Vol. 33 (2002) *ACTA PHYSICA POLONICA B* No 12

CP AND CPT VIOLATION IN K^0 DECAYS* **

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(Received September 18, 2002)

The job of the experimentalist is to give numbers, with as many digits behind the decimal point as possible.

R. Feynman

Physics is an experimental science!

S. Glashow

PACS numbers: 13.25.Es

1. Introduction

Almost 40 years after the discovery of CP violation, in 1964, by Christenson, Cronin, Fitch and Turlay [1], experiments are now in a position to rule out the first, *ad hoc*, model of Wolfenstein [2], in which a new, super-weak interaction with a change of two units of strangeness ($\Delta S=2$) is responsible for CP violation, and to confirm — within experimental and theoretical errors — the idea, formulated in 1973 by Kobayashi and Maskawa [3], that CP violation is due to an imaginary coupling constant in the quark mixing matrix.

Why did it take so long? To a large extent the reason is that the effects are extremely small, limited to the K^0–$\bar{K}^0$ system until the construction of asymmetric B factories, and very hard to measure. In these lectures I would like not only to present the latest results for the kaon system but also to recall a part of the history. The development of an adequate instrumentation took not only years, but decades. I will not discuss CP violation in the B system. Instead I will spend some time on CPT violation, which comes as a byproduct of the experiments on the K^0 system. In discussing theoretical

* Presented at the XLII Cracow School of Theoretical Physics, Zakopane, Poland, May 31–June 9, 2002.

** Work supported by the German Ministry of Science and Education (BMBF) under contract 056Si74.

issues I will mostly rely on the article by Buras and Fleischer [4] and on the book by Bigi and Sanda [5]. A further reference for the reader is the volume "*CP violation*" edited by Jarlskog [6].

2. CP violation in K^0 decays

2.1. CP violation in state mixing

K^0 and $\bar{K}^0$ are quark–antiquark bound states of s and d quarks $K^0 = (\bar{s}d)$, with strangeness 1 and $\bar{K}^0 = (s\bar{d})$, with strangeness -1. The W which mediates the decay of the s quark can couple to a lepton pair or to a quark pair, leading to semileptonic or to hadronic final states (see Fig. 1).

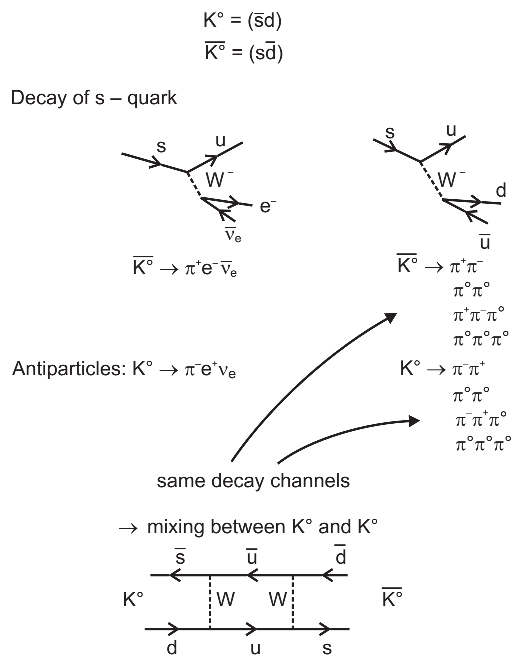

Fig. 1. Decays and mixing of K^0 and $\bar{K}^0$.

The sign of the lepton in semileptonic decays tells the strangeness of the decaying kaon. The hadronic decays are identical for K^0 and $\bar{K}^0$. Therefore, there is a coupling mechanism for K^0 to $\bar{K}^0$ via intermediate hadronic states. This coupling can be described, as in the classical case of two coupled pendula, by coupled differential equations. It is convenient to write them in matrix form, with a two-component state vector

$$\Psi = \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}.$$

At times sufficiently long compared to $1/m_K$ the most general form of a linear equation in Ψ and, therefore, compatible with the superposition principle, is

$$i\frac{d\Psi}{dt} = H\Psi = (M - i\Gamma)\Psi\,, \tag{1}$$

where $M = M^*$ and $\Gamma = \Gamma^*$ are two Hermitian 2×2 matrices, into which any 2×2 matrix H can be decomposed. The eigenstates of this equation are the short-lived meson K_S and the long-lived meson K_L

$$K_\mathrm{S} = \frac{1}{\sqrt{2(1+|\varepsilon_\mathrm{S}|^2)}}\,(K_1 + \varepsilon_\mathrm{S} K_2)\ ,$$

$$K_\mathrm{L} = \frac{1}{\sqrt{2(1+|\varepsilon_\mathrm{L}|^2)}}\,(K_2 + \varepsilon_\mathrm{L} K_1)\ ,$$

where K_1 and K_2 are the CP-even and CP-odd eigenstates

$$K_1 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K^0}\right)\ ,$$

$$K_2 = \frac{1}{\sqrt{2}}\left(K^0 - \bar{K^0}\right)\ .$$

In the last equations it is assumed that the choice CP $|K^0\rangle = |\bar{K^0}\rangle$ was made. If CP were conserved in the decay, K_S and K_L would be eigenstates of CP, implying

$$\varepsilon_\mathrm{S} = \varepsilon_\mathrm{L} = 0\,.$$

Conversely, if ε is not zero, then there is CP violation. This CP violation by state mixing is called **indirect CP violation**.

In general, the solution of the eigenvalue problem associated with Eq. (1) relates ε_S and ε_L to the elements of the matrices M and Γ

$$\varepsilon = \frac{\varepsilon_\mathrm{S} + \varepsilon_\mathrm{L}}{2} = \frac{\Gamma_{12} - \Gamma_{12}^* + i(M_{12} - M_{12}^*)}{\gamma_\mathrm{S} - \gamma_\mathrm{L} - 2i(m_\mathrm{L} - m_\mathrm{S})}\,, \tag{2}$$

$$\delta = \frac{\varepsilon_\mathrm{S} - \varepsilon_\mathrm{L}}{2} = \frac{\Gamma_{11} - \Gamma_{22} + i(M_{11} - M_{22})}{\gamma_\mathrm{S} - \gamma_\mathrm{L} - 2i(m_\mathrm{L} - m_\mathrm{S})}\,. \tag{3}$$

If CPT is conserved in the decay, masses and lifetimes of particles and antiparticles are the same, *i.e.* $\delta = 0$. We will for the moment assume that this is the case; experimental evidence will be discussed below. Since the relative phases of K^0 and $\bar{K^0}$ can be arbitrarily chosen, it is convenient to make a choice. In the Wu–Yang phase convention [7] the decay amplitude A_0 is defined to be real and positive.

$$\langle 2\pi|_0|H|K^0\rangle = A_0 \exp(i\delta_0)\,,$$

where δ_0 is the $\pi\pi$ scattering phase shift in the $I = 0$ state at the mass of the kaon. Since A_0 is by far the dominant decay amplitude, this convention has the consequence that ε is small; it also implies that Γ_{12} is practically real

$$\Gamma_{12} = \Sigma_f \langle K^0|H|f\rangle\ \langle f|H|\bar{K}^0\rangle$$

and the phase of ε is, according to (2), equal to

$$\phi_\varepsilon = \tan^{-1}\left(\frac{2(m_{\rm L} - m_{\rm S})}{\gamma_{\rm S}}\right) = (43.5 \pm 0.1)^\circ\,.$$

This is also called the "superweak" phase.

The time evolution of an initial K^0 state can now easily be obtained. The $K_{\rm S}$ and $K_{\rm L}$ states change with time as

$$K_{\rm S}(t) = K_{\rm S}(0)\exp\left(-i\,m_{\rm S}t - \frac{\gamma_{\rm S}}{2}\,t\right)\,,$$
$$K_{\rm L}(t) = K_{\rm L}(0)\exp\left(-i\,m_{\rm L}t - \frac{\gamma_{\rm L}}{2}\,t\right)\,.$$

Neglecting for simplicity the CP violating terms proportional to ε we have

$$K^0(0) \propto K_{\rm S}(0) + K_{\rm L}(0)$$

and

$$K^0(t) \ \propto\ \left(K^0 + \bar{K}^0\right)\exp\left(-i\,m_{\rm S}\,t - \frac{\gamma_{\rm S}}{2}\,t\right) + \left(K^0 - \bar{K}^0\right)\exp\left(-i\,m_{\rm L}t - \frac{\gamma_{\rm L}}{2}\,t\right)\,.$$

The observation of

$$K^0 \to \pi^- e^+ \nu_e$$

filters out the K^0 component

$$N^+(t) = \left|\langle \pi^- e^+ \nu_e | K^0(t)\rangle\right|^2\,,$$

$$\propto \exp(-\gamma_{\rm S}t) + 2\exp(-\bar{\gamma}\,t)\cos(m_{\rm L} - m_{\rm S})t + \exp(-\gamma_{\rm L}t)\,.$$

The second term (the interference term) changes sign, if the $\bar{K}^0$ component is filtered out, or if the initial state is $\bar{K}^0$.

Inclusion of the CP violating terms in the $K_{\rm S}$ and $K_{\rm L}$ states changes little in this interference pattern, with an important exception; since

$$K_{\rm L}^0 \propto K_2 + \varepsilon K_1 \propto K^0(1+\varepsilon) - \bar{K}^0(1-\varepsilon)$$

the K^0 and $\bar{K}^0$ components in the long-lived kaon are no longer equal and, therefore, an asymmetry in the semileptonic decay rates persists. For $t \to \infty$ we have

$$\frac{N^+(t) - N^-(t)}{N^+(t) + N^-(t)} = 2{\rm Re}\,\varepsilon\,,$$

irrespective of whether the initial state was K^0 or $\bar{K}^0$.

The first precision measurement of ${\rm Re}\,\varepsilon$ with a magnetic spectrometer, pioneering the use of multiwire proportional chambers, which can run at much higher rate than spark chambers, was made by a CERN–Heidelberg group [8], led by Steinberger. The setup of this experiment is shown in Fig. 2. Gas Cerenkov counters were used for the identification of electrons. The charge asymmetry as a function of proper time is also shown in Fig. 2. After the interference has damped out, an asymmetry, *i.e.* a predominance of positive over negative electrons, persists. This observation demonstrates in a striking way the effect of CP violation: it shows up as an asymmetry

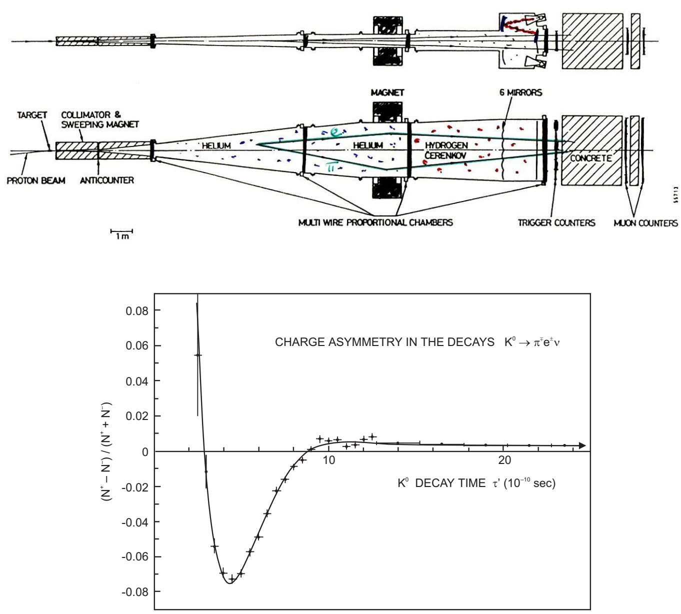

Fig. 2. Setup and leptonic asymmetry in the CERN–Heidelberg experiment.

between particles and antiparticles (the symmetry required by CPT invariance remains valid; to demonstrate it, a comparison of $K^0(t)$ and $\bar{K}^0(-t)$ would be necessary).

2.2. CP violation in $K^0 \to 2\pi$ decays

The state of two pions ($\pi^+\pi^-$ or $\pi^0\pi^0$) is an eigenstate of the CP operator with CP $= +1$. In the absence of CP violation the K_{L}^0 has CP $= -1$. It can, therefore, not decay into two pions. The discovery of CP violation [1] consisted in the demonstration that K_{L} does decay into two pions. The setup of this famous experiment is shown in Fig. 3. It consists of a double arm

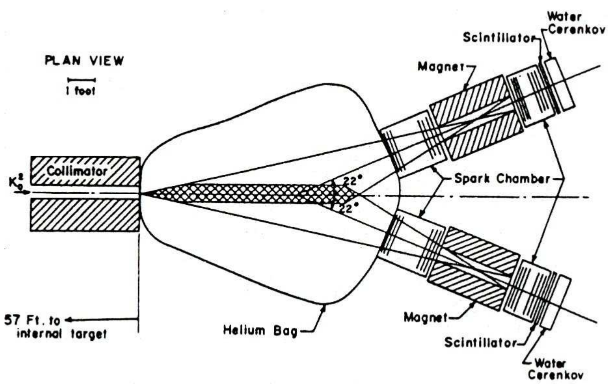

Fig. 3. Setup of the experiment in which CP violation was discovered.

magnetic spectrometer with optical spark chambers. The $K_{\mathrm{L}}^0 \to \pi^+\pi^-$ decay is demonstrated by a signal of about 50 events with the kaon invariant mass and no momentum transverse to the beam (Fig. 4). In general, the decay of the K^0 to two pions is described by two weak amplitudes, A_0 and A_2. Since there are strong final state interactions, it is necessary to separate the weak decay amplitudes from the subsequent strong interaction. The Watson theorem states that this can be done, using the S matrix for the strong scattering of two pions in an s state, which is $e^{i\delta}$, where δ is the phase shift. Since the strong interaction conserves I spin, the weak decay amplitudes A_I are given as transitions to two pions in a definite I spin state.

$$\begin{aligned} \langle 2\pi|_I|H|K^0\rangle &= A_I \exp(i\delta_I)\,, \\ \langle 2\pi|_I|H|\bar{K}^0\rangle &= \bar{A}_I \exp(i\delta_I)\,, \qquad\qquad I = 0,\, 2\,. \end{aligned}$$

With CPT symmetry the amplitudes $\bar{A}_I$ for $\bar{K}^0$ decays are related to those of K^0 decays by

$$\bar{A}_I = A_I^*\,.$$

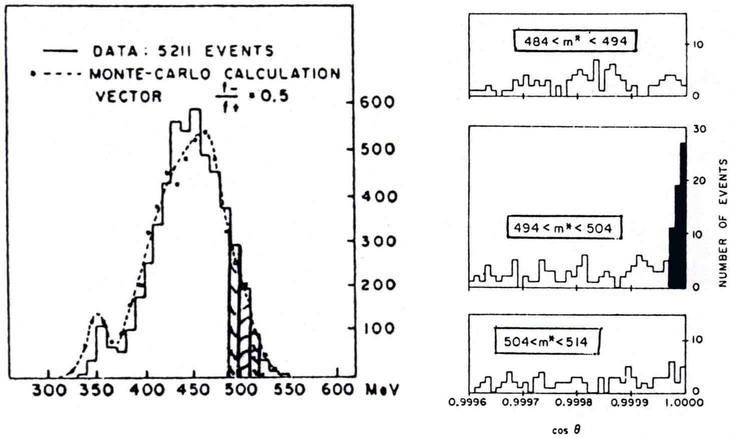

Fig. 4. Invariant $\pi\pi$ mass and $\cos\theta$ for events around the K-mass in the experiment of Christenson *et al.*, [1].

Assuming no CPT violation, there are then two complex amplitudes to be measured, A_0 and A_2. One phase is arbitrary; in the Wu–Yang convention, A_0 is defined as real and positive. If there is a phase difference between A_0 and A_2, then there is T violation in the transition (T conservation, *i.e.* invariance under $t \to -t$ requires all coupling constants to be relatively real). Assuming CPT symmetry, T violation implies CP violation. This CP violation in the transition is also called **direct CP violation**. It is linked to a non-zero value of ε' with

$$\begin{aligned}\varepsilon' &= \frac{i}{\sqrt{2}}\,\mathrm{Im}\left(\frac{A_2}{A_0}\right)\exp i(\delta_2-\delta_0)\,,\\ \Phi_{\varepsilon'} &= \frac{\pi}{2}+\delta_2-\delta_0 \approx (45\pm 15)^\circ\,. \end{aligned}\tag{4}$$

Due to the interference between $K_{\mathrm{S}} \to 2\pi$ and $K_{\mathrm{L}} \to 2\pi$ decays one can measure two complex amplitudes

$$\begin{aligned}\eta^{+-} &= \frac{\langle\pi^+\pi^-|H|K_{\mathrm{L}}\rangle}{\langle\pi^+\pi^-|H|K_{\mathrm{S}}\rangle}\,,\\ \eta^{00} &= \frac{\langle\pi^0\pi^0|H|K_{\mathrm{L}}\rangle}{\langle\pi^0\pi^0|H|K_{\mathrm{S}}\rangle}\,,\end{aligned}$$

In the Wu–Yang phase convention they are related to ε (Eq. (2)) and ε' (Eq. (4)) in good approximation by

$$\begin{aligned}\eta_{+-} &= \varepsilon+\varepsilon'\,,\\ \eta_{00} &= \varepsilon-2\varepsilon'\,.\end{aligned}$$

One can also take the attitude to define ε by these equations. It is then the amplitude ratio of transitions to the $I = 0$ state of two pions. Obviously its phase is then not a matter of choice. As we will see in Sec. 5.1, the phase of this amplitude ratio is defined by unitarity. In the absence of CPT violations it is equal to the "superweak" phase. Without ε', for example in the superweak model [2], the relative rates of $\pi^+\pi^-$ and $\pi^0\pi^0$ decays are the same for K_L and K_S; the K_L decays to two pions only because of the presence of the K_1 component (essentially K_S) in its wave function. The best way to measure ε' is, therefore, to measure the double ratio

$$R = \frac{\dfrac{\Gamma(K_\mathrm{L} \to \pi^0\pi^0)}{\Gamma(K_\mathrm{S} \to \pi^0\pi^0)}}{\dfrac{\Gamma(K_\mathrm{L} \to \pi^+\pi^-)}{\Gamma(K_\mathrm{S} \to \pi^+\pi^-)}} = 1 - 6\,\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) .$$

This is in fact what experiments have been doing. The main difficulty is the measurement of $K_\mathrm{L} \to 2\pi^0$ because photons are not easy to measure, and the background from the 200 times more abundant $K_\mathrm{L} \to 3\pi^0$ decay is overwhelming. Unfortunately also, the effect is a small difference between large numbers. For a precision of 10^{-3} in R one needs millions of events in all four decay channels. Nobody has come up with an equivalent of a Wheatstone bridge where the instrument shows zero if there is exact balance.

3. Theoretical expectations

The basic assumption [3], to be tested experimentally, is, that CP violation is due to a complex coupling constant in the quark mixing matrix V (also called the CKM matrix). The idea is, in a few words, that the W coupling is not diagonal in the 3×3 dimensional space of $+2/3$ charged quarks and $-1/3$ charged quarks, but contains off-diagonal elements. The basic observation is, that with three quark flavours the (supposedly) unitary mixing matrix contains 4 parameters — the relative quark phases are unmeasurable — one of which is a phase. The unitarity condition between the first and the third column of the mixing matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\,,$$

can be viewed as a triangle in the complex plane. In the Wolfenstein parametrisation of the mixing matrix

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\rho e^{-i\phi} \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\phi}) & -A\lambda^2 & 1 \end{pmatrix},$$

with

$$\lambda \equiv \sin\theta_c = 0.220 \pm 0.002\,,$$

the unitary condition reads as

$$V_{ub}^* - \lambda\, V_{cb} + V_{td} = 0\,.$$

With $\lambda\, V_{cb}$ as base, the tip of the triangle has the coordinates $A\lambda^3\rho e^{i\phi}$. CP violating amplitudes are proportional to the area of the triangle (it can be shown that all six possible unitary triangles have the same area).

It is straightforward to relate $K^0 - \bar{K}^0$ mixing to the elements of the CKM matrix by calculating the "box" diagram of Fig. 1. Denoting

$$\begin{aligned} \lambda_i &= V_{is}V_{id}^*\,, \qquad\qquad i = c, t \\ x_i &= \frac{m_i^2}{m_W^2}\,, \end{aligned}$$

the calculation with free quarks of masses m_q gives

$$\begin{aligned} \mathrm{Im}\, M_{12} &= \frac{G_F^2}{12\pi^2} F_K^2 B_K m_K M_W^2 \\ &\times \left[\mathrm{Im}\left(\lambda_c^2\right)\eta_1 S_0(x_c) + \mathrm{Im}\left(\lambda_{\mathrm{t}}^2\right)\eta_2 S_0(x_{\mathrm{t}}) + 2\mathrm{Im}\left(\lambda_c\lambda_{\mathrm{t}}\right)\eta_3 S_0(x_{\mathrm{t}}, x_c)\right], \end{aligned}$$

where the three terms correspond to intermediate states with cc, tt and ct (the couplings V_{ud} and V_{us} are real in the standard parametrisation). The "bag" constant B_K is defined by

$$\left\langle K^0 \middle| (sd)_{V-A}(sd)_{V-A} \middle| K^0 \right\rangle = \frac{4}{3} B_K F_K^2 m_K\,.$$

In the "vacuum insertion approximation", where the matrix element on the left side is replaced by

$$\left\langle K^0 \middle| (sd)_{V-A} \middle| \right\rangle \left\langle \middle| (sd)_{V-A} \middle| K^0 \right\rangle$$

one has B_K=1. S_0 are elementary functions of quark masses with the values

$$\begin{aligned} S_0(x_c) &= 3\times 10^{-4}\,, \\ S_0(x_{\mathrm{t}}) &= 2.7\,, \\ S_0(x_{\mathrm{t}}, x_c) &= 2.7\times 10^{-3}\,. \end{aligned}$$

QCD corrections have been calculated to be [4]

$$\begin{aligned}\eta_1 &= 1.38 \pm 0.20\,,\\ \eta_2 &= 0.57 \pm 0.01\,,\\ \eta_3 &= 0.47 \pm 0.04\,.\end{aligned}$$

The most difficult part is the calculation of the constant B_K. The current model estimates of B_K are summarised in [4] as

$$B_K = 0.75 \pm 0.15\,.$$

The relation between ε, ρ and ϕ is then a hyperbola in the complex plane, shown in Fig. 5 together with the current best values of $|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$ [9] and $|V_{td}/V_{ts}| < 0.22$ [10] and a preliminary measurement of $\sin 2\beta = 0.75 \pm 0.10$ [11]. A discussion of these results is beyond the scope of this lecture. What is important to stress, however, is that the error bar on ε comes from theory, not from experiment.

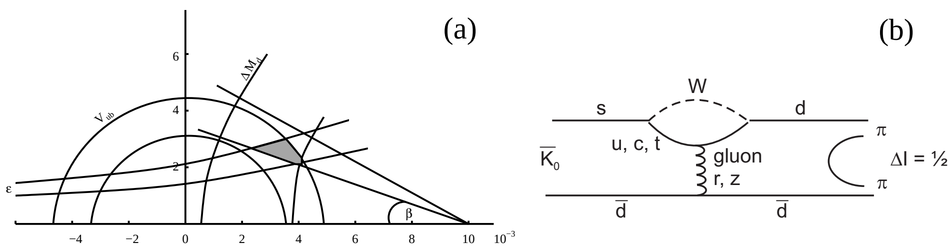

Fig. 5. (a) Unitarity triangle. (b) Penguin graph.

The calculation of ε' is more difficult. The "penguin" diagrams of Fig. 5 contribute an imaginary amplitude to the $\Delta I = 1/2$ transition of $K^0 \to \pi\pi$. The result for ε'/ε depends strongly on the mass of the top quark, which was not known around 1980, when a new series of experiments was planned. The predictions for ε'/ε from these times [12] went up to 1 %. As the lower limit of the top mass increased, it became clear that electroweak penguin diagrams, with exchange of γ and Z instead of gluon exchange, had to be considered. It is difficult, and certainly beyond the competence of the author, to summarise the various theoretical approaches. It is perhaps best to show the predictions of one group (Fig. 6) which were made before the recent experimental results became available, and to indicate in an approximate formule the dependence of the result on various parameters of the theory. According to the Munich group one has

$$\frac{\varepsilon'}{\varepsilon} = 13 \times 10^{-4} \left(\frac{\mathrm{Im}\lambda_{\mathrm{t}}}{1.34\times10^{-4}}\right) \left(\frac{110\mathrm{MeV}}{m_{\mathrm{s}}}\right)^2 \left[B_6 - 0.4 B_8 \left(\frac{m_{\mathrm{t}}}{165\mathrm{GeV}}\right)^{2.5}\right] \frac{\Lambda_{\mathrm{QCD}}}{340\mathrm{MeV}}\,.$$

The parameters and their uncertainties are approximately:

$$\begin{aligned}
m_{\mathrm{s}} &= (110 \pm 20)\,\mathrm{MeV}\,, \\
B_6 &= 1.0 \pm 0.3\,, \\
B_8 &= 0.8 \pm 0.2\,, \\
\Lambda_{\bar{M}S} &= (340 \pm 50)\,\mathrm{MeV}\,, \\
\mathrm{Im}\,\lambda_{\mathrm{t}} &= (1.34 \pm 0.22) \times 10^{-4}\,.
\end{aligned}$$

The errors are large and not symmetric around the central value. Therefore, the Munich Group has decided to give something like a probability distribution for ε'/ε, shown in Fig. 6, with an average of

$$\frac{\varepsilon'}{\varepsilon} \approx \left(7.7^{+6.0}_{-3.5}\right) \times 10^{-4}\,.$$

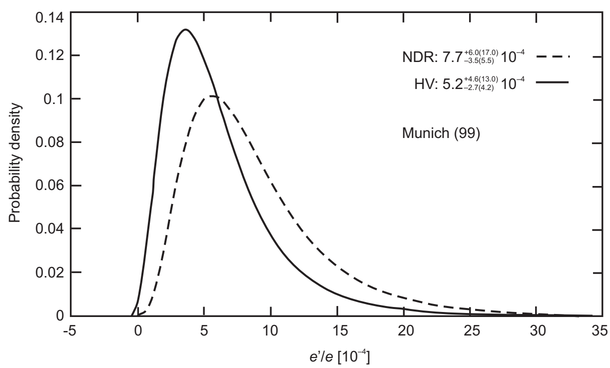

Fig. 6. Predictions for ε'/ε from the Munich group [13] for two regularisation schemes.

4. Measurements of ε'/ε

Right after the discovery of CP violation in the $K_{\mathrm{L}} \to \pi^+\pi^-$ decay, experiments were started to look for the $K_{\mathrm{L}} \to \pi^0\pi^0$ decay. The initial detectors for γ rays were spark chambers with enough material to convert photons but thin enough to provide information on the photon direction and energy. The energy resolution from spark counting is, however, very limited and turned out to be not adequate for a clean discrimination between $K^0 \to 2\pi^0$ and $K^0 \to 3\pi^0$ decays.

Another instrumentation was clearly required. In Fig. 7 the techniques for γ energy measurements as of 1965 are compared [14]. It is apparent that, apart from pair spectrometers, total absorption lead glass counters promised to be powerful instruments. The two most precise experiments on the ratio $\Gamma(K_{\rm L} \to \pi^0\pi^0)/\Gamma(K_{\rm L} \to \pi^+\pi^-)$ in the early 70's make use of these techniques. Both of them sacrifice detection efficiency in favour of a clean signal.

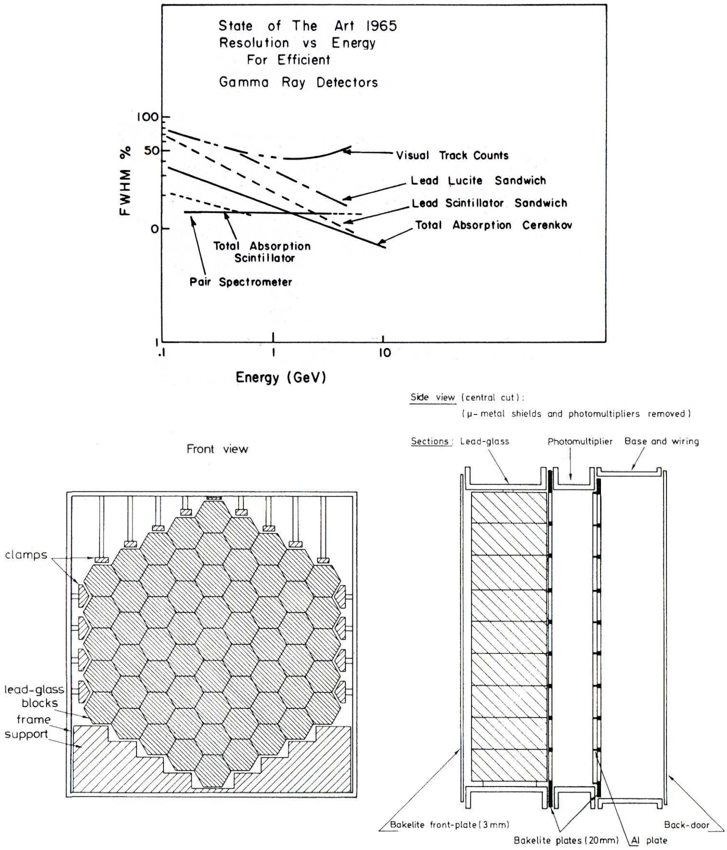

Fig. 7. Top: State of the art of γ detection in 1965. Bottom: Lead glass counter of the Aachen–CERN–Torino Collaboration.

The experiment of a Princeton group led by Cronin [15] is based on the idea that a precise measurement of the energy and direction of a single γ ray and only a conversion point measurement of the other γ rays is sufficient to obtain a signal, because the transverse momentum of the well measured γ ray can be restricted to $p_{\rm T} > 170\,{\rm MeV}/c$, beyond the range of γ's from $K^0 \to 3\pi^0$ decays. The apparatus and the signal in the 4γ invariant mass are shown in Fig. 8.

In the experiment of the Aachen–CERN–Torino group, led by Rubbia [16], the photon energies are all measured in hexagonal lead glass counters with 16 cm inner diameter; the necessary spatial resolution is obtained

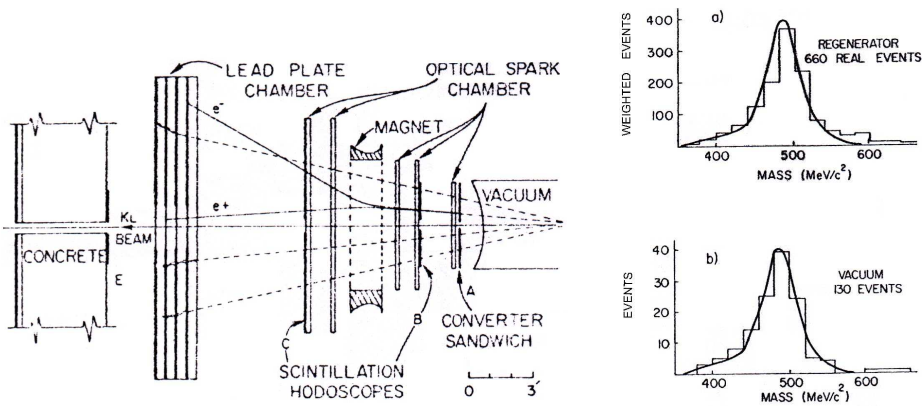

Fig. 8. Setup and $K_{\mathrm{L}} \to 2\pi^0$ signal in the Princeton experiment.

from earlier conversion of photons and from measurement of the direction of at least two of them in wire spark chambers. The setup and the resulting signal are shown in Fig. 9.

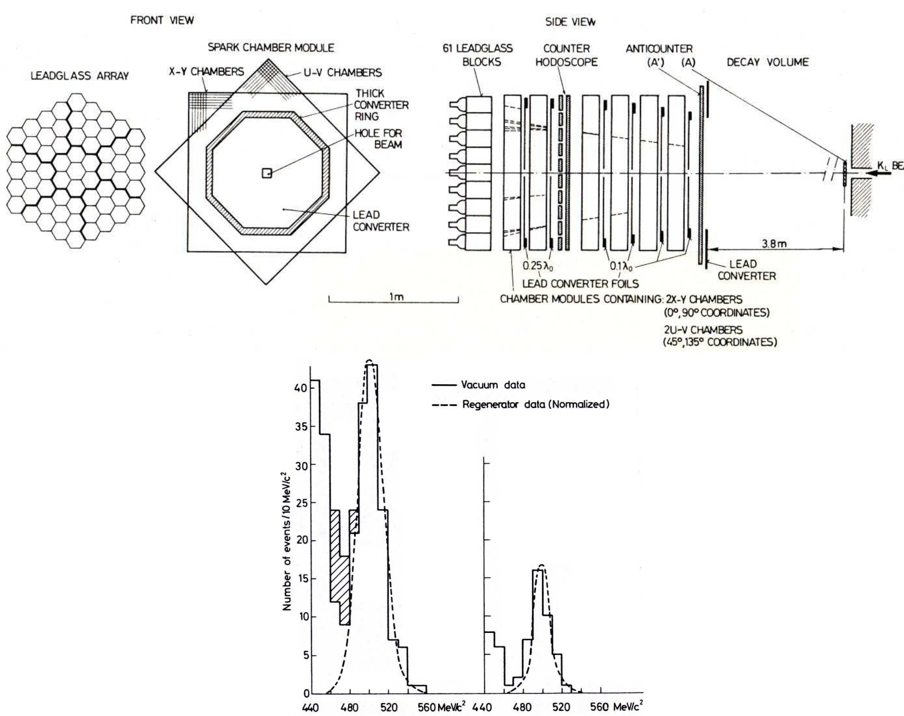

Fig. 9. Setup and $K_{\mathrm{L}} \to 2\pi^0$ signal in the Aachen–CERN–Torino experiment.

In both of these experiments the $K_{\rm L}$ decays are compared to $K_{\rm S}$ decays with $K_{\rm S}$ produced in a regenerator. The same regenerators are used in separate experiments on $K^0 \to \pi^+\pi^-$ decays, so that effectively $|\eta_{00}/\eta_{+-}|^2$ is measured. The results of these experiments

$$|\eta_{00}/\eta_{+-}| = 1.03 \pm 0.07 \qquad \text{Ref. [15]}$$
$$|\eta_{00}/\eta_{+-}| = 1.00 \pm 0.06 \qquad \text{Ref. [16]}$$

are compatible with only indirect CP violation.

When loop diagrams in $K \to 2\pi$ decays were studied in the late 70's in an attempt to understand why $\Delta I = 1/2$ transitions are favoured, it was also observed that loop diagrams lead to direct CP violations, large enough to be measurable eventually. These observations triggered a new round of experiments at the sites of the high energy proton accelerators, Fermilab and CERN. An advantage of high energies is that in total absorption calorimeters, used for γ detection, the resolution improves with energy. The experiments, E731 at Fermilab and NA31 at CERN, profit from this fact. They differ, however, in many respects.

The E731 Collaboration uses a double beam, with a regenerator in one of the beams move-able from one beam to the other burst by burst. A magnetic spectrometer is used for charged particle measurement and a lead glass counter array for photons, with holes for the two beams (Fig. 10). Initially one of the photons was required to convert in a thin (0.1 rad.l.) lead foil near the end of the decay region in order to allow a reconstruction of the decay point coordinates transverse to the beam. The longitudinal position of a $K \to 2\pi^0$ decay is not measured in these experiments. It is reconstructed from kinematics, using the mass constraint of the $K \to 4\gamma$ or $\pi^0 \to 2\gamma$ decay. The transverse position is useful in the Fermilab experiment because, together with the centre of gravity of the photons in the lead glass, it defines

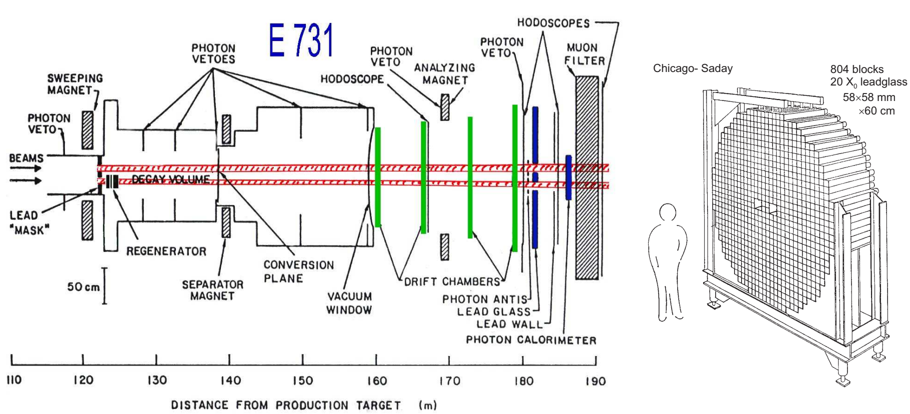

Fig. 10. Layout and lead-glass counter of the Fermilab E731 experiment.

the line of flight of the kaon. As illustrated in Fig. 11, in the regenerator beam there is always a certain amount of diffractively regenerated K_S which have to be subtracted from the forward regenerated kaons. Another method, used later, is to measure the amount of diffraction in charged ($K \to \pi^+\pi^-$) decays and, based on this, to calculate the diffractive background in the neutral decays, using the known resolution of the γ detector. The detection efficiency for neutral decays is then much higher. In the centre of gravity distribution (Fig. 11) there is a halo from this diffraction around the regenerator beam which extends into the vacuum beam. The amount of halo is well predicted by the Monte Carlo simulation based on diffraction in charged decays. The experiment was initially run separately for charged and neutral modes, but later also in a common mode, measuring all four decay modes, which enter in double ratio, at the same time.

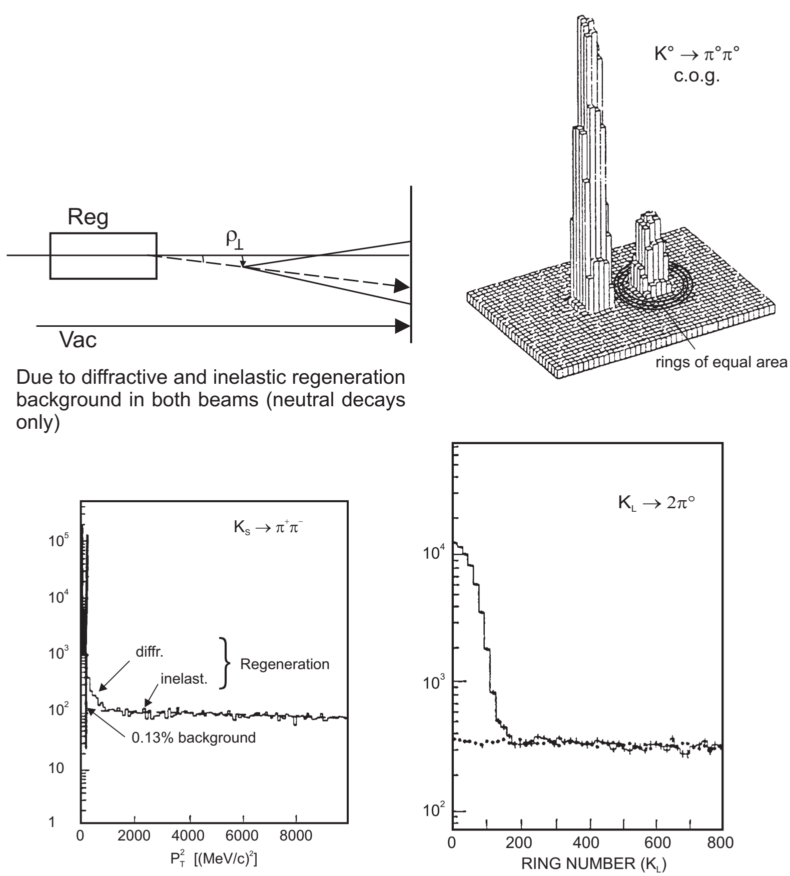

Fig. 11. Diffraction regeneration and its subtraction in charged and neutral decays in experiment E731.

The CERN experiment NA31 does not use a regenerator. Instead the proton beam is transported to a $K_{\rm S}$ target and collimator station next to the decay region (see Fig. 12). The experiment can run either with a $K_{\rm S}$ beam or with a $K_{\rm L}$ beam but not with both at the same time. The $K_{\rm S}$ target is movable on a train with 40 stations over a distance of 50 m. In this way a 50 m long decay volume can be used in $K_{\rm L}$ decays, where the detection efficiency is calibrated with $K_{\rm S}$ decays.The beam is transported in a vacuum beam-pipe throughout the detector to avoid neutron and photon interactions. There is no magnetic spectrometer for the charged decays;

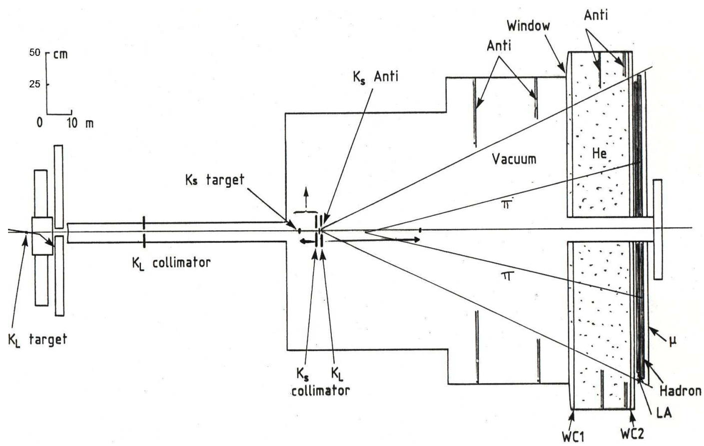

Fig. 12. Layout of the NA31 experiment.

without magnet the acceptance for the $K \to 2\pi^0$ decay can be very large. Two proportional wire chambers, with drift time measurements added, are used to measure the straight trajectories of charged pions. The pion energy is measured in a iron-scintillator sandwich-like hadron calorimeter. Only the energy ratio from this calorimeter is necessary; the kaon energy is obtained from the opening angle with the kaon mass as a constraint. Three-body decays are subtracted with the distance of the target from the decay plane as a measure of the transverse momentum imbalance. The electromagnetic calorimeter consists of a liquid-argon-lead-plate sandwich (Fig. 13). This technique was introduced in the '70 's by Willis in an ISR-experiment [17]. Several reasons are in favour of it. At the cost of some complication with cryogenics it offers the advantage that γ energy measurement is based on charge collection without any amplification process. A liquid-argon detector is easy to calibrate electronically, because the liquid is the same for all channels. There is no problem with radiation damage, in contrast for example to lead-glass. The argon suffers from oxygen contamination; therefore, a mild purification is necessary. Argon is not so expensive that an occasional refill is out of question.

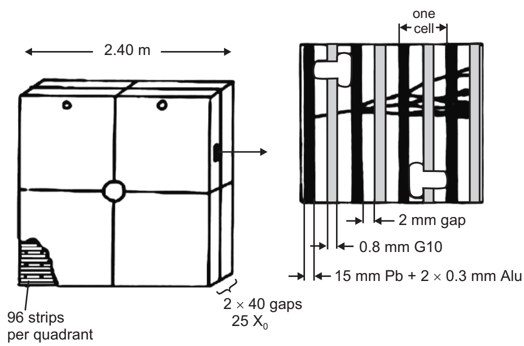

Fig. 13. Liquid Argon detector of the NA31 experiment.

In the meantime many liquid-argon detectors are in operation; there is sufficient experience with the technology.

The first result was given by the NA31 Collaboration [18], $\varepsilon'/\varepsilon = (33 \pm 11) \times 10^{-4}$, later improved by additional running to [19] $\varepsilon'/\varepsilon = (23 \pm 6.5) \times 10^{-4}$. The E731 experiment [20] found no significant deviation from zero: $\varepsilon'/\varepsilon = (7.4 \pm 5.9) \times 10^{-4}$. Preliminary versions of the final results were first presented at the 1991 LP-HEP conference in Geneva. A comparison (Fig. 14) suggested that the two experiments were not really incompatible, and that the superweak model was ruled out by about three standard deviations.

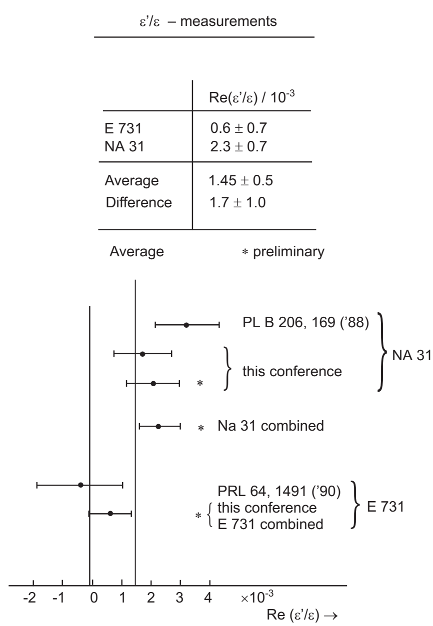

Fig. 14. Comparison of E731 and NA31 results at the LP-HEP conference 1991.

A three standard deviation effect in a key experiment with some skepticism of the opponents about the evaluation of systematic errors is, however, not satisfactory for the experimentalist if he can see a way to do better. It was clear that this would not be an easy task because both statistical and systematic errors had to be reduced. In both groups, however, a hard core of physicists decided to make a new attempt.

The NA48 Collaboration at CERN found a way to run K_{L} and K_{S} beams at the same time, reducing the difference in accidental background to a negligible level. The K_{S} target is located only 6.8 cm from the centre of the K_{L} beam, and the two beams converge to the centre of the calorimeter 100 m downstream of the K_{S} target (see Fig. 15). The illumination of the detector by the decay products is, therefore, almost the same for the two beams, with the exception of a noticeable difference at the first wire chamber, 20 m in front of the calorimeter. The chambers are placed midway between the beams to reduce this effect. Since in neutral decays the transverse position of the decay point cannot be measured, another method is necessary to distinguish K_{L} and K_{S} decays. It consists in a time coincidence of the event time and the time of a proton in the primary proton beam to the K_{S} target. The proton time is measured with scintillation counters. To reduce the intensity in each counter to an acceptable level, the beam area is shared among 12 horizontal and 12 vertical counters. They sit like blades in the beam, displaced longitudinally on a precision ladder. They are said to "tag" K_{S} events. Time is measured with 1 GHz flash ADCs, with a precision of 200 ps.

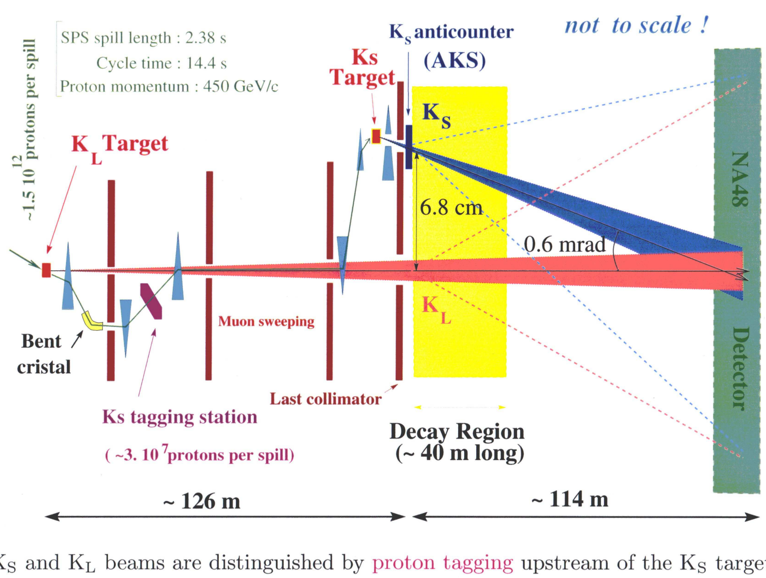

Fig. 15. Setup in the experiment NA48.

This is also the time resolution provided by scintillator hodoscopes behind the magnetic spectrometer. The intensity of the proton beam to the $K_{\rm S}$ target is only 10^{-5} of that to the $K_{\rm L}$ target. Such a small fraction can be picked up from the primary proton beam in the dump behind the $K_{\rm L}$ target and deviated back to the $K_{\rm L}$ beam line, first by channelling in a bent Si crystal, and then by bending magnets. Between these elements sit the tagging counters.

The electromagnetic calorimeter in NA48 is no more a sampling calorimeter as in NA31, but a pure liquid-krypton calorimeter, with improved energy resolution (see Fig. 16). Since a spatial resolution of less than 1 mm is required, the calorimeter is divided in 2×2 cm wide and 125 cm long cells. This tower structure, pointing to the decay region, is realized by individual Cu–Be strips, guided by four intermediate vetronite frames. The strips run through windows in these frames and are stretched between front and back in a 4.8° zig-zag path. The central anode strip between the cathode borders of a cell is read out in the back of the calorimeter. The calorimeter is calibrated electronically, or with electrons from Ke3 decays, or with $\pi^0 s$ from the $K_{\rm S}$ target, or, in special runs, with π^0 s from a polyethylene target

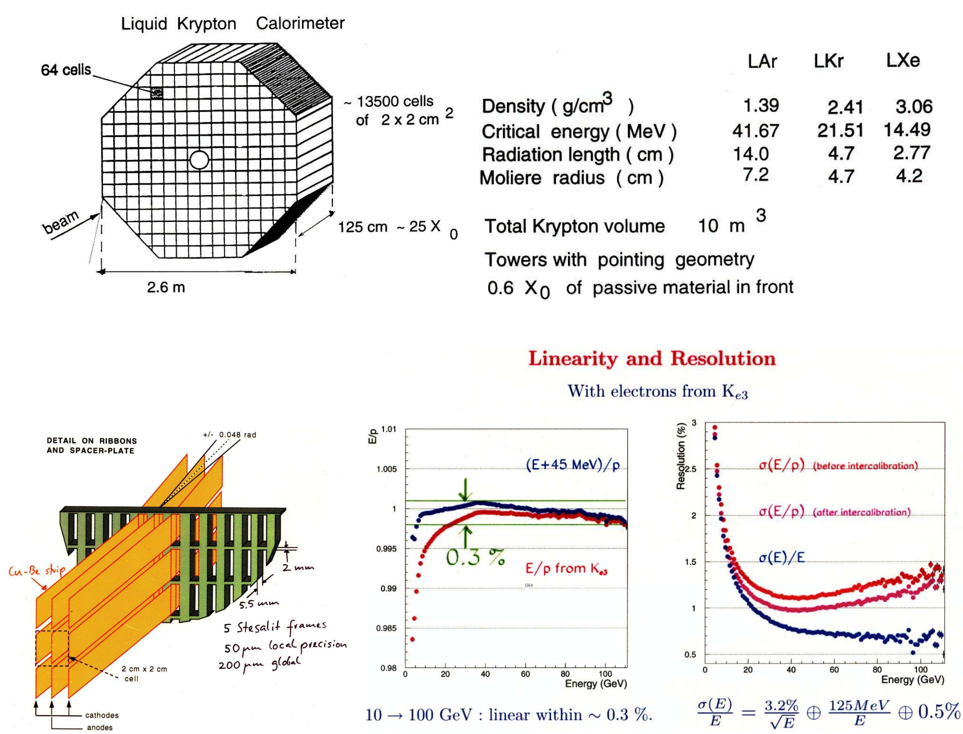

Fig. 16. Liquid Krypton calorimeter of NA48 and its energy resolution.

placed in the decay volume and struck by a charged pion beam. In these π^0 calibrations the known distance to the calorimeter, in $K_{\rm S}$ runs it is the distance from an anti-counter at the begin of the decay volume, fixes the opening angle of the γs from a π^0 decay. The energy scale is then linked to the π^0 mass, with a precision of 2×10^{-4}. With an accuracy of 10^{-4} also the transverse length scale in the wire chambers is known. Energy measurement is reduced in this way to distance measurement. In the wire chambers of the spectrometer sufficient redundancy against inefficiency or malfunctioning is provided by eight wire planes per chamber, with four different wire orientations. The NA48 setup is completed by the hadron calorimeter of NA31, used in conjunction with the liquid-krypton calorimeter for a total energy trigger, and by three planes of muon counters separated by iron walls.

The different decay-time distributions of $K_{\rm S}$ and $K_{\rm L}$ events are equalised in the analysis by weighting the $K_{\rm L}$ events, with a cut at 3.5 $K_{\rm S}$ lifetimes (Fig. 17). This technique, first applied by Cronin [21], avoids a Monte Carlo

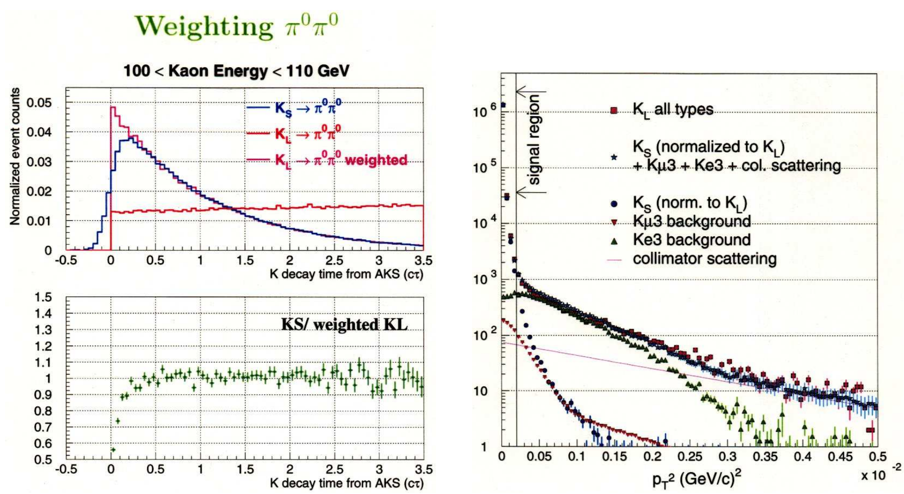

Fig. 17. Weighting $K_{\rm L} \to 2\pi^0$ events and background subtraction $K_{\rm L} \to \pi^+\pi^-$ events in the NA48 experiment.

efficiency determination at the price of an about 20% loss in statistical accuracy. Very detailed studies of systematic effects (see Fig. 18) showed that, apart from small background subtractions, there are only small corrections to the raw double ratio, mainly due to geometrical acceptance differences.

In the KTeV experiment at Fermilab similar calibration techniques are used. As in E731 there are two beams, one with a movable regenerator, now totally active, *i.e.* consisting of scintillator, in order to veto inelastic interactions. Instead of the lead-glass counters pure CsI crystals are used, with 2.5 cm width near the beams and 5 cm width further out (see Fig. 19).

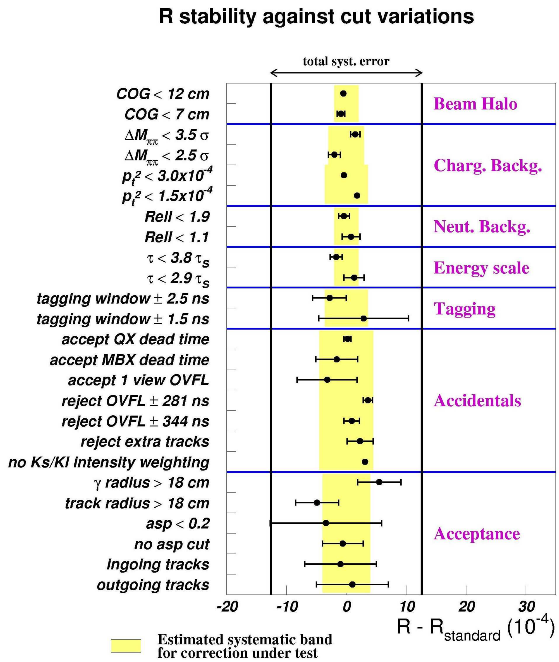

Fig. 18. Systematic errors in the experiment NA48.

The KTeV experiment was first to announce a new result [22], $\varepsilon'/\varepsilon = (28.0 \pm 3.0 \pm 2.8) \times 10^{-4}$, based on 20 % of their statistics, not very compatible with the result of E731. NA48 came later, with $\varepsilon'/\varepsilon = (18.5 \pm 7.3) \times 10^{-4}$ from a short run in 1997 [23], and $\varepsilon'/\varepsilon = (15.0 \pm 2.7) \times 10^{-4}$ from runs in 1998 and 1999 [24], with the combined result $\varepsilon'/\varepsilon = (15.3 \pm 2.6) \times 10^{-4}$. Because accidental effects in a high intensity beam are potentially most dangerous, the NA48 Collaboration had decided, long before any result was known, that a run at a different intensity would be desirable. This run, with about 30% reduced intensity and a 5 sec instead of a 3 sec spill, took place in 2001. The chambers had been rewired because of an accident in the fall of '99. The result, $\varepsilon'/\varepsilon = (13.7 \pm 3.1) \times 10^{-4}$ is fully compatible with the previous value. The final number from NA48 is then

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (14.7 \pm 2.2) \times 10^{-4}.$$

The KTeV Collaboration gave a new preliminary result at the KAON 2001 Conference in Pisa [25], based on more statistics and an improved analysis, $\varepsilon'/\varepsilon = (20.7 \pm 1.5 \pm 2.3) \times 10^{-4}$.

A reanalysis of the published data gave $\varepsilon'/\varepsilon = (23.2 \pm 3.0 \pm 3.2) \times 10^{-4}$. This has raised the question if their systematic error is not in fact underestimated. A "world average" would, therefore, have to be made with some care.

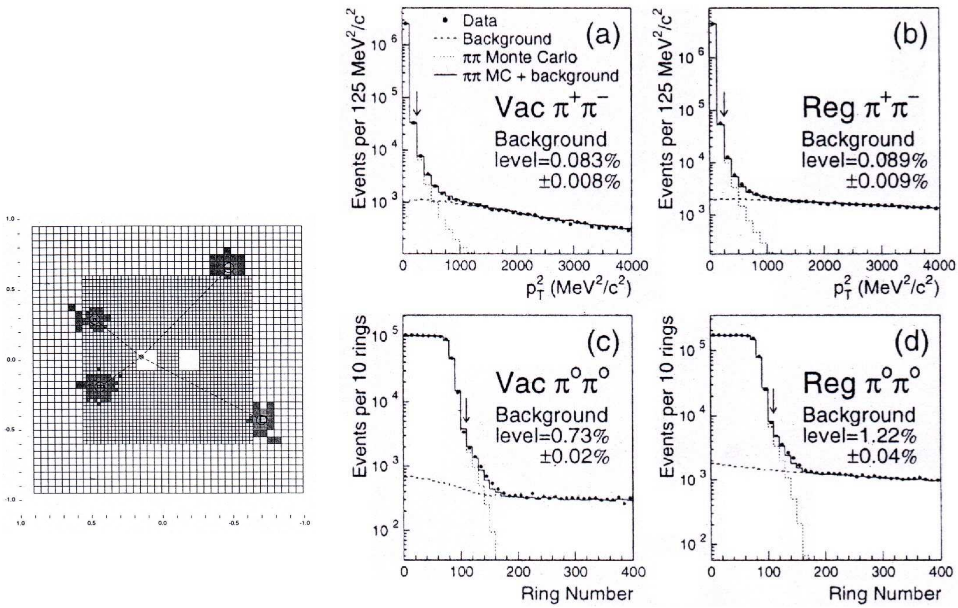

Fig. 19. CsI counter and $K_{\mathrm{L}} \to 2\pi$ signals in the KTeV experiment.

Needless to say, however, that both experiments agree, that the superweak model is ruled out.

5. CPT violation

5.1. Definition of measurable quantities

As for CP violation, there are two possibilities: CPT violation in the mixing of K^0 and $\bar{K}^0$ states and CPT violation in the decay amplitudes. In state mixing, CPT violation is characterised by a difference between ε_{S} and ε_{L}, *i.e.* a nonzero value of the (complex) parameter (see Eq. (3))

$$\delta = \frac{\varepsilon_{\mathrm{S}} - \varepsilon_{\mathrm{L}}}{2} = \frac{\Gamma_{11} - \Gamma_{22} + i(M_{11} - M_{22})}{\gamma_{\mathrm{S}} - \gamma_{\mathrm{L}} - 2i(m_{\mathrm{L}} - m_{\mathrm{S}})} \,.$$

The real matrix elements Γ_{ii} and M_{ii} are the decay rates and masses of K^0 and $\bar{K}^0$. If there is just a lifetime difference between K^0 and $\bar{K}^0$, then δ is oriented along the "superweak" direction, if there is only a mass difference, δ points orthogonal to this direction.

In the decay to two pions there are two possible CPT violating amplitudes. For convenience we define them with suitable factors to make the connection to observable quantities more transparent. We define

$$\begin{aligned}\alpha_0 &= \frac{A_0 - \bar{A}_0}{A_0 + \bar{A}_0}\,,\\ \alpha_2 &= \frac{1}{\sqrt{2}}\frac{A_2^* - \bar{A}_2}{(A_0 + \bar{A}_0)}\exp i(\delta_2 - \delta_0)\,,\end{aligned}$$

and choose a phase convention [26], in which A_0 and $\bar{A}_0$ have the same phase, such that α_0 is real. Since CPT violation is small, this represents only a small rotation of the K^0 and $\bar{K}^0$ state vectors from the Wu–Yang convention; A_0 is still approximately real. In the absence of CPT violation α_0 and α_2 both vanish. The observable amplitudes η_{+-} and η_{00} are then

$$\begin{aligned}\eta_{+-} &= \varepsilon + \varepsilon' - \delta + \alpha_0 + \alpha_2\,,\\ \eta_{00} &= \varepsilon - 2\varepsilon' - \delta + \alpha_0 - 2\alpha_2\,,\\ \eta_{00} - \eta_{+-} &= 3(\varepsilon' + \alpha_2)\,,\\ \tfrac{2}{3}\eta_{+-} + \tfrac{1}{3}\eta_{00} &= \varepsilon - \delta + \alpha_0\,,\\ \varepsilon_0 &= \varepsilon - \tilde{\delta}\,,\\ \tilde{\delta} &= \delta - \alpha_0\,,\end{aligned}$$

For later use we have defined here two additional quantities, the barycentre of η_{+-} and η_{00}, $\varepsilon_0 = \frac{2}{3}\eta_{+-} + \frac{1}{3}\eta_{00}$, and $\tilde{\delta}$. As evident from these equations, only the combinations $\varepsilon' + \alpha_2$ and $\tilde{\delta} = \delta - \alpha_0$, not ε' or δ itself, can be determined.

Let us see what further information can be obtained from unitarity. Unitarity in this context means, that for a decaying particle the rate at which it disappears equals the rate at which the decay products appear. The transition between two quantum states is described by a transition amplitude. If a state is a superposition of several other states, the amplitudes add. Applying these principles to a superposition of K_S and K_L states with arbitrary coefficients α and β, we have

$$-\frac{d}{dt}\Big|\alpha K_\mathrm{S} + \beta K_\mathrm{L}\Big|^2 = \Sigma_f\,|\alpha\langle f|H|K_\mathrm{S}\rangle + \beta\langle f|H|K_\mathrm{L}\rangle|^2\,.$$

Hence we obtain three equations

$$\begin{aligned}\gamma_\mathrm{L} &= \Sigma_f\,|\langle f|H|K_\mathrm{L}\rangle|^2\,,\\ \gamma_\mathrm{S} &= \Sigma_f\,|\langle f|H|K_\mathrm{S}\rangle|^2\,,\\ \left(\frac{\gamma_\mathrm{S}}{2}+\frac{\gamma_\mathrm{L}}{2}+i\,(m_\mathrm{L}-m_\mathrm{S})\right)\,2\,(\mathrm{Re}\,\varepsilon - i\,\mathrm{Im}\,\delta)\langle K_\mathrm{L}|K_\mathrm{S}\rangle &= \Sigma_f\,\eta_f\gamma_f\,. \qquad (5)\end{aligned}$$

The first two equations state that the total decay rate is equal to the sum of the partial decay rates. The third equation is less transparent. This equation is called the Bell–Steinberger relation [27]. Separating out here the 2π states with $I = 0$, and denoting the CP violating amplitude in each channel f with η_f we have

$$\left(\frac{\gamma_{\rm S}}{2} + i(m_{\rm L} - m_{\rm S})\right) 2\ ({\rm Re}\,\varepsilon - i{\rm Im}\,\delta) = \varepsilon_0\gamma_{\rm S} + \Sigma'_f\,\eta_f\gamma_f$$

or

$$\left(1 + i\,\frac{2(m_{\rm L} - m_{\rm S})}{\gamma_{\rm S}}\right)\ ({\rm Re}\,\varepsilon - i{\rm Im}\,\delta) = \varepsilon_0 + \eta\,,$$

where η is the sum over CP violating terms in other than the $I = 0$ two pion decays. This term is close to zero; the largest uncertainty is from η_{000}. From this relation one obtains Imδ with good precision, without any assumption on the CPT violating amplitudes α_0 and α_2.

Additional information is necessary to determine Reδ.

The extraction of CPT violating quantities from the measurements of η_{+-}, η_{00}, η and the charge asymmetry in semileptonic $K_{\rm L}$ decays, Re$\,\varepsilon_{\rm L}$, is perhaps best illustrated graphically. In Fig. 20 the distances between the end points of η_{+-}, η_{00} *etc.*, are highly exaggerated for sake of clarity. In reality all lines nearly coincide and end up in a single point within measurement errors.

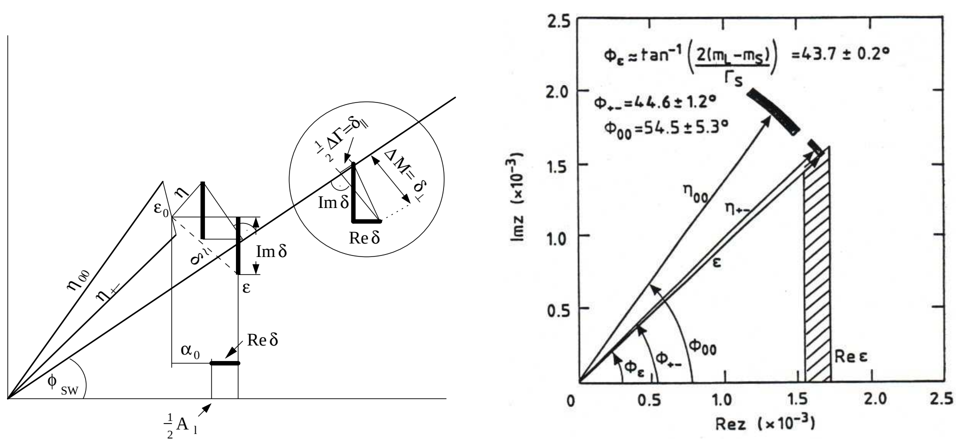

Fig. 20. Left: CP and CPT violating amplitudes. Right: Wu–Yang triangle in 1988 (schematic).

From the barycentre ε_0 one obtains a new vector $\varepsilon_0+\eta$ by adding the contribution η of CP violations in other than 2π decays. In general this vector has an argument which is different from the superweak phase. Its component orthogonal to the superweak line can be split into a real and an imaginary part. The imaginary part is the desired quantity Imδ. To construct ε, one notices that $\mathrm{Re}\varepsilon$ is identical to the real part of the projection of $\varepsilon_0 + \eta$ to the superweak line. Then $\mathrm{Im}\varepsilon = \mathrm{Im}\varepsilon_0 + \mathrm{Im}\delta$. $\mathrm{Re}\delta$ is obtained from the difference between $\mathrm{Re}\varepsilon$ and the charge asymmetry A_l in semileptonic decays of K_{L}: $A_l = 2\mathrm{Re}\,\varepsilon_{\mathrm{L}} = 2\mathrm{Re}\,(\varepsilon - \delta)$. The components of δ transverse and parallel to the superweak direction are $m_{K^0} - m_{\bar{K}^0}$ and $\Gamma_{K^0} - \Gamma_{\bar{K}^0}$, respectively.

5.2. Experiments on CPT violation

The unambiguous sign of CPT violation is a phase difference between either η_{+-} and η_{00} or between one of them and the superweak phase. The situation of the so-called Wu–Yang triangle (the relation between η_{+-}, η_{00} and ε') as of 1988 is shown in Fig. 20. It is clear that the experiments set up to measure $|\eta_{00}/\eta_{+-}|$ were also to make an effort to improve the phase measurements. This was done both by the Fermilab and the CERN experiments. In both experiments the phase is extracted from the interference between the K_{S} and K_{L} amplitudes in K^0 (or $\bar{K}^0$) $\to 2\pi$ decays. A mixture of K^0 and $\bar{K}^0$ different from that in K_{L} is obtained behind a regenerator in the Fermilab experiment and directly from the proton target in the CERN experiment. In order to eliminate the dependence on detection efficiency the CERN experiment [28] run with two distances (Far and Near) between target and decay region; from the ratio of events as a function of time one can extract the phases without corrections for acceptance (see Fig. 21).

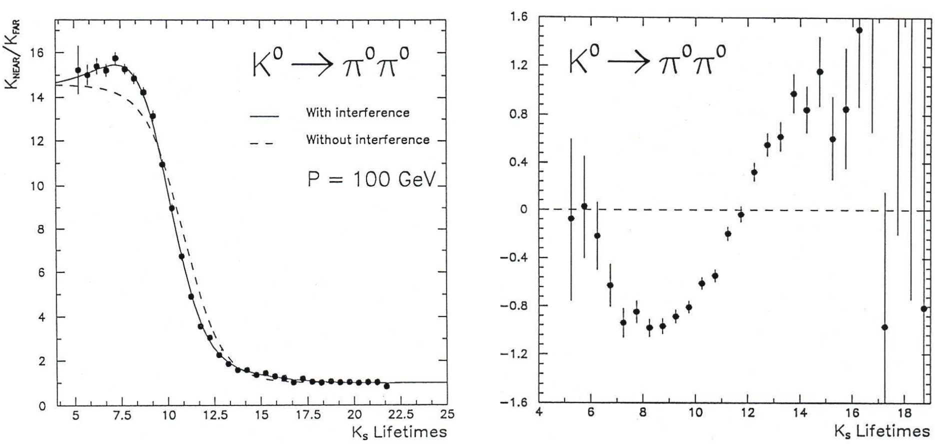

Fig. 21. Near/Far ratio in NA31 and extracted interference term.

The Fermilab experiment [29] does not measure the weak phases ϕ_{00} and ϕ_{+-} directly, but it measures their difference to the phase ϕ_ρ of the regeneration amplitude (see Fig. 22). Since the initial state behind the regenerator is $K_{\mathrm{L}} + \rho K_{\mathrm{S}}$, one has

$$I_{2\pi} \propto |\rho|^2 \exp(-\gamma_{\rm S} t) + 2|\rho||\eta| \exp\left(-\frac{\gamma_{\rm S}}{2}\, t\right) \times \cos\left(\Delta m\, t + \phi_\rho - \phi\right) + |\eta|^2 \exp(-\gamma_{\rm L} t)\,.$$

The expression for the regeneration amplitude,

$$\rho = i\pi N L g\, \frac{f - \bar{f}}{k}$$

contains a geometric factor g and a factor related to the forward scattering amplitudes f for K^0 and $\bar{f}$ for $\bar{K}^0$.

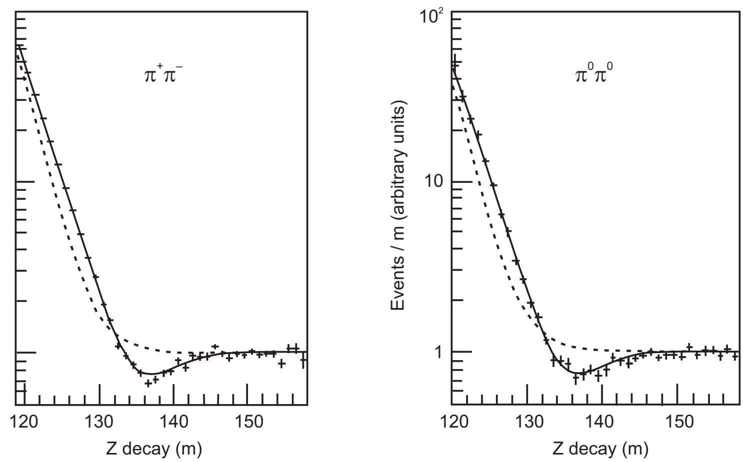

Fig. 22. Phase measurements in the Fermilab experiment E773.

Under the assumption of a single trajectory exchange one obtains from Regge theory

$$\arg\left(\frac{f - \bar{f}}{k}\right) = -(2 - a)\frac{\pi}{2}\,,$$

where a is the exponent of the momentum dependence of the regeneration amplitude

$$\frac{f - \bar{f}}{k} \propto p_k^{-a}\,.$$

With the measured values $a = 0.572 \pm 0.007$ and $|g - 1| < 0.2$ the regeneration phase is $\phi_\rho \approx -40^0$, shifting the interference pattern to earlier times, a favourable situation as far as counting rates are concerned. The systematic error on ϕ_ρ has been the subject of some controversy [30]; the measurement of the phase difference $\phi_{00} - \phi_{+-}$ is, of course, unaffected by the regeneration phase. In both experiments the phase is correlated with Δm, as evident from Eq. (6), albeit to a different degree in the CERN and Fermilab experiments, because the time of highest sensitivity is different. This correlation is illustrated in Fig. 23.

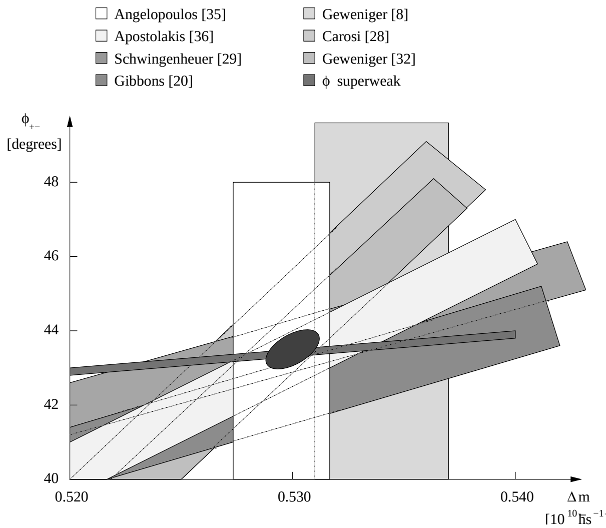

Fig. 23. Correlation between Δm and ϕ_{+-}.

The numerical results are summarised in Table I. Within errors there is, therefore, no CPT violation. The smallest error is that from E773

$$\phi_{00} - \phi_{+-} = (-0.3 \pm 0.9)^\circ \,.$$

One may wonder, then, if it is possible to measure Δm independently. This can be done in semileptonic decays and also in 2π decays, by the "gap" method, first proposed by Fitch [31]. The 2π intensities behind a regenerator

TABLE I

Phase measurements.

	NA31	E773
Φ_{+-}	$(46.8 \pm 1.4 \pm 0.7)^\circ$	$(43.5 \pm 0.58 \pm 0.49)^\circ$
Φ_{00}	$(47.1 \pm 2.1 \pm 1.0)^\circ$	
$\Phi_{00} - \Phi_{+-}$	$(0.2 \pm 2.6 \pm 1.2)^\circ$	$(-0.3 \pm 0.9)^\circ$

are compared for two locations of a second regenerator placed upstream at a distance d. The $K_{\rm S}$ amplitude regenerated in the upstream part adds to the amplitude of the downstream part, but with a phase difference, since $K_{\rm S}$ and $K_{\rm L}$ oscillate with different frequencies. In the gap between the regenerators they develop a phase difference $\Delta m\, t = \Delta m\, d/\beta$, if $\beta = v/c$ is their velocity. Comparing measurements with different d (for example $\Delta m\, t = 0$ and $\Delta m\, t = \pi/2$) the mass difference Δm is obtained. The best measurement is from the CERN–Heidelberg experiment [32], in very good agreement with their result from semileptonic decays [33]. This measurement is also shown in Fig. 23.

A more precise measurement of Δm and ϕ_{+-} is possible if the initial strangeness of the kaon is known. This is the case in the CP–LEAR experiment. Here the source of kaons is antiproton–proton annihilations at rest. In a small fraction (0.2 %) of cases the annihilation reaction is

$$\bar{p}p \to \pi^+ K^- K^0 \,, \qquad \text{or} \qquad \bar{p}p \to \pi^- K^+ \bar{K^0} \,.$$

The initial strangeness of the kaon is tagged by the charged kaon. In this case the mass difference Δm can be extracted from the charge asymmetry in semileptonic K^0 or $\bar{K^0}$ decays. The difference in the 2π decay rates of K^0 and $\bar{K^0}$ as a function of time gives a very precise determination of ϕ_{+-} (the equivalent "dilution" factor is 2 instead of ≈ 0.35 in high energy neutral beams).

The CP–LEAR experiment [34] uses cylindrical drift chambers in a solenoid for charged particle detection, a liquid Cerenkov counter for π–K discrimination and a lead-streamer-tube sandwich calorimeter for γ detection (see Fig. 24). The Δm [35] and ϕ_{+-} [36] measurements are shown in Fig. 25;

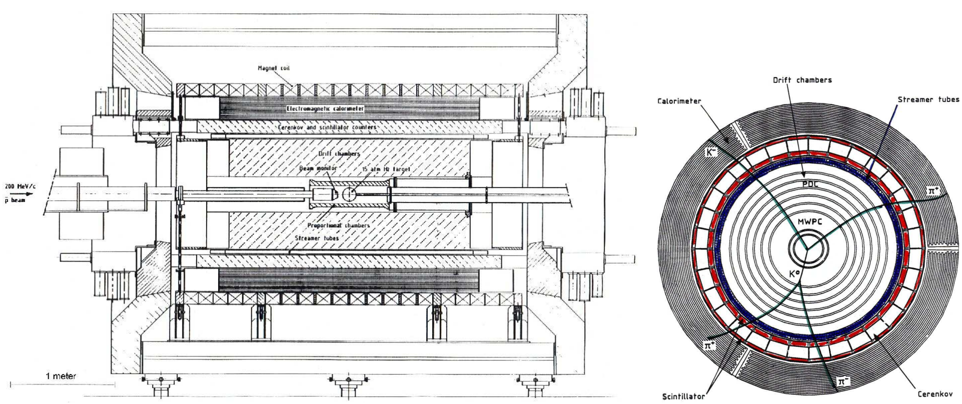

Fig. 24. Side and end view of the CP–LEAR experiment.

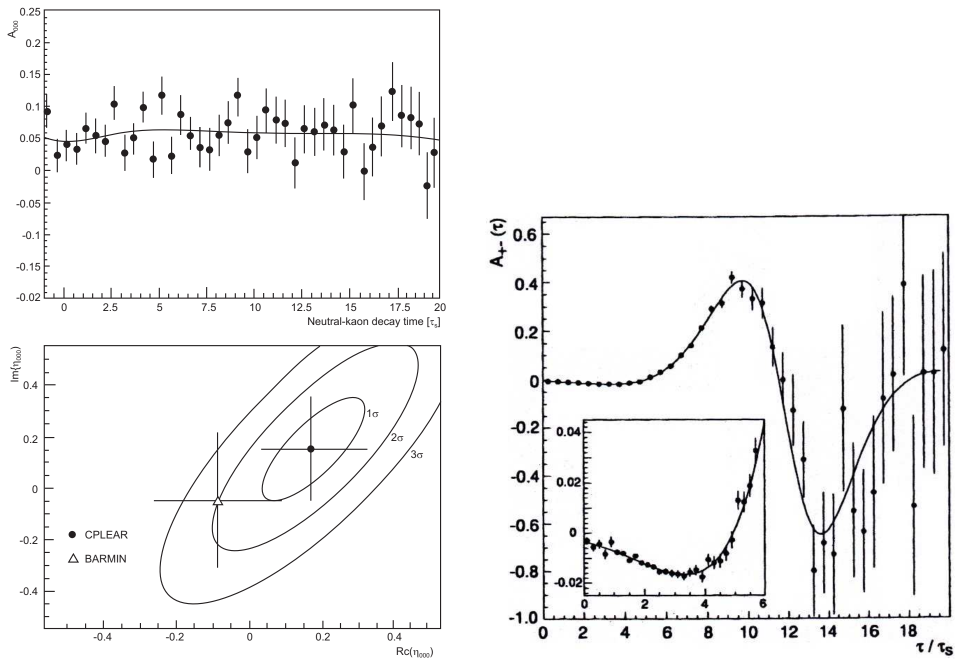

Fig. 25. η_{000} and ϕ_{+-} measurements from CP–LEAR.

they are also given for comparison in Fig. 23. The main result is that ϕ_{+-} agrees with the superweak phase $\phi_{\rm SW}$ within an error of $0.7°$. Within this error there is no CPT violation.

To convert the measured phases into limits on mass and lifetime differences between K^0 and $\bar{K}^0$ we go back to Fig. 20. Without any assumption on CPT violation in the 2π decay amplitudes one can use unitarity (the Bell–Steinberger relation) to restrict Imδ. It is then necessary to know CP violation amplitudes in common decay channels of K^0 and $\bar{K}^0$ other than 2π. The largest uncertainty comes from η_{000}, for which the CP–LEAR Collaboration has given [37]

$$\mathrm{Im}(\eta_{000}) = 0.15 \pm 0.20\,, \qquad \mathrm{Re}(\eta_{000}) = 0.18 \pm 0.16\,,$$

on the basis of the difference in $K \to 3\pi^0$ rates between initial K^0 and $\bar{K}^0$ states. This represents hardly an improvement over the old bubble chamber result of Barmin *et al.* [38] as seen in Fig. 25.

The reason is a large amount of background, due to poor resolution and detection efficiency in the CP–LEAR γ detector [39] (see Fig. 26). In the original proposal [40] a much better performance was anticipated on the basis of a CsI calorimeter. This was later [41] given up in favour of a lead-streamer-tube sandwich for reasons of cost effectiveness. We can see again,

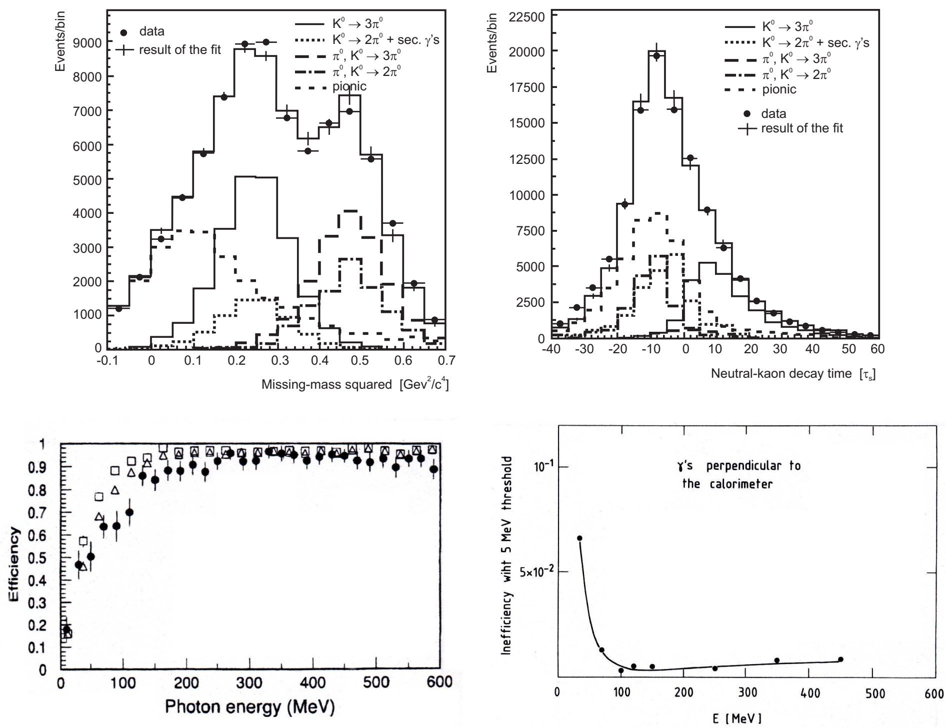

Fig. 26. CP–LEAR: $3\pi^0$ signal and background; γ detection efficiency in data and proposal.

perhaps not for the last time, that measuring CP violation in neutral decays of the K^0 is difficult.

The results on CPT violation are summarised in Table II.

$$\begin{aligned}
\Phi_{\varepsilon_0} - \Phi_{\mathrm{SW}} &= (-0.2 \pm 0.6)^\circ\,, \\
\varepsilon_{0\perp} &= |\eta_{+-}|\,(\Phi_{\varepsilon_0} - \Phi_{\mathrm{SW}}) &&= (-0.8 \pm 2.4) \times 10^{-5}\,, \\
\mathrm{Re}\,\varepsilon_0 &= |\eta_{+-}|\cos(\Phi_\varepsilon) &&= (1.66 \pm 0.02) \times 10^{-3}\,, \\
\eta_\perp &= (-1.7 \pm 4.8) \times 10^{-5}, \\
\mathrm{Im}\,\delta &= 0.7(\varepsilon_0 + \eta)_\perp &&= (-1.7 \pm 3.7) \times 10^{-5}\,.
\end{aligned}$$

The smallest upper limit is on Imδ

$$|M_{K^0} - M_{\bar{K}^0} + 0.5(\Gamma_{K^0} - \Gamma_{\bar{K}^0})| < 6 \times 10^{-19}\,\mathrm{GeV}.$$

The error could be improved (by not more than a factor of 2), if the upper limit on η_{000} were reduced. The NA48 Collaboration is aiming at that, in runs with a higher intensity K_{S} beam.

TABLE II

CPT violation in numbers.

		Main experiment
$\Phi_{00} - \Phi_{+-}$	$(-0.3 \pm 0.9)^\circ$	E773
Φ_{+-}	$(43.4 \pm 0.5)^\circ$	CPLEAR
Δm	$(0.5295 \pm 0.0020) \times 10^{10} \hbar s^{-1}$	CPLEAR
Φ_{SW}	$(43.5 \pm 0.1)^\circ$	(PDG)
$\mathrm{Im}(\eta_{000})$	0.15 ± 0.20	CPLEAR
$\mathrm{Re}(\eta_{000})$	0.18 ± 0.16	CPLEAR
A_{lept}	$(3.32 \pm 0.075) \times 10^{-3}$	KTeV
$\lvert\eta_{+-}\rvert$	$(2.276 \pm 0.017) \times 10^{-3}$	(PDG)

For a measurement of Reδ additional information is necessary. The most obvious procedure is to compare charge asymmetries in semileptonic decays for different mixtures of K^0 and $\bar{K}^0$. This was done by CP–LEAR [42]; by a suitable choice of variables they could avoid to be sensitive to a possible CPT violation in semileptonic decays. Their result is

$$\mathrm{Re}\,\delta = (3.0 \pm 3.3 \pm 0.6) \times 10^{-4} \,.$$

Statistically more significant are charge asymmetries in K_{L} decays from the high energy experiments. The most precise number was given recently [43] by the KTeV Collaboration, based on about 300 million events.

$$A_e = (3.322 \pm 0.058(\mathrm{stat.}) \pm 0.047(\mathrm{syst.})) \times 10^{-3} \,.$$

The combined error is 0.075 $\times 10^{-3}$, a factor of 2.4 better than the best previous result.

Let y characterise the CPT violation in semileptonic decays

$$y = \frac{\bar{A}^* - A}{\bar{A}^* + A} \,,$$

where

$$A = \langle e^+ \pi^- \nu | K^0 \rangle \,, \qquad \bar{A} = \langle e^- \pi^+ \bar{\nu} | \bar{K}^0 \rangle \,,$$

are the decay amplitudes. Neglecting a possible CPT violation in $\Delta Q = -\Delta S$ transitions, one has then

$$A_{\mathrm{lept}} = 2\mathrm{Re}(\varepsilon - \delta - y) = 2\mathrm{Re}(\varepsilon_0 - \alpha_0 - y) \,.$$

The best limit is then for the sum of CPT violating amplitudes

$$\alpha_0 + \mathrm{Re}\, y = (0 \pm 4) \times 10^{-5}\,.$$

Assuming $\mathrm{Re}\, y = 0$ one obtains from

$$\begin{aligned} \mathrm{Re}\,\delta &= \alpha_0 + \mathrm{Re}\,(\varepsilon - \varepsilon_0)\,, \\ \mathrm{Re}\,\delta &= (5 \pm 8) \times 10^{-5}\,, \end{aligned}$$

a factor of 4 more precise than the result quoted before. The CP–LEAR result can be used to give limits on α_0 and $\mathrm{Re}\, y$ individually,

$$|\mathrm{Re}\, y| < 8 \times 10^{-4}\,,$$

$$|\alpha_0| < 8 \times 10^{-4}\,.$$

None of these limits on CPT violation amplitudes reaches down to the observed CP violating amplitude

$$\mathrm{Re}\,\varepsilon' = 2.6 \times 10^{-6}\,.$$

The most sensitive test on CPT invariance limits $\mathrm{Im}\delta$, and thereby mass and lifetime differences between K^0 and $\bar{K}^0$, as given above

$$\left| M_{K^0} - M_{\bar{K}^0} + 0.5(\Gamma_{K^0} - \Gamma_{\bar{K}^0}) \right| < 6 \times 10^{-19}\,\mathrm{GeV}\,.$$

6. Summary and conclusions

The possible origin of CP violation has been further constraint by the uncontroversial observation of "direct" CP violation in the $K \to 2\pi$ decay. The data are not in conflict with the idea that CP violation is due to a non-trivial phase in the quark mixing matrix.The predictions are, however, not (or not yet?) as precise as the data.

There is no sign of CPT violation in K^0 decay. Masses and lifetimes of K^0 and $\bar{K}^0$ agree to within one part in 10^{18}.

The author wishes to thank Prof. A. Białas for his kind invitation and hospitality at this school and Prof. M. Praszałowicz for the excellent organisation. It was a great pleasure to come back after 25 years, to meet some of the old friends and to discuss physics and other topics. I want also to acknowledge the very fruitful collaboration over many years with the members of the NA31 and NA48 teams. Special thanks go to my collaborator Dr. Michał Ziółkowski for his help in the preparation of these lectures.

REFERENCES

[1] J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turlay, *Phys. Rev. Lett.* **13**, 138 (1964).

[2] L. Wolfenstein, *Phys. Rev. Lett.* **13**, 562 (1964).

[3] M. Kobayashi, K. Maskawa, *Progr. Theor. Phys.* **49**, 652 (1973).

[4] A.J. Buras, R. Fleischer *Heavy Flavours II*, Eds. A.J. Buras and M. Lindner, World Scientific.

[5] I.I. Bigi, A.I. Sanda, *CP Violation*, Cambridge University Press, 2000.

[6] *CP Violation*, C. Jarlskog, Ed. World Scientific, 1989.

[7] C.S. Wu, C.N. Yang, *Phys. Rev. Lett.* **13**, 180 (1964).

[8] C. Geweniger *et al., Phys. Lett.* **B48**, 483 (1974).

[9] CLEO Collaboration, A. Bornheim *et al.*, CLNS 01/1767, CLEO 01-23.

[10] J. Drees, Proc. of the XX International Symposium on Lepton and Photon Interactions at High Energies, Rome, July 23–28, 2001.

[11] BABAR Collaboration, B. Aubert *et al.*, SLAC-PUB-9153 (2002).

[12] J. Ellis, M.K. Gaillard, D.V. Nanopoulos, *Nucl. Phys.* **B109**, 213 (1976); F.J. Gilman, M.B. Wise, *Phys. Lett.* **B83**, 83 (1979).

[13] S. Bosch *et al.*, Techn. Univ. Muenchen preprint TUM-HEP-347/99.

[14] D. Luckey, Proc. of the International Symposium on Electron and Photon Interactions at High Energies, Hamburg, June 8–12, 1965, p. 397.

[15] M. Banner *et al., Phys. Rev. Lett.* **28**, 1957 (1972).

[16] M. Holder *et al., Phys. Lett.* **B40**, 141 (1972).

[17] W.J. Willis, V. Radeka, *Nucl. Instrum. Methods* **120**, 21 (1974).

[18] H. Burkhardt *et al., Phys. Lett.* **B206**, 169 (1988).

[19] G.D. Barr *et al., Phys. Lett.* **B317**, 233 (1993).

[20] L.K. Gibbons *et al., Phys. Rev. Lett.* **70**, 1203 (1993).

[21] J. Cronin, talk presented at the 1984 SLAC Summer Institute on Particle Physics.

[22] KTeV Collaboration, A. Alavi-Harati *et al., Phys. Rev. Lett.* **83**, 22 (1999).

[23] NA48 Collaboration, A. Lai *et al., Phys. Lett.* **B465**, 335 (1999).

[24] NA48 Collaboration, A. Lai *et al., Eur. Phys. J.* **C22**, 231 (2001).

[25] A. Glazov, KTeV Collaboration, Proc. of the KAON 2001 International Conference on CP Violation, Pisa, June 2001.

[26] K.R. Schubert *et al., Phys. Lett.* **B31**, 662 (1970).

[27] J.S. Bell, Proc. of the Conference on Elementary Particles, Oxford 1965, p. 195.

[28] R. Carosi *et al., Phys. Lett.* **B237**, 303 (1990).

[29] B. Schwingenheuer *et al., Phys. Rev. Lett.* **74**, 4376 (1995).

[30] See discussion in the 2000 edition of the Particle Physics Group, *Eur. Phys. J.* **C15**, 522 (2000).

[31] J.H. Christenson *et al., Phys. Rev.* **B140**, 74 (1965).
[32] C. Geweniger *et al., Phys. Lett.* **B52**, 108 (1974).
[33] S. Gjesdal *et al., Phys. Lett.* **B52**, 113 (1974).
[34] R. Adler *et al., Nucl. Instrum. Methods* **A379**, 76 (1996).
[35] A. Angelopoulos *et al., Phys. Lett.* **B444**, 38 (1998).
[36] A. Apostolakis *et al., Phys. Lett.* **B458**, 545 (1999).
[37] A. Angelopoulos *et al., Phys. Lett.* **B425**, 391 (1998).
[38] V.V. Barmin *et al., Phys. Lett.* **B128**, 129 (1983).
[39] R. Adler *et al., Nucl. Instrum. Methods* **A390**, 293 (1997).
[40] CP–LEAR proposal CERN/PSCC/P 82 (1985).
[41] CP–LEAR addendum CERN/PSCC/P 82, Add.1 (1985).
[42] A. Angelopoulos *et al., Phys. Lett.* **B444**, 52 (1998).
[43] KTeV Collaboration, A. Alavi-Harati *et al.*, *Phys. Rev. Lett.* **88**, 181601 (2002).