarized parton distributions at the input scale of $Q^2 = 4.0$ GeV^2 is:

$$x\Delta q_i(x, Q_0^2) = \eta_i A_i x^{a_i}(1-x)^{b_i}(1 + \gamma_i x + \rho_i x^{\frac{1}{2}}) . \qquad (2)$$

The normalization constant A_i is defined such that η_i is the first moment of $\Delta q_i(x, Q_0^2)$. The densities to be fitted are Δu_v, Δd_v, $\Delta \bar{q}$, and ΔG.

Assuming SU(3) flavor symmetry the first moments of Δu_v and Δd_v can be fixed by the SU(3) parameters F and D measured in neutron and hyperon β–decays to $\eta_{u_v} = 0.926$ and $\eta_{d_v} = -0.341$. The sea-quark distribution $\Delta\bar{q}$ was assumed to be described according to SU(3) flavor symmetry. Given the present accuracy of the data we set a number of parameters to zero, namely $\rho_{u_v} = \rho_{d_v} = 0$, $\gamma_{\bar{q}} = \rho_{\bar{q}} = 0$, and $\gamma_G = \rho_G = 0$. Furthermore, we adopted the following two parameter relations: $a_G = a_{\bar{q}} + c$, with $0.5 < c < 1.0$ and $(b_{\bar{q}}/b_G)(\text{pol}) = (b_{\bar{q}}/b_G)(\text{unpol})$. These relations were essential to respect positivity for $\Delta\bar{q}$ and ΔG and to achieve the expected similar low–x behavior for both parton densities. No positivity constraint was assumed for Δu_v and Δd_v. In addition $\Lambda_{\rm QCD}$ was determined.

The gradients of the polarized parton densities w.r.t. the fitted parameters needed for the Gaussian error propagation are calculated analytically at the input scale Q_0^2 in MELLIN–N space. Their values at Q^2 are then given by evolution. For more details see Ref. [1].

To treat all data sets on the same footing only the statistical errors were used. To be able to calculate the fully correlated 1σ error bands via Gaussian error propagation only fits ending with a positive–definite covariance matrix were accepted. We allowed for a relative normalization shift between the different data sets within the normalization uncertainties quoted by the experiments. Thereby the main systematic uncertainties of the data were taken into account. The relative normalization shifts being fitted once and then fixed enter as an additional term (penalty) for each data set in the χ^2–expression.

3. Results

The results reported here are based on 435 data points of asymmetry data, *i.e.* g_1/F_1 or A_1, above $Q^2 = 1.0$ GeV^2, the world statistics published so far (see Ref. [1] for a list of references). The fits are performed on g_1 which is evaluated from the asymmetry data using parametrizations for F_2 [5] and R [6]. The data do not constrain the four parameters γ_{u_v}, γ_{d_v}, $b_{\bar{q}}$, and b_G well enough. Their values have been fixed as obtained in the first minimization. The NLO polarized parton densities at the input scale are presented in Fig. 1 (ISET = 3, see Ref. [1]).

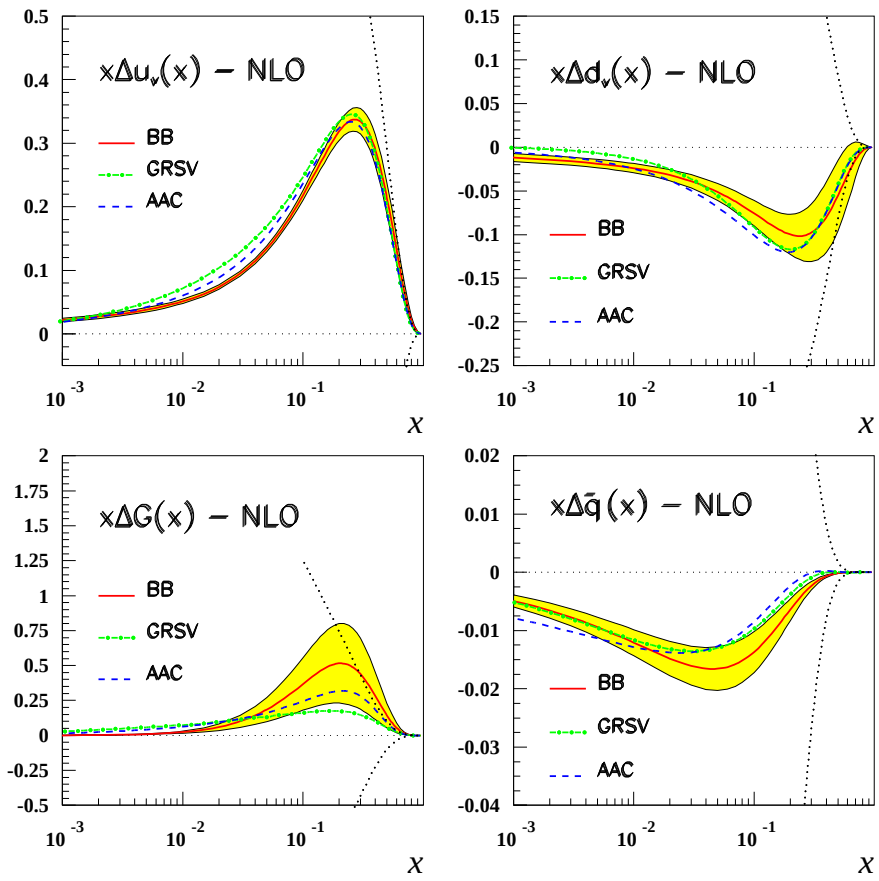


Fig. 1. Polarized parton distributions at the input scale $Q_0^2 = 4.0$ GeV^2 (solid line) compared to results obtained by GRSV (dashed–dotted line) [7], AAC (dashed line) [8]. The shaded areas represent the fully correlated 1σ error bands calculated by Gaussian error propagation [1]. The dark dotted lines correspond to the positivity bounds given by the unpolarized densities.

The current data constrain Δu_v at best, followed by Δd_v, $\Delta\bar{q}$, and ΔG. Mainly the lack of data at low x leads to the broader error bands for the latter two densities. The comparison with results from other QCD analyses shows the variation of the outcoming parton distributions when using different parametrizations at the input scale. The measured structure function g_1^p is well described both as function of x and Q^2. The derived parton densities and their error bands have been evolved to Q^2 values up to $10,000$ GeV^2. One finds that Δu_v stays positive, Δd_v remains negative although less constraint than Δu_v, and ΔG stays positive even within the error band for the range $Q^2 \geq 4\,\mathrm{GeV}^2$. All three densities evolve towards smaller values of x. The sea-quark density $\Delta\bar{q}$ is negative for $Q^2 \leq 10^4\,\mathrm{GeV}^2$ and remains negative within errors for $x \leq 5 \times 10^{-2}$ up to $Q^2 = 10^4\,\mathrm{GeV}^2$, but changes sign for larger values of x.

In determining α_s the QCD–parameter Λ_QCD was fitted. The impact of the variation of both the renormalization and the factorization scale on the value of α_s was investigated. The following result was obtained (for ISET = 3, see Ref. [1], as well for more details):

$$\alpha_\mathrm{s}(M_Z^2) = 0.113 \;{}^{+0.004}_{-0.004}\;(\mathrm{stat})\;{}^{+0.004}_{-0.004}\;(\mathrm{fac})\;{}^{+0.008}_{-0.005}\;(\mathrm{ren})\;.$$

This value is compatible within 1σ with the world average of 0.118 ± 0.002 [9], although our central value is somewhat lower. It is also compatible with results from other QCD analyses of polarized and unpolarized data.

TABLE I

Moments of the NLO parton densities (for ISET = 3, see [1]) at $Q^2 = 4$ GeV^2 and from recent lattice simulations at the scale $\mu^2 = 1/a^2 \sim 4$ GeV^2.

Δf	n	QCD results	Lattice results	
		NLO moments at $Q^2 = 4$ GeV^2	QCDSF	LHPC/ SESAM
Δu_v	–1	0.926 ± 0.071	0.889(29)	0.860(69)
	0	0.163 ± 0.014	0.198(8)	0.242(22)
	1	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	–1	-0.341 ± 0.123	−0.236(27)	−0.171(43)
	0	-0.047 ± 0.021	−0.048(3)	−0.029(13)
	1	-0.015 ± 0.009	−0.028(2)	0.001(25)
Δu–Δd	–1	1.267 ± 0.142	1.14(3)	1.031(81)
	0	0.210 ± 0.025	0.246(9)	0.271(25)
	1	0.070 ± 0.011	0.069(9)	0.115(49)

In recent lattice simulations [10] low moments for the polarized parton densities Δu_v, Δd_v, and $\Delta u - \Delta d$ were determined. In Table I these moments are compared with the ones extracted from our NLO polarized densities. The first moment is denoted with $n = -1$. For the $n = 0, 1$ values of the QCDSF Collaboration no continuum extrapolation was performed. The values compare within errors. Unlike in the unpolarized case where a strong m_π–dependence is expected in the lattice extrapolation the comparison here suggests a flat behavior, which also has been found in [11] very recently.

4. Conclusions

A QCD analysis in LO and NLO of the current world–data on polarized structure functions was performed. New parametrizations of the polarized parton densities including their fully correlated 1σ error bands were derived.These parameterizations are available as a fast `FORTRAN`–routine which makes their application possible in Monte Carlo simulations. The value determined for $\alpha_s(M_Z^2)$ is compatible within 1σ with the world average. Comparing the lowest moments with values from lattice simulations the errors improved during recent years and the values became closer.

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