FRAGMENTATION FUNCTIONS AND THEIR ROLE IN DETERMINING THE POLARIZED PARTON DENSITIES*

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(Received May 22, 2002)

We extract the pion fragmentation functions and their uncertainties from a judicious choice of e^+e^- and semi-inclusive DIS data. These are used to study the error propagation in the extraction of polarized parton densities from semi-inclusive DIS asymmetries. We conclude that the uncertainties on polarized PDs have been underestimated in the past.

PACS numbers: 13.88.+e

1. Introduction

Flavour separation of the unpolarized Parton Densities (PDs) relies heavily on the neutrino CC reactions. Determination of the gluon density relies on the large range of Q^2 available. Neither of these possibilities exist for the polarized PDs. Thus, although the data from polarized DIS studies are now of superb quality, they, in principle, only provide information on the

* Presented at the X International Workshop on Deep Inelastic Scattering (DIS2002) Cracow, Poland, 30 April–4 May, 2002.

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combinations $\Delta q(x) + \Delta\bar{q}(x)$ and on $\Delta G(x)$, and both $\Delta s(x) + \Delta\bar{s}(x)$ and $\Delta G(x)$ are poorly known.

Ultimately, a neutrino factory and TESLA-N would remedy this, but for the next decade progress will rely upon HERMES and COMPASS semi-inclusive (SIDIS) data.

Extraction of the polarized PDs from polarized SIDIS requires a good knowledge of Fragmentation Functions (FFs). We derive values of the pion FFs and a realistic assessment of their uncertainties. We then study how the uncertainties of the FFs generate errors on the extracted polarized PDs. We conclude that previous studies have underestimated these errors.

For simplicity we employ a LO formalism [1]. A strategy for a simplified NLO treatment is given in [2].

2. Polarized DIS — current status

Remember that, in principle, we can only obtain information on $\Delta u(x) + \Delta\bar{u}(x)$, $\Delta d(x) + \Delta\bar{d}(x)$, $\Delta s(x) + \Delta\bar{s}(x)$ and $\Delta G(x)$ from present day polarized DIS. Moreover, whereas $(\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})$ is directly determined from $g_1^p(x, Q^2) - g_1^n(x, Q^2)$, the separation of ΔG, $\Delta s + \Delta\bar{s}$ and $(\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d})$ from each other relies partly on evolution — not very efficacious with the presently available range of Q^2 — and partly on the hyperon β-decay sum rule

$$a_8 \equiv \int_0^1 dx \left\{ \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2\left(\Delta s + \Delta\bar{s}\right)\right\} = 3F - D = 0.58\,. \quad (1)$$

All the modern analyses [3] agree quite well on the determination of $(\Delta u + \Delta\bar{u})$ and $(\Delta d + \Delta\bar{d})$ (results are often presented for Δu_V, Δd_V and Δ_{sea}, but this is quite artificial and usually incorporates some assumptions like $\Delta\bar{u} = \Delta\bar{d} = \Delta s$ *etc.*), but there is substantial disagreement about $(\Delta s + \Delta\bar{s})$ and ΔG; though all suggest a non-zero, positive first moment $\int dx \Delta G(x)$.

Eq. (1) assumes that the β-decays of the hyperon octet respect flavour SU(3) symmetry, a belief challenged by some authors. We have therefore studied the effect of varying the value of a_8, (1), between the extreme estimates 0.40 and 0.86 found in literature [4]. Very likely this is too extreme to be realistic, but the huge effect on the poorly determined $(\Delta s + \Delta\bar{s})$ and ΔG, as seen in Fig. 1 is instructive. It underlines the importance of other sources of information on the polarized PDs and this, in the foreseeable future, means polarized SIDIS at HERMES and COMPASS, which, as mentioned, requires an accurate knowledge of the FFs to which we now turn.

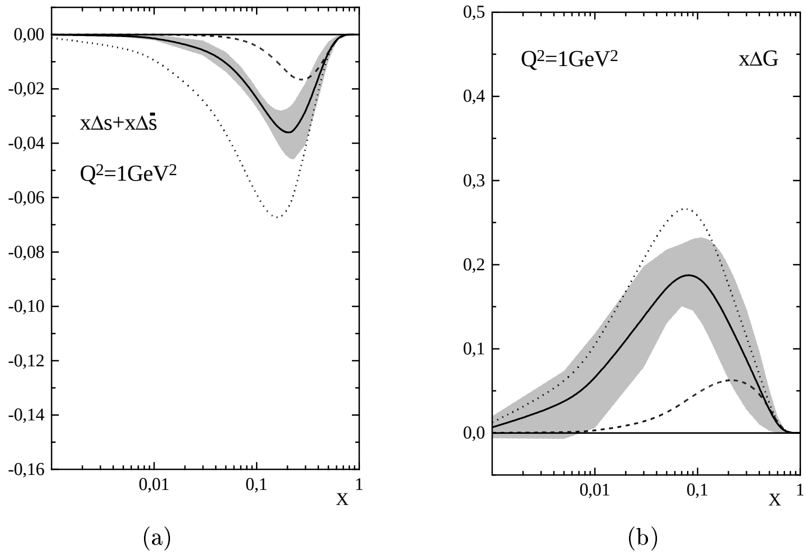

Fig. 1. Variation in $\Delta s + \Delta\bar{s}$ and ΔG arising from varying the value of a_8 in (1) away from its SU(3) value of 0.58 (solid line and error band); $a_8 = 0.86$ (dotted line); $a_8 = 0.40$ (dashed line).

3. Extraction of the fragmentation functions

For a long time it was thought that the majority of FFs were well determined by the $e^+e^- \to hX$ reaction. Then in 2000/2001 three papers appeared which all described essentially the same e^+e^- data equally well, but with significantly different individual flavour FFs [5]. In other words (and in retrospect this is not surprising) $e^+e^- \to hX$ does not provide a sensitive method of flavour separation.

Despite this we have been able to extract the pion FFs with considerable accuracy, by combining recent HERMES data on $\pi^\pm$ multiplicities on protons with a judicious choice of information from $e^+e^- \to \pi^\pm X$ [6]. The latter involves a subtle trick which allows us to avoid the above mentioned ambiguities in the FFs derived from $e^+e^- \to hX$.

There are 3 independent FFs for pions: $D_u^{\pi^+}$, $D_d^{\pi^+}$ and $D_s^{\pi^+}$. All the others can be related to these via charge conjugation invariance or isotopic spin invariance. The HERMES data on $\pi^\pm$ multiplicities on protons provides us with 2 equations for 3 unknowns (the $\tilde{\sigma}$ differ from the cross sections σ by a standard kinematic factor – see [6]):

$$\begin{aligned}\tilde{\sigma}^h &= \tfrac{4}{9}\left[u(x)D_u^h(z) + \bar{u}(x)D_{\bar{u}}^h(z)\right]\\ &\quad+\tfrac{1}{9}\left[d(x)D_d^h(z) + \bar{d}(x)D_{\bar{d}}^h(z) + s(x)D_s^h(z) + \bar{s}(x)D_{\bar{s}}^h(z)\right], \qquad (2)\end{aligned}$$

where $h = \pi^\pm$ and where we assume known unpolarized PDs.

To obtain a third relation we note that in $e^+e^- \to hX$ at low Q^2 the coupling to $q\bar{q}$ pairs is electromagnetic, with

$$e_u^2 = \frac{4}{9} > e_d^2 = e_s^2 = \frac{1}{9}. \tag{3}$$

However at $Q^2 = M_Z^2$ the electroweak couplings $\hat{e}_q^2$ are such that

$$\hat{e}_u^2 < \hat{e}_d^2 = \hat{e}_s^2, \qquad \text{with} \qquad \frac{\hat{e}_u^2}{\hat{e}_d^2} \approx \frac{3}{4}. \tag{4}$$

Hence there must exist a "magic" energy Q_0^2 where

$$\hat{e}_u^2(Q_0^2) = \hat{e}_d^2(Q_0^2) = \hat{e}_s^2(Q_0^2). \tag{5}$$

At this energy

$$\tilde{\sigma}_{e^+e^-}^{\pi^+} \sim D_u^{\pi^+} + D_d^{\pi^+} + D_s^{\pi^+} + D_{\bar{u}}^{\pi^+} + D_{\bar{d}}^{\pi^+} + D_{\bar{s}}^{\pi^+} \sim D_\Sigma^{\pi^+} \tag{6}$$

where $D_\Sigma^{\pi^+} = 2\left(D_u^{\pi^+} + D_d^{\pi^+} + D_s^{\pi^+}\right)$ is the flavour singlet FF.

Unfortunately there is no data at the magic energy $\sqrt{Q_0^2} = 78.4$ GeV, but it is close enough to M_{Z^0} to be able to argue that $D_\Sigma^{\pi^+}$ is very well determined by the combination of FFs measured at the Z^0, namely,

$$D_\Sigma^{\pi^+} = \frac{43}{77} D_{\text{measured}}^{\pi^+ + \pi^-}\,(1.00 \pm 0.02), \tag{7}$$

i.e. $D_\Sigma^{\pi^+}$ is known accurately at $Q^2 = M_Z^2$. However it has to be evolved down to Q^2_{HERMES}. This involves mixing with the poorly known gluon FF D_G^π. Consequently we end up knowing $D_\Sigma^{\pi^+}$ at Q^2_{HERMES} to $\pm 10\%$.

Knowing the value of $D_\Sigma^{\pi^+}$ provides us with a third equation so that we can now solve for the individual $D_q^{\pi^+}$:

$$\begin{aligned}
D_u^{\pi^+} - D_d^{\pi^+} &= \frac{9\left(R_p^{\pi^+} - R_p^{\pi^-}\right)\tilde{\sigma}_p^{\text{DIS}}}{4u_V - d_V}, \\
D_u^{\pi^+} + D_d^{\pi^+} &= \frac{9\left(R_p^{\pi^+} + R_p^{\pi^-}\right)\tilde{\sigma}_p^{\text{DIS}} - 2s\, D_\Sigma^{\pi^+}}{4(u + \bar{u} - s) + d + \bar{d}}, \\
D_s^{\pi^+} &= \frac{-18\left(R_p^{\pi^+} + R_p^{\pi^+}\right)\tilde{\sigma}_p^{\text{DIS}} + [4(u+\bar{u}) + d + \bar{d}]\, D_\Sigma^{\pi}}{2\,[4(u + \bar{u} - s) + d + \bar{d}\,]},
\end{aligned} \tag{8}$$

where $R^h \equiv \sigma^h/\sigma^{\text{DIS}} = \tilde{\sigma}^h/\tilde{\sigma}^{\text{DIS}}$. Note that $D_u - D_d$ is independent on D_Σ, that $D_u + D_d$ depends weakly on it, and that D_s is most sensitive to it. This is reflected in the uncertainties shown on the $D_q^{\pi^+}$ derived from (8) (see left of Fig. 2).

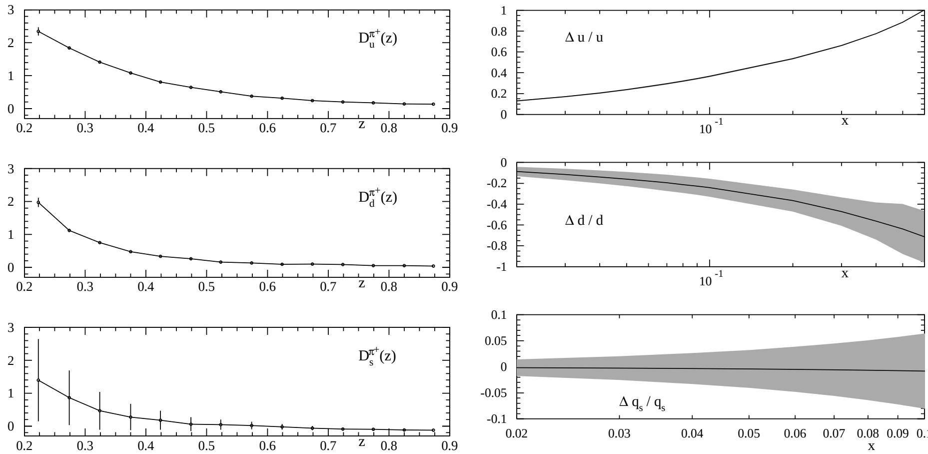

Fig. 2. The extracted fragmentation functions (left) with their uncertainties. Polarized parton densities (right) extracted from fake, error-free data, as explained in the text — the uncertainties arise solely from the realistic experimental errors on our FFs.

4. Implications for the polarized parton densities

At present the asymmetry data on $\pi^\pm$ SIDIS using a polarized target are not yet available. In order, therefore, to study the error propagation, we generate perfect (error-free) fake proton DIS and SIDIS asymmetry data, and then analyze it, following closely the methods advocated by HERMES.

Thus we construct the flavour f purities $(h = \pi^+, \pi^-)$:

$$P^h_{q_f}(x) = \frac{e^2_{q_f}\, q_f(x)\, \int dz D^h_{q_f}(z)}{\sum_{f'} e^2_{q f'}\, q_{f'}(x)\, \int dz D^h_{q_{f'}}(z)}, \qquad P^{\rm DIS}_{q_f}(x) = \frac{e^2_{q_f}\, q_f(x)}{\sum_{f'} e^2_{q_{f'}}\, q_{f'}(x)}. \quad (9)$$

The measured asymmetries integrated over z, are then given, in LO, by

$$\left\langle \Delta A^h_p(x) \right\rangle \equiv \frac{\int dz\, \Delta\tilde{\sigma}^h(x,z)}{\int dz\, \tilde{\sigma}^h(x,z)} = \sum_{q,\bar{q}} P^h_q(x) \left(\frac{\Delta q(x)}{q(x)} \right), \quad h = \pi^\pm, {\rm DIS}\,. (10)$$

We have data on proton DIS and proton $\pi^\pm$ SIDIS at each x, *i.e.* 3 pieces of information on the LHS of (10). On the RHS there are 6 unknown $\Delta q/q$. Again, following HERMES, we make the somewhat bizarre assumption

$$\Delta\bar{u}/\bar{u} = \frac{\Delta\bar{d}}{\bar{d}} = \frac{\Delta s}{s} = \frac{\Delta\bar{s}}{\bar{s}} \equiv \frac{\Delta q_s}{q_s} \qquad (11)$$

and solve for $\Delta u/u$, $\Delta d/d$ and $\Delta q_s/q_s$. The results are shown in Fig. 2. We see that even with our fake, error-free DIS and $\pi^\pm$ SIDIS data, the uncertainties in the FFs induce significant uncertainties in both $\Delta d/d$ and $\Delta q_s/q_s$.

5. Conclusions

We have explained why the large uncertainty in our knowledge of $(\Delta s(x) + \Delta\bar{s}(x))$ and $\Delta G(x)$, and the total lack of knowledge of the separate valence and sea polarized densities, cannot be remedied by purely DIS experiments, at least not until far in the future when a neutrino factory and TESLA-N will be built.

All progress on the polarized PDs thus rests upon the HERMES and COMPASS polarized SIDIS measurements, and the utility of these, in turn rests upon an accurate knowledge of the FFs. We have used a judicious combination of HERMES $\pi^{\pm}$ multiplicity and $e^{+}e^{-}$ data to evaluate the 3 independent pion FFs $D_u^{\pi^+}$, $D_d^{\pi^+}$, $D_s^{\pi^+}$ and their uncertainties. Using these we have generated fake error-free DIS and SIDIS asymmetry data and extracted the polarized PDs and their uncertainties from these data, following the methods advocated by HERMES.

E.L. is grateful to the Leverhulme Trust for an Emeritus Fellowship, the Foundation for Fundamental Research on Matter (FOM) and the Dutch Organization for Scientific research (NWO), and to P. Mulders for hospitality at the Vrije University, Amsterdam. E.C.'s work was supported by the Bulgarian National Science Foundation, Grant Ph-1010. S.K.'s work supported by the National Science Foundation under Grant PHY-0070443.

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