ESTIMATE OF THE COLLINS FUNCTION IN A CHIRAL INVARIANT APPROACH*

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We estimate the Collins function at a low energy scale by calculating the fragmentation of a quark into a pion at the one-loop level in the chiral invariant model of Manohar and Georgi. We give a useful parametrization of our results and we briefly discuss different spin and/or azimuthal asymmetries containing the Collins function and measurable in semi-inclusive DIS and e^+e^- annihilation.

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The Collins function $H_1^\perp$, describing the fragmentation of a transversely polarized quark into an unpolarized hadron [1], plays an important role in studies of the nucleon spin structure. Although $H_1^\perp$ is T-odd, it can be nonzero due to final state interactions. In fact, we were able to show for the first time that a nonvanishing Collins function can be obtained in a field theoretical approach through a consistent one-loop calculation of the fragmentation process [2].

To obtain a reasonable estimate of the Collins function, we calculate it in a chiral invariant approach at a low energy scale [3]. We use the model of Manohar and Georgi [4], which incorporates chiral symmetry and its spontaneous breaking, two important aspects of QCD at low energies.

The Collins function enters several spin and azimuthal asymmetries in one-particle inclusive DIS. Of particular interest is the transverse single spin asymmetry, where $H_1^\perp$ appears in connection with the transversity distribution, for which we predict effects of the order of 10%. In principle in semi-

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inclusive DIS only the shape of the Collins function can be studied, while a purely azimuthal asymmetry in e^+e^- annihilation allows one to measure also its magnitude. For this asymmetry we obtain effects of the order of 5%.

1. Model calculation of the Collins function

Considering the fragmentation process $q^*(k) \to \pi(p)X$ we define the Collins function, which depends on the longitudinal momentum fraction z of the pion and the transverse momentum $k_{\rm T}$ of the quark, as [5]

$$\frac{\varepsilon_{\rm T}^{ij} k_{{\rm T}\,j}}{m_\pi} H_1^\perp(z, z^2 k_{\rm T}^2) = \frac{1}{4z} \int dk^+ \, {\rm Tr}\left[\Delta(k,p) i\sigma^{i-}\gamma_5\right]\Big|_{k^-=p^-/z}\,, \qquad (1)$$

with the correlation function

$$\Delta(k,p) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} {\rm e}^{+ik\cdot\xi} \langle 0|\,\psi(\xi)|\pi, X\rangle\langle\pi, X|\bar\psi(0)|0\rangle\,. \qquad (2)$$

To get an estimate of the Collins function at a low energy scale we make use of the chiral invariant model of Manohar and Georgi [4]. The model contains massive constituent quarks and Goldstone bosons as effective degrees of freedom. The Lagrangian of the model reads

$$\mathcal{L} = \bar\psi\,(i\partial\!\!\!/ + V\!\!\!/ - m + g_A A\!\!\!/\gamma_5)\,\psi\,. \qquad (3)$$

The Manohar–Georgi model describes the valence part of the normal unpolarized fragmentation function D_1 fairly well [3].

While at tree level the Collins function is zero, the situation changes if rescattering corrections are included in $\Delta(k,p)$. In a consistent one-loop calculation the diagrams in Fig. 1 contribute to $H_1^\perp$. Because of the presence of nonvanishing imaginary parts in these diagrams, we obtain a nonvanishing Collins function.

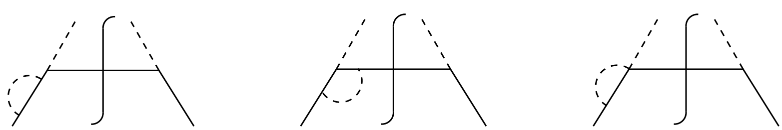

Fig. 1. One-loop corrections to the fragmentation of a quark into a pion contributing to the Collins function. The Hermitian conjugate diagrams are not shown.

The z behaviour of asymmetries containing $H_1^\perp$ is typically governed by the ratio

$$\frac{H_1^{\perp(1/2)}(z)}{D_1(z)} \equiv \frac{\pi}{D_1(z)}\, z^2 \int dk_{\rm T}^2\, \frac{|\vec{k}_{\rm T}|}{2\,m_\pi}\, H_1^\perp(z, z^2 k_{\rm T}^2)\,. \qquad (4)$$

In our model, it turns out that this ratio is approximately equal to

$$\frac{H_1^{\perp(1/2)}(z)}{D_1(z)} \approx \frac{\langle|\vec{K}_\mathrm{T}|\rangle(z)}{2zm_\pi}\frac{H_1^{\perp}(z)}{D_1(z)} \approx \frac{\sqrt{0.9\langle K_\mathrm{T}^2\rangle(z)}}{2zm_\pi}\frac{H_1^{\perp}(z)}{D_1(z)}\,. \tag{5}$$

For convenience of use, the result of our model can be roughly parametrized by means of a simple analytic form, *i.e.*

$$\frac{H_1^{\perp(1/2)}(z)}{D_1(z)} \approx 0.316\,z + 0.0345\,\frac{1}{1-z} - 0.00359\,\frac{1}{(1-z)^2}. \tag{6}$$

Fig. 2 shows the result of our model together with the parametrization we suggested. The ratio $H_1^{\perp(1/2)}/D_1$ is clearly increasing with increasing z; this feature is largely independent of the parameter choice in our approach [3].

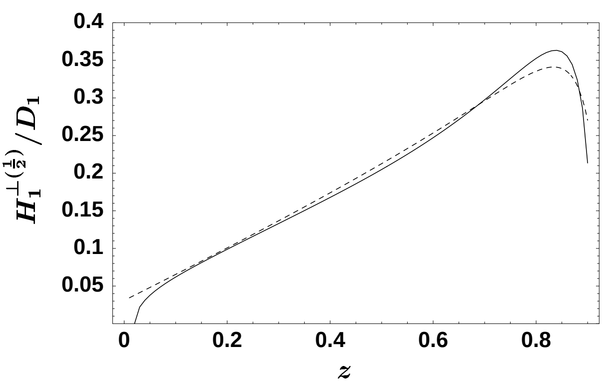

Fig. 2. Model result for the ratio $H_1^{\perp(1/2)}/D_1$ as a function of z and the simple analytic parametrization of Eq. (6).

2. Observables

In semi-inclusive DIS, three important single spin and azimuthal asymmetries containing the Collins function exist. Neglecting quark mass terms one has [5]

$$\langle\sin\phi_h\rangle_\mathrm{UL} \propto \frac{1}{Q}\Big[\Big(c_1\,h_L(x) + c_2\,h_1(x)\Big)H_1^{\perp(1/2)}(z)\; + \text{other terms}\Big], \tag{7}$$

$$\langle\sin\phi_h\rangle_\mathrm{LU} \propto \frac{1}{Q}\,e(x)\,H_1^{\perp(1/2)}(z)\,, \tag{8}$$

$$\langle\sin(\phi_h+\phi_S)\rangle_\mathrm{UT} \propto h_1(x)\,H_1^{\perp(1/2)}(z)\,. \tag{9}$$

where only the asymmetry numerators are written down. ϕ_h (ϕ_S) is the azimuthal angle of the produced hadron (target spin); the subscript UL, *e.g.*, indicates an unpolarized beam and a longitudinally polarized target; h_1 denotes the transversity distribution and h_L,e denote two twist-3 distributions; c_1 and c_2 are kinematical factors.

From the asymmetry (7) it is difficult to draw any conclusion on the Collins function. However, if we assume that the *other terms* in (7) are small, then the z dependence of the asymmetry should be almost entirely due to the Collins function. In Fig. 3 we compare the parametrization (6) of our results with HERMES data on $\langle \sin\phi_h \rangle_{\rm UL}$ [6] and preliminary data on the same asymmetry from CLAS [7]. The agreement of the z shape is remarkable. Note that we arbitrarily normalized our curve to take into account the unknown distribution functions and prefactors.

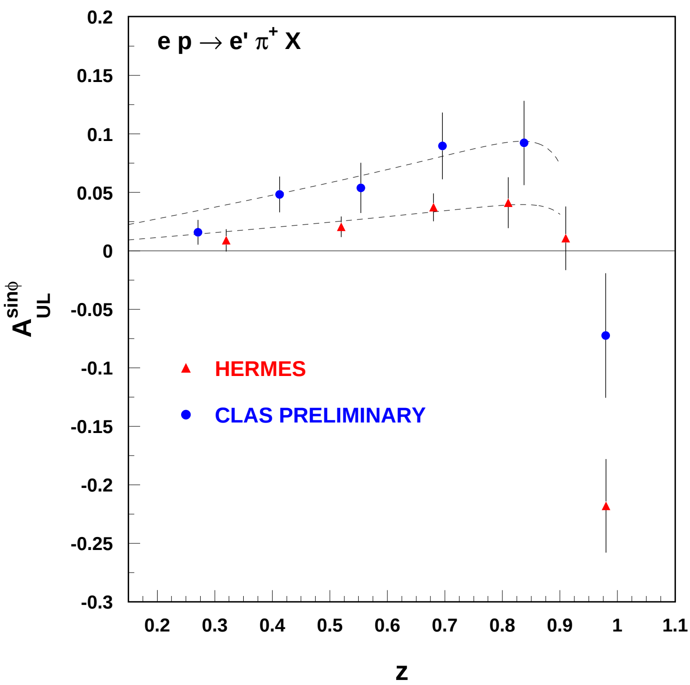

Fig. 3. Comparison between the results of our model and data from the HERMES and CLAS experiments.

The transverse spin asymmetry in Eq. (9) is one of the most promising observables to measure h_1. In fact, it is one of the ambitions of the HERMES Collaboration to extract the transversity distribution from this observable. However, without any knowledge on the Collins function it is impossible to evaluate the magnitude of the asymmetry. Using our model and two extreme assumptions on the transversity distribution, we obtain values up to about 10% for the asymmetry (left panel of Fig. 4), which therefore should be observable. Our results support the intention of extracting the transversity in this way.

At present, data on semi-inclusive DIS allow us to extract in an assumption-free way only the shape of $H_1^\perp$. The azimuthal asymmetry [3, 8]

$$\langle P_{h\perp}^2 \cos 2\phi \rangle_{e^+e^-} = \frac{2\,\sin^2\theta}{1+\cos^2\theta}\,\frac{H_1^{\perp(1)}(z_1)\,\bar{H}_1^{\perp(1)}(z_2)}{\Big(D_1(z_1)\,\bar{D}_1^{(1)}(z_2) + D_1^{(1)}(z_1)\,\bar{D}_1(z_2)\Big)}\,, \quad (10)$$

accessible in e^+e^- annihilation into two hadrons, can provide information on the magnitude of $H_1^\perp$ as well. As shown in the right panel of Fig. 4, our result for this asymmetry is of the order of 5%. Possible accurate measurements of this observable at BABAR or BELLE would be very useful to better pin down the Collins function from the experimental side.

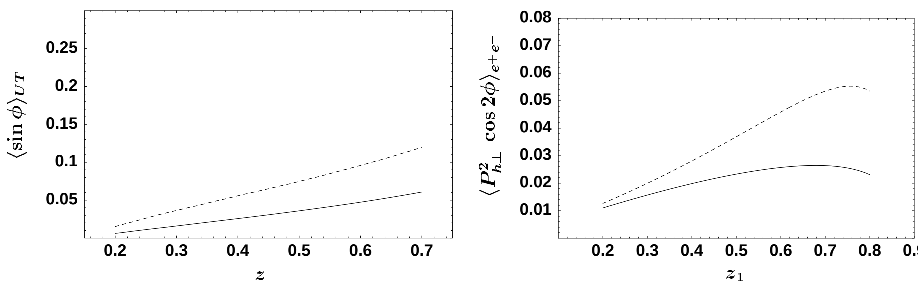

Fig. 4. Left panel: spin asymmetry $\langle \sin\phi \rangle_{UT}$ of Eq. (9) as a function of z, assuming $h_1 = g_1$ (solid line), and assuming $h_1 = (f_1 + g_1)/2$ (dashed line). Right panel: azimuthal asymmetry $\langle P_{h\perp}^2 \cos 2\phi \rangle_{e^+e^-}$ of Eq. (10), integrated over the angular range $0 \le \theta \le \pi$, and over the z_2 ranges $0.2 \le z_2 \le 0.8$ (solid line) and $0.5 \le z_2 \le 0.8$ (dashed line).

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