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PION STRUCTURE AT HIGH AND LOW ENERGIES IN CHIRAL QUARK MODELS*

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Low and high energy properties of the pion are reviewed in the framework of chiral quark models. Particular emphasis is put on the simplest version of the SU(2) NJL model as prototype. The role of gauge invariance in this kind of calculations is stressed. The results are used as initial conditions for perturbative QCD evolution equations. At leading order the quark model scale is $\mu_0 \sim 320$MeV as determined from the pion distribution functions and the pion distribution amplitudes.

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1. Introduction and motivation

The structure and dynamics of hadrons is known to be a difficult problem because the underlying QCD theory in terms of quark and gluon degrees of freedom is actually very complicated [1]. There are, however, well known situations where some helpful simplifications take place.

At high energies asymptotic freedom sets in, and perturbative QCD becomes applicable. The renormalization group equations allow, on the basis of the Operator Product Expansion (OPE), to relate hadronic matrix elements of quark and gluon operators at different scales [2]. There are many situations where this scale dependence has been predicted and in some cases tested, particularly in the study of high energy inclusive [3] and exclusive processes [4], but perturbative QCD says nothing about the value of these matrix elements at a given scale.

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At very low energies, spontaneous chiral symmetry breaking of the QCD Lagrangian dominates, and the would-be Goldstone bosons in the massless quark limit become the lightest states in the hadronic spectrum. They are naturally identified with the pion, and it can be shown that at low energies the interaction among pions becomes weak. Actually, at energies of the order of the pion mass, Chiral Perturbation Theory (ChPT) can be applied, in terms of low energy constants which correspond to certain QCD Green's functions, and many successful results have been found [5,6]. The values of the low energy constants cannot be computed themselves from ChPT.

As the previous considerations suggest, one may try to exploit chiral symmetry to study pion properties not only at low but also at high energies both in exclusive or inclusive channels. From a theoretical viewpoint, π mesons are particularly distinguished hadrons, since most of their low energy properties follow the patterns dictated by the chiral symmetry. Actually, we do not expect to understand the properties of any hadron better than the pion, as Chiral Perturbation Theory suggests. Recent studies have shown, for instance, that ChPT can be applied to compute chiral corrections of hadronic matrix elements contributing to the deep inelastic scattering [7] (in much the same way as chiral corrections are computed for $\pi\pi$ scattering). However, nothing is said about the value of those matrix elements in the chiral limit.

The calculation of pion matrix elements of quark and gluon operators contributing to the high energy exclusive and inclusive processes requires some non-perturbative method. Along these lectures it will be shown how this can be done with the help of low energy chiral quark models, which account for spontaneous chiral symmetry breaking, supplemented with QCD evolution equations, which automatically implement QCD logarithmic radiative corrections.

Chiral Quark Models share with QCD several features, mainly chiral symmetry (realized in the Goldstone phase) and explicit quark degrees of freedom. As we will see below they are useful since they can make quantitative predictions which, in many cases, agree rather well with experiment. Unfortunately, there is not a unique model, but many and almost as many versions as authors working in the field. So, for practical reasons we will be using the Nambu–Jona-Lasinio model [8] for quark degrees of freedom [9]. It turns out that, when dealing with low energy properties of the pion, model details seem rather irrelevant, so most people have not paid particular attention to their different versions. This seems not to be the case for properties which are extracted from high energy inclusive and exclusive processes where the model details and/or their different versions turn out to produce quite different results.

Given the fact that QCD is a theory with quarks and gluons, one obvious question one should ask is what is the scale at which a (chiral) quark model is naturally defined taking into account that its only explicit degrees of freedom are quarks. So far, we only know of one possible quantitative answer [10,11], namely the adequate scale is defined by requiring that quarks carry all momentum of the hadron. In a field theoretical chiral quark model this is a trivial statement, because there are no explicit gluons. This assumption seems consistent with QCD in perturbation theory since it turns out that the gluon contribution to the energy-momentum tensor becomes smaller as the running scale goes down until it eventually vanishes. The remaining quark contribution can be separated into sea (flavor singlet) and valence (flavor non-singlet) pieces. The precise value suggested by perturbative calculations is in the range, $\mu = 300$–$350\,\mathrm{MeV}$. Of course, the separation of the gluon, sea-quark and valence-quark momentum fractions is not a renormalization group invariant operation. Thus, if such a matching procedure between QCD and the chiral quark is meaningful, it depends on the particular renormalization scheme.

2. Low energy models from high energies

QCD is an asymptotically free theory, which means that the universal coupling constant among quarks and gluons becomes small as the renormalization scale goes to infinity, and hence perturbation theory becomes applicable. For definiteness, we use LO evolution for the running strong coupling constant [3],

$$\alpha(\mu) = \left(\frac{4\pi}{\beta_0}\right) \frac{1}{\log\left(\mu^2/\Lambda_{\mathrm{QCD}}^2\right)}\,, \tag{1}$$

with $\beta_0 = 11C_{\mathrm{A}}/3 - 2N_{\mathrm{F}}/3$, $C_{\mathrm{A}} = 3$ and N_{F} being the number of active flavors, which we take equal to three. We take for concreteness $\Lambda_{\mathrm{QCD}} = 226$ MeV, which for $\mu = 2\mathrm{GeV}$ yields $\alpha = 0.32$ [12]. In perturbative QCD calculations, $\alpha/2\pi$ is the expansion parameter. Renormalization group invariance requires a very specific dependence for any set of observables, O on the scale

$$O(\mu) = U(\mu,\mu_0)O(\mu_0)\,, \tag{2}$$

where $U(\mu,\mu_0)$ is a linear matrix operator, fulfilling cocycle properties typical of evolution equations,

$$U(\mu_1,\mu_2)U(\mu_2,\mu_3) \;=\; U(\mu_1,\mu_3)\,, \tag{3}$$

$$U(\mu,\mu) \;=\; 1\,. \tag{4}$$

In the limit $\mu,\mu' \to \infty$ the operators are explicitly calculable in perturbation theory. Obviously, conserved quantities do not depend on the evolution scale.

A particularly interesting operator in QCD is the energy-momentum tensor $\theta^{\alpha\beta}$, which due to Poincare invariance is a conserved quantity and hence renormalization invariant. Its matrix element between pion states fulfills

$$\langle \pi | \theta^{\alpha\beta} | \pi \rangle = 2 p^\alpha p^\beta \, . \tag{5}$$

For the QCD Lagrangian the energy momentum tensor can be separated into three contributions [3]

$$\theta^{\alpha\beta} = \theta^{\alpha\beta}_G + \theta^{\alpha\beta}_{q,\mathrm{S}} + \theta^{\alpha\beta}_{q,\mathrm{NS}} \, , \tag{6}$$

where $\theta^{\alpha\beta}_G$,$\theta_{q,\mathrm{S}}$ and $\theta^{\alpha\beta}_{q,\mathrm{NS}}$ are the gluon, quark-singlet and quark-non-singlet (or valence) contributions respectively fulfilling

$$\begin{aligned} \langle \pi | \theta^{\alpha\beta}_G | \pi \rangle |_\mu &= 2 p^\alpha p^\beta G_1(\mu) \, , \\ \langle \pi | \theta^{\alpha\beta}_{q,\mathrm{S}} | \pi \rangle |_\mu &= 2 p^\alpha p^\beta S_1(\mu) \, , \\ \langle \pi | \theta^{\alpha\beta}_{q,\mathrm{NS}} | \pi \rangle |_\mu &= 2 p^\alpha p^\beta V_1(\mu) \, , \end{aligned} \tag{7}$$

where $G_1(\mu)$, $S_1(\mu)$ and $V_1(\mu)$ are the gluon, sea quark and valence quark momentum fractions of the pion. In Deep Inelastic Scattering (DIS) it can be shown [3] that if $q_i(x,\mu)$, $\bar{q}_i(x,\mu)$ and $G(x,\mu)$ represent the probability density of finding a quark, antiquark and gluon, respectively, with momentum fraction x at the scale μ ($\mu^2 = Q^2$ in DIS), then one has

$$G_1(\mu) = \int_0^1 dx x G(x,\mu) \, , \tag{8}$$

$$S_1(\mu) = \int_0^1 dx x S(x,\mu) \, , \tag{9}$$

$$V_1(\mu) = \int_0^1 dx x V(x,\mu) \, , \tag{10}$$

where, for π^+, $V = u_\pi - \bar{u}_\pi + \bar{d}_\pi - d_\pi$ and $S = u_\pi - \bar{u}_\pi - \bar{d}_\pi + d_\pi$. These momentum fractions depend on the scale, μ, and fulfill the momentum sum rule

$$V_1(\mu) + S_1(\mu) + G_1(\mu) = 1 \tag{11}$$

as a consequence of the energy-momentum tensor conservation. In perturbation theory it is verified that due to radiative corrections $G_1(\mu)$ and $S_1(\mu)$ decrease as the scale μ goes down. On the contrary, the non-singlet contribution to the energy momentum tensor evolves as

$$\frac{V_1(\mu)}{V_1(\mu_0)} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_1^{\rm NS}/2\beta_0} , \tag{12}$$

where $\gamma_1^{\rm NS}/2\beta_0 = 32/81$ for $N_{\rm F} = N_c = 3$. The value of $V_1(\mu)$ has been extracted from the analysis of high energy experiments. In Ref. [13] it was found that at $Q^2 = 4{\rm GeV}^2$ valence quarks carry 47% of the total momentum fraction in the pion, *e.g.*, for π^+,

$$V_1 = \left\langle x \left(u_\pi - \bar{u}_\pi + \bar{d}_\pi - d_\pi\right)\right\rangle = 0.47 \pm 0.02 \qquad \text{at} \qquad \mu^2 = 4{\rm GeV}^2 \, . \tag{13}$$

Downwards LO evolution yields that for a given reference scale, μ_0,

$$V_1(\mu_0) = 1 \, , \qquad G_1(\mu_0) + S_1(\mu_0) = 0 \, . \tag{14}$$

The scale μ_0 so defined is called the quark model point for obvious reasons. At LO the scale turns out to be

$$\mu_0 = 313^{+20}_{-10}\,{\rm MeV} \, . \tag{15}$$

This is admittedly a rather low scale, but one can still hope that the typical expansion parameter $\alpha(\mu_0)/2\pi \sim 0.34 \pm 0.04$ makes perturbation theory meaningful. Since such an approach was first suggested [10] that is all one can do for the moment[1]. There are more uncertainties to Eq. (15). For instance, if the point for the quark model is defined by $G_1(\mu_G) = 0$, then at LO the scale is $\mu_G \sim 350{\rm MeV}$. The determination of μ_0 given by Eq. (15) is model independent. In these lectures we will show how this determination not only leads to a successful description of non-singlet pionic parton distribution functions in certain versions of the chiral quark model, but also that the number is in quantitative agreement with other determinations.

3. Chiral symmetry and chiral quark models

For the SU(2) up and down quarks, chiral symmetry is the invariance of the QCD Lagrangian under the transformations [6]

$$q(x) \quad \to e^{i\gamma} q(x) \, , \tag{16}$$

$$q(x) \quad \to e^{i\vec{\alpha}\cdot\vec{\tau}} q(x) \, , \tag{17}$$

$$q(x) \quad \to e^{i\vec{\beta}\cdot\vec{\tau}\gamma_5} q(x) \, , \tag{18}$$

[1] Actually, in the case of the nucleon these low scales produce negative gluon densities, if one takes $V_1(\mu = 2{\rm GeV}) = 0.40$ and hence violate positivity of parton distribution functions as well as unitarity of structure functions [14, 15].

where $q(x)$ represent Dirac spinor fields with $N_{\rm F}=2$ flavors and N_c colors where $\vec{\tau}$ are the isospin Pauli matrices. As a consequence of this symmetry there exist baryon, vector and axial Noether currents

$$J^{\mu}_{\rm B}(x) = \bar{q}(x)\gamma^{\mu}q(x)\,, \qquad \text{Baryon current} \tag{19}$$

$$J^{\mu,a}_{\rm V}(x) = \bar{q}(x)\gamma^{\mu}\frac{\tau_a}{2}q(x)\,, \qquad \text{Vector current} \tag{20}$$

$$J^{\mu,a}_{\rm A}(x) = \bar{q}(x)\gamma^{\mu}\gamma_5\frac{\tau_a}{2}q(x)\,, \qquad \text{Axial current} \tag{21}$$

respectively. Conservation of the Vector Current (CVC) and Partial Conservation of the Axial Current (PCAC) imply that

$$\partial_\mu J^{\mu}_{\rm B}(x) = 0\,, \tag{22}$$

$$\partial_\mu J^{\mu,a}_{\rm V}(x) = 0\,, \tag{23}$$

$$\partial_\mu J^{\mu,a}_{\rm A}(x) = m\bar{q}(x)\, i\gamma_5\tau_a\, q(x)\,, \tag{24}$$

with m denoting the average current quark mass (we neglect isospin breaking effects). A chiral quark model is any chirally invariant dynamical field theoretical model containing only explicit quark degrees of freedom and fulfilling the conservation laws (22), (23) and (24). Unfortunately, there is no such a thing as *the* chiral quark model. This is probably the reason why there exists a proliferation of models with many variants. Regardless of their differences all these models are characterized by the three ways of chiral symmetry breaking in QCD: explicit, spontaneous and anomalous. These are non trivial requirements, which imply constraints on the regularization methods in local models or equivalently on the high energy behavior of non-local models. In fact non-local models have been preferred because they provide a more natural explanation of the anomalous $\pi^0\to\gamma\gamma$ decay rate [16], but the calculations are cumbersome. In addition, they are formulated in Euclidean space and their extrapolation to compute matrix elements contributing to high energy processes is subtle as it is on the lattice. For instance, the calculation of structure functions requires either a continuation of the non-local model to Minkowki space or the determination of some moments in the Euclidean region and subsequent reconstruction of the parton distribution. While the first possibility turns out to be extremely difficult to explore, the second alternative cannot be used to pin down the $x\to 1$ which depends on the asymptotic behavior of the moments. On the contrary, local models, although less realistic, require specifying a suitable regularization which may be directly formulated in Minkowski space.

3.1. NJL Lagrangian

For the purpose of our discussion we think it is useful to review the SU(2) NJL model [8, 9]. This model has extensively been used in the past and there exist many reviews on the subject. So, although we aim at a self-contained discussion of pion properties, our presentation will be necessarily sketchy in order to provide the main ideas. Nevertheless, we will stress those points where differences with other authors become important. The NJL Lagrangian in Minkowski space is given by [8, 9]

$$\mathcal{L}_{\rm NJL} \ = \ \bar{q}(i\partial\!\!\!/ - m)q + \frac{G}{2}\left((\bar{q}q)^2 + (\bar{q}\vec{\tau}i\gamma_5 q)^2\right) \,, \tag{25}$$

where $q(x)$ is a Dirac spinor field with $N_c = 3$ colors and $N_{\rm F} = 2$ flavors, G is the coupling constant with dimension $[G] = M^{-2}$ and m the average mass of the current up and down quarks ($\sim 7\,{\rm MeV}$ at the scale $\mu \sim 1\,{\rm GeV}$). With the exception of the term with the current quark mass m, this Lagrangian is invariant under the $\mathrm{U}_B(1) \otimes \mathrm{SU}(2)_{\rm R} \otimes \mathrm{SU}(2)_{\rm L}$ chiral group, with a $\mathrm{U}(1)_B \otimes \mathrm{SU}(2)_V$ subgroup. Thus, the currents (19), (20) and (21), and their conservation laws (22), (23) and (24) are satisfied, as in QCD. A consequence of Poincaré invariance is the conservation of the energy momentum tensor

$$\theta^{\mu\nu} = \frac{i}{2}\bar{q}\left(\gamma^\mu \partial^\nu + \gamma^\nu \partial^\nu\right) q - g^{\mu\nu}\mathcal{L} \,, \tag{26}$$

which only contains quark degrees of freedom. Thus, we expect in this model that for any hadron, and in particular for the pion

$$\langle \pi | \theta^{\mu\nu} | \pi \rangle = 2p^\mu p^\nu \,. \tag{27}$$

Since the coupling constant, G, has a negative energy dimension, the Lagrangian (25) above is not renormalizable by power counting and it is usually interpreted as a theory with a finite cut-off, which we generically denote by Λ. This means that results depend crucially on the value of Λ and some fine tuning will be invoked. The need of a cut-off is not a serious problem from a physical point of view because we know that at high energies, much above the cut-off, the effective interaction should be replaced by the underlying QCD interactions in terms of explicit quarks and gluons. Roughly speaking, this feature is present in more sophisticated chiral quark models with non-local interactions.

The real problem derived from the high-energy suppression has to do with the fact that we want to preserve the symmetries of the Lagrangian, namely gauge and chiral symmetry, without removing the cut-off by taking the limit $\Lambda \to \infty$, *i.e.* keeping the high energy suppression. This implies that not every regularization, although it may accommodate some particular prejudices, may be considered as suitable. This point will be illustrated below. A full discussion on regularization methods in the NJL model can be found in Ref. [17].

In these lectures we will work, as usual, in the one-quark loop approximation. This is equivalent to the leading order of the large N_c expansion, taking GN_c = constant. In this way we comply not only with chiral symmetry constraints, but also with large N_c requirements. The study of higher orders in a $1/N_c$ expansion can be traced from Refs. [18–20].

3.2. Pauli–Villars regularization

In a cut-off theory most of the discussion is related to the regularization, and hence gets a bit technical. We will use throughout these lectures the Pauli–Villars (PV) regularization method [21] which has already been used for several applications in the NJL model, like ChPT [22], finite temperature [23], solitons [24], structure functions [25, 27, 28], correlation functions [26] and distribution amplitudes [29]. To our knowledge this is the most reliable regularization method of the NJL model so far fulfilling many desirable features, among others preserving gauge invariance and working directly in Minkowski space (see the recent discussion in Ref. [28]). The only difference with the standard PV method of QED is that in the NJL model this type of regularization is applied at the constituent quark mass level (see below). It is surprising that given the advantages of the method, it has been used so few times as compared with other methods such as Euclidean O(4) cut-off or Schwinger's Proper Time. A potential disadvantage of the method is that it provides a negative spectral strength due to the PV subtractions, and thus positivity of some physical quantities, like form factors or distribution functions might not be fulfilled. Although this is not excluded, in all the calculations presented, we explicitly see that this is not the case.

In the NJL model this method has been used mainly within the context of the bosonization or auxiliary field method (see *e.g.* Ref. [24]), but at the one loop level and for the processes we are considering here, it corresponds to the following replacement under the momentum integral:

$$\frac{1}{\not{k}_1 - M} \cdots \frac{1}{\not{k}_N - M} \to \sum_i c_i \left\{ \frac{\not{k}_1 + M}{k_1^2 - M^2 - \Lambda_i^2} \cdots \frac{\not{k}_N + M}{k_N^2 - M^2 - \Lambda_i^2} \right\} \quad (28)$$

c_i and Λ_i are the same suitable coefficients for all one loop graphs, fulfilling $c_0 = 1$ and $\Lambda_0 = 0$ and chosen in such a way as to make one loop integrals finite. In the graphs with an odd number of Dirac γ_5's the regulator has to be removed at the end of the calculation.

In the NJL there appear quadratic and logarithmic ultraviolet divergences. To illustrate the method let us consider the integral

$$I = -i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2} \,, \tag{29}$$

which is quadratically ultraviolet divergent, as can be seen by integrating first over k_0 component (using Feynman's $M^2 \to M^2 - i0^+$ prescription) and then over the $\vec{k}$ component. Using the PV subtractions, it becomes

$$I_{\rm PV} = -i \sum_i c_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2 - \Lambda_i^2} \,. \tag{30}$$

Evaluating the integral by any method, one sees that at least the following two conditions are required to render the one loop Feynman integrals finite

$$\sum_i c_i = 0 \,, \qquad \sum_i c_i \Lambda_i^2 = 0 \,. \tag{31}$$

Thus, at least two subtractions are needed. Solving the system of equations in terms of Λ_1 and Λ_2 provides two unknown parameters. To reduce them to only one the coincidence limit $\Lambda_1 \to \Lambda_2 = \Lambda$ is taken yielding the rule

$$\sum_i c_i f(\Lambda_i^2) = f(0) - f(\Lambda^2) + \Lambda^2 f'(\Lambda^2) \,. \tag{32}$$

This calculation already illustrates a very general feature of the PV method, calculations may be done directly in Minkowski space, and there is no need to go to Euclidean space. This is a computational advantage in the study of high energy processes as we will see below. To simplify the notation we will implicitly assume the use of such a regulator in what follows.

3.3. Chiral symmetry breaking

To illustrate the method let us consider the Dyson–Schwinger equation for the quark propagator, $S(p)$, (see Fig. 1).

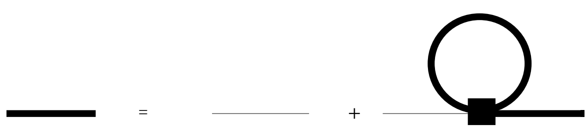

Fig. 1. Schwinger–Dyson equation for the quark propagator. The thick line represents the full propagator, whereas the thin line stands for the free quark propagator. The full blob is the irreducible two particle amplitude.

$$S(p) = S_0(p) + S(p)(-i) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[S(k)G\right] S_0(p)\,, \tag{33}$$

where $S(p)$ and $S_0(p)$ are the full and free quark propagators, respectively. G is the four-point kernel, which in the lowest order approximation becomes the coupling constant and can be taken out of the integral. Thus the full propagator acquires a constant self-energy, $\Sigma(p) = M$. Computing the Dirac, flavor and color traces one gets the so-called gap equation in the convenient form

$$M\left(1 - 4GN_{\mathrm{F}}N_c I\right) = m\,, \tag{34}$$

where Eq. (30) is understood for the integral I. For $m = 0$ there are two solutions (the Wigner alternative)

- Wigner phase. Corresponding to $M = 0$.
- Goldstone phase. $M \neq 0$ and hence

  $$1 = 4GN_{\mathrm{F}}N_c I\,, \tag{35}$$

  which is called the gap equation in the limit $m \to 0$ because it provides a gap of width $2M$ in the quark spectrum.

For $m \neq 0$ the gap equation is Eq. (34) and one has to choose the solution which goes to Eq. (35) for $m \to 0$.

3.4. Quark condensate

In the chirally broken phase, one has a quark condensate ($\langle\bar{q}q\rangle \equiv \langle\bar{u}u + \bar{d}d\rangle$)

$$\langle\bar{q}q\rangle = (-i)\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\frac{1}{\not{k} - M} = -8N_c M I = -\frac{M-m}{G} \neq 0\,. \tag{36}$$

The standard accepted value from QCD sum rules [30] is $\langle\bar{q}q\rangle_\mu = \langle\bar{u}u + \bar{d}d\rangle_\mu = -2(240 \pm 10\,\mathrm{MeV})^3$ at the scale $\mu = 1\,\mathrm{GeV}$. At the leading order in the QCD evolution one has

$$\frac{\langle\bar{q}q\rangle_\mu}{\langle\bar{q}q\rangle_{\mu_0}} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\frac{\gamma_{\bar{q}q}}{2\beta_0}}\,, \qquad \gamma_{\bar{q}q} = 8\,. \tag{37}$$

If we use the scale of Eq. (15) we get an enhancement factor ~ 2 and hence

$$\langle\bar{q}q\rangle_{\mu_0} = -(380 \pm 20\mathrm{MeV})^3\,. \tag{38}$$

According to our point of view it is this value that should be compared to the model calculation, and not to the one at $\mu = 1\,\mathrm{GeV}$. Of course, one may argue that LO evolution is not sufficient to go to such low scales. All we can do to check the consistency of the approach is to provide, as we do below, other determinations of the scale μ_0.

3.5. The pion as quark–antiquark bound state

In the spontaneously broken phase, the constituent quark mass, $M \neq 0$. As a consequence of Goldstone's theorem there must exist massless pseudoscalar particles, which one identifies with the pions. This can actually be obtained by solving the Bethe–Salpeter equation in the pseudoscalar–isovector channel for quark–antiquark scattering (see Fig. 2) and checking

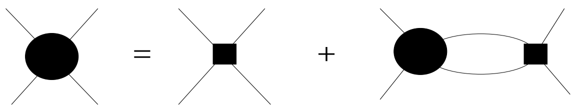

Fig. 2. Bethe–Salpeter equation for quark–antiquark scattering. The square blob represents the two particle irreducible quark–antiquark potential, and the round blob is the full T-matrix. The lines correspond to constituent quarks (thick lines in Fig. (1)).

that there is a bound state pole which becomes massless as the current quark mass goes to zero, $m \to 0$. In the lowest order approximation, where the kernel is approximated by a constant, the Bethe–Salpeter amplitude for π^+ becomes

$$T_{\alpha\beta;\gamma\delta} = \left(\gamma_5 \tau^+\right)_{\alpha\gamma} \left(\gamma_5 \tau^+\right)_{\beta\delta} t(P)\,, \tag{39}$$

where $t(P)$ is a number depending only on the total momentum, given by

$$t(P) = G + G(-i) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\gamma_5 \frac{1}{\not{k} - M} \gamma_5 \frac{1}{\not{P} - \not{k} - M}\right] t(P)\,. \tag{40}$$

In writing the polarization operator, a particular choice of momentum routing has been made, but in a gauge invariant regularization such as PV, one may safely shift the integration variable by any amount of the external momentum P. Euclidean O(4) cut-offs do not preserve this property. Using the PV regulators one gets

$$t(p)^{-1} = G^{-1} - 8N_c I + 4N_c p^2 F\left(p^2\right)\,, \tag{41}$$

where I is defined by Eq. (29) and the one-loop function, $F(p^2)$, is given by

$$F(p^2) = (-i) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - M^2} \frac{1}{(k-p)^2 - M^2}\,. \tag{42}$$

Using the Feynman trick for the two propagators, this function can be rewritten as

$$F(p^2) \;=\; \int_0^1 dx F(p^2, x)\,, \tag{43}$$

where

$$F(p^2,x) = (-i)\int \frac{d^4k}{(2\pi)^4}\frac{1}{\left[k^2-M^2+x(1-x)p^2+i\varepsilon\right]^2}$$
$$= -\frac{1}{(4\pi)^2}\sum_i c_i \log\left[M^2+\Lambda_i^2-x(1-x)p^2\right] . \qquad (44)$$

In the chiral limit, $m=0$, the gap equation (35) can be used, and the first terms in the r.h.s. cancel, producing a massless quark–antiquark bound state. However, for $p^2 > 4M^2$, the function $F(p^2)$, and hence the t-matrix, develops an imaginary part indicating a lack of confinement. For $m_\pi^2 \ll 4M^2$ we get a deeply bound state, and thus hope confinement not to be essential. For $M = 300\text{MeV}$ one has $m_\pi^2/(4M^2) \sim 0.06$.

For a finite quark mass we have a pole at $p^2 = m_\pi^2$,

$$\frac{m}{GM} = 4N_c m_\pi^2 F\left(m_\pi^2\right) . \qquad (45)$$

The pion coupling of a composite pion state to a quark–antiquark pair is given by the residue of the t-matrix at the pion pole (see Fig. 3)

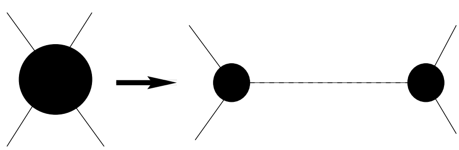

Fig. 3. Pion pole contribution to the Bethe–Salpeter quark–antiquark scattering amplitude. The small solid blob represents the Bethe–Salpeter pion quark–antiquark amplitude. Solid lines are constituent quarks.

$$g_{\pi qq}^{-2} = \frac{dt^{-1}(p^2)}{dp^2}\Big|_{p^2=m_\pi^2} = 4N_c\frac{d\left[p^2F(p^2)\right]}{dp^2}\Big|_{p^2=m_\pi^2} . \qquad (46)$$

So that the composite canonically quantized pion propagator is

$$D(p^2) = \frac{t(p^2)}{g_{\pi qq}^2} \to \frac{1}{p^2-m_\pi^2} , \qquad (p^2\to m_\pi^2) . \qquad (47)$$

3.6. Pion weak decay

Using the bound state solution of the Bethe–Salpeter equation for the pion, one can compute the pion weak decay defined as

$$\langle 0|J_{\rm A}^{\mu,b}(0)|\pi^a(p)\rangle = i f_\pi p^\mu \delta^{ab} \tag{48}$$

yielding (see Fig. 4)

$$f_\pi = 4N_c M g_{\pi qq} F(m_\pi^2) \tag{49}$$

and thus

$$g_{\pi qq} f_\pi = M \frac{F(m_\pi^2)}{\left[m_\pi^2 F(m_\pi^2)\right]'}\,, \tag{50}$$

which, in the chiral limit yields the Goldberger–Treiman relation at the quark level, $g_{\pi qq} f_\pi = M$. Combining Eqs. (36), (45) and (49) one gets

$$-m\langle \bar{q}q\rangle = m_\pi^2 f_\pi^2 \left(\frac{M-m}{g_{\pi qq} f_\pi}\right)\,, \tag{51}$$

which at the lowest order in the chiral expansion becomes the well known Gell-Mann–Oakes–Renner relation.

Eqs. (49) and (46) are usually employed to fix the parameters in the broken phase, taking for $f_\pi = 92$MeV and $m_\pi = 139.6$MeV. For a given constituent quark mass, M, one can compute the cut-off Λ. The gap equation allows to determine the coupling constant G. For instance, using the PV regularization given by Eq. (32), for $M = 280$MeV one obtains $\Lambda = 871$MeV and $\langle \bar{q}q\rangle = -(290\text{MeV})^3$. From this number one may deduce that, since it resembles the numerical value of the condensate at the scale $\mu = 1$GeV, the model scale is around 1GeV. Of course, the numerical values may change in a different version of the PV scheme (like *e.g.* with $\Lambda_1 \neq \Lambda_2$, or adding more terms in the sum). Fortunately, as we will see below, the results for some high energy properties are insensitive to the choice of parameters, up to unimportant chiral corrections.

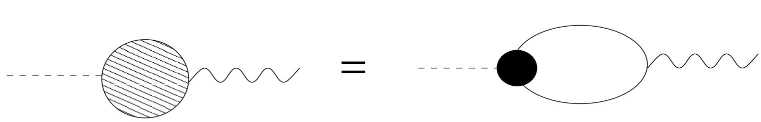

Fig. 4. Pion (dashed line) weak decay into an axial current (wavy line). The solid blob represents the Bethe–Salpeter pion quark–antiquark amplitude. Solid lines are constituent quarks.

3.7. Pion wave function

In principle, the Bethe–Salpeter pion–quark–antiquark wave function is given by (we take π^+)

$$\chi_P(k) = \frac{i}{\not{P}/2 + \not{k} - M} \left(-g_{\pi qq}\gamma_5 \tau^+\right) \frac{i}{\not{P}/2 - \not{k} - M} \tag{52}$$

but PV regularization has to be understood because, at least formally,

$$f_\pi P_\mu = (-i) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\chi_P(k) \frac{\tau^+}{2} \gamma^\mu \gamma_5\right) \tag{53}$$

and the integral is logarithmically divergent if Eq. (52) is taken literally. This problem is typical in chiral quark models and has been the cause of much confusion. Regularization is only easily applied at the level of closed quark lines[2]. We will see more of this later, but it has always been a weak point in quark-loop calculations. One obvious choice to regularize open lines is to enforce consistency with the closed lines, although at first sight this procedure looks a bit arbitrary. Another possibility to open a regularized quark line, but starting from a closed quark line, is to consider a physical process involving photons and pions in the high energy limit and extract the leading power behavior. Prominent examples which will be analyzed below are deep inelastic scattering from which the pion structure functions can be deduced in the Bjorken limit or the pion transition $\gamma^* \to \pi^0\gamma$-form factor in the limit of high photon virtualities and fixed photon asymetry from which the pion distribution amplitude may be derived.

3.8. Pion electromagnetic form factor

The pion electromagnetic form factor is probably the simplest case where one can illustrate some of the points we want to make. For a charged pion $\pi^+ = u\bar{d}$ the electromagnetic form factor is defined as

$$\langle \pi^+(p') | J_\mu^{\mathrm{em}}(0) | \pi^+(p) \rangle = e\Gamma_\mu^{\mathrm{em}}(p', p) = (p^\mu + p'^\mu) e F_\pi^{\mathrm{em}}(q^2) \tag{54}$$

and can be computed using Fig. 5. For on-shell pions and in the chiral limit the result is rather simple

$$F_\pi^{\mathrm{em}}(q^2) = \frac{4N_c M^2 F(q^2)}{f_\pi^2}\,, \tag{55}$$

[2] This is actually a good reason to use bosonization schemes; there the concept of an open quark line does not appear.

Fig. 5. Electromagnetic pion form factor in the NJL model. The closed loops are Pauli–Villars regularized. Wavy lines are photons, solid lines are constituent quarks and dashed lines composite pions.

where $F(p^2)$ is given by Eq. (42). The pion form factor is obviously properly normalized, $F_\pi^{\rm em}(0)=1$, as it should be due to the gauge invariance of the Pauli–Villars regularization. The mean-square pion radius reads

$$\langle r^2 \rangle_\pi^{\rm em} = -6 \frac{dF_\pi^{\rm em}(q^2)}{dq^2}\Big|_{q^2=0} = \frac{N_c}{4\pi^2 f_\pi^2} \sum_i c_i \frac{M^2}{M^2+\Lambda_i^2} , \tag{56}$$

a result explicitly depending on the regularization. Numerically one has a reduction of about 25% for $M = 300$MeV due to the finite cut-off effect, $(\langle r^2 \rangle_\pi^{\rm em})^{1/2} = (0.58\,{\rm fm}) \times 0.75 = 0.50\,{\rm fm}$, to be compared with the experimental number, 0.66 fm. Two points are worth stressing here. Firstly, it is clear that the finite cut-off corrections go in the wrong direction. On the other hand, one should not expect a perfect agreement with the experimental number, since one expects a sizeable contribution from the pion loops ($1/N_c$ corrections in the model), as suggested by ChPT [5]. There is a way one can get rid of the cut-off corrections; one can simply split the charge contribution at $q^2=0$, which should be one, and compute the difference using a unregularized quark loop. This is equivalent to write down a once subtracted dispersion relation for $F_\pi^{\rm em}(q^2)$. For the unregularized quark loop only, one has the asymptotic behavior for $q^2 \to -\infty$

$$F_\pi^{\rm em}(q^2) \sim \log\left(\frac{-q^2}{M^2}\right) , \tag{57}$$

without power corrections. For the PV regularized NJL model, at high Euclidean momentum one has instead a cut-off independent relation, which is a pure power correction[3],

$$q^2 F_\pi^{\rm em}(q^2) \to -\frac{M\langle \bar q q\rangle}{2f_\pi^2} \sim 0.34\,{\rm GeV}^2 \tag{58}$$

for $M = 300$MeV and the PV method, Eq. (32). Up to the measured momentum transfers, $-2\,{\rm GeV}^2 < q^2 < -6\,{\rm GeV}^2$ this value is very similar to the experimental average number 0.38 ± 0.04 [31]. However, it is in principle

[3] For a Euclidean O(4) cut-off one gets a logarithmic behavior $\log(Q^2/\Lambda^2)/Q^2$ [33].

not clear whether this number should directly be compared to experiment, without taking into account QCD radiative corrections. Another point of view is to reject the validity of the model to such high energies, matching instead to the known QCD result at LO [32] (see also Ref. [4])

$$q^2 F_\pi^{\rm em}(q^2) \to 16\pi\alpha(-q^2) f_\pi^2 \,, \tag{59}$$

which for $q^2 = -4{\rm GeV}^2$ yields, 0.13, a too small value as compared to the experimental one[4]. We may identify both coefficients at the model scale, μ_0, yielding the result

$$\frac{\alpha(\mu_0)}{2\pi} = -\frac{M_{\mu_0}\langle\bar{q}q\rangle_{\mu_0}}{64\pi^2 f_\pi^4} \,. \tag{61}$$

In this relation we have stressed the fact that also the constituent quark mass is fixed at the model scale, μ_0. Actually, we may use relations (61) and (15) to get

$$M_{\mu_0} = 300 \pm 80{\rm MeV} \,. \tag{62}$$

Although this value should be considered a crude estimate it provides a *reasonable* value for the constituent quark mass at the model scale. Conversely, if we assume $M = 300{\rm MeV}$ then $\alpha(\mu_0)/(2\pi) \sim 0.5 \pm 0.2$, a compatible value with the momentum fraction estimate.

4. Pion distribution functions

4.1. Deep inelastic scattering

Inclusive lepton–hadron scattering is described in terms of the hadronic tensor, $W_{\mu\nu}(p,q)$, and can be obtained from the imaginary part of the forward Compton amplitude for virtual photons as follows [11]

$$\begin{aligned} W_{\mu\nu}(p,q) &= \frac{1}{2\pi}{\rm Im}\, T_{\mu\nu}(p,q) = W_1(q^2, p\cdot q)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \\ &+ \frac{W_2(q^2,p\cdot q)}{m_\pi^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) \,, \end{aligned} \tag{63}$$

[4] Actually, this expression can be obtained from

$$-Q^2 F_\pi^{\rm em}(-Q^2) \to 16\pi\alpha\left(Q^2\right) f_\pi^2 \left(\int_0^1 dx\, \frac{\varphi_\pi(x,Q)}{6x(1-x)}\right)^2 \tag{60}$$

with $\varphi_\pi(x,\mu)$ the pion distribution amplitude at the scale μ, when the asymptotic wave function, $\varphi_\pi(x,\infty) = 6x(1-x)$ is substituted [4] (see also below).

where

$$T_{\mu\nu}(p,q) = i\int d^4x e^{iq\cdot x}\left\langle \pi(p)|T\left\{J_\mu^{\rm em}(x)J_\nu^{\rm em}(0)\right\}|\pi(p)\right\rangle . \tag{64}$$

Here, $m_{\rm P}^2 = p^2$ is the mass of the pseudoscalar meson and q is the momentum of the virtual photon. In the Bjorken limit, one obtains [11] ($x = -q^2/(2p\,q)$),

$$W_1(x,Q^2) \to F_1(x)\,, \tag{65}$$

$$W_2(x,Q^2) \to F_2(x)\,, \tag{66}$$

where the Callan–Gross relation $F_2(x) = 2xF_1(x)$ is fulfilled, as a consequence of the spin 1/2 nature of quarks. The structure function is given by

$$F_1(x) = \frac{1}{2}\sum_{i=u,d} e_i^2\left[\bar{q}_i(x) + q_i(x)\right]. \tag{67}$$

where $e_u = 2/3$ and $e_d = -1/3$ are the quark charges and $q_i(x)$ and $\bar{q}_i(x)$ are the momentum fraction distribution of the different quark species.

Introducing Light Cone (LC) coordinates $x = (x^+, x^-, \vec{x}_\perp)$ with $x^\pm = x^0 \pm x^3$ the analysis of Ref. [11] yields

$$F_1(x) = -\frac{i}{4\pi}\int \frac{dk^- d^2\vec{k}_\perp}{(2\pi)^3}{\rm tr}\left[\hat{Q}^2\gamma^+\chi(p,k)\right]\Big|_{k^+=p^+=m_\pi x}\,, \tag{68}$$

where $\hat{Q} = {\rm diag}(e_u, e_d)$ is the charge operator and the forward quark–target scattering amplitude is defined

$$\chi(p,k) = -i\int d^4\xi\, e^{i\xi\cdot k}\langle p|T\{q(\xi)\bar{q}(0)\}|p\rangle\,. \tag{69}$$

$\chi(p,k)$ corresponds to the unamputated vertex. Eq. (68) holds under the assumption of scaling and finiteness in the Bjorken limit.

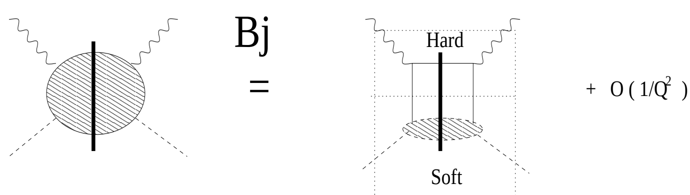

Fig. 6. The Compton amplitude in the Bjorken limit and its relation with the quark–target scattering in the parton model. The high and low energy contributions are also explicitly displayed.

Logarithmic scaling violations due to perturbative QCD radiative corrections [3], relate in a linear fashion the leading twist contribution to structure functions at a given reference scale, μ_0, to the scale of interest, μ, (in DIS one takes $Q^2 = \mu^2$)

$$F_1(x,\mu) = U(\mu,\mu_0)F_1(x,\mu_0)\,, \tag{70}$$

where Eq. (4) is satisfied for the linear matrix integral operator U. Asymptotically, QCD predicts [3]

$$F_2(x,Q^2) = 2xF_1(x,Q^2) \to \frac{\sum_i e_i^2}{1+2(N_c^2-1)/N_{\rm F}}\,\delta(x)\,. \tag{71}$$

Actually, the pion structure functions cannot be measured directly since there are no pion targets. Instead, the parton structure of the pion has been analyzed [13] and a simple parameterization at $Q^2 = 4{\rm GeV}^2$ has been given. The valence quark distributions extracted in Ref. [13] from Drell–Yan experiments [34] seem well determined, whereas the gluon distribution as obtained from the analysis of prompt photon emission data [35] is less well determined. A recent analysis [36] of the ZEUS dijet data seems to favor the gluon distributions of Ref. [13]. This is why, in determining the low–energy scale of our model, we use the valence momentum fraction of 47% found in Ref. [13] at $4{\rm GeV}^2$ instead of 40 % found in Ref. [52, 53] using some additional assumptions [54].

The results found long ago [37] and usually adopted in DIS calculations are the so-called counting rules. They establish that there is a relation between the asymptotic behavior of the electromagnetic pion form factor for large Euclidean momenta and the structure function behavior as $x \to 1$, namely

$$F_\pi^{\rm em}(q^2) \sim 1/(q^2)^n\,, \qquad\qquad F_1(x) \sim (1-x)^{2n-1}\,. \tag{72}$$

In QCD $F_\pi^{\rm em}(q^2)$ has a logarithmic radiative correction, Eq. (59) so we may take $n \sim 1$, which implies $F(x) \sim 1-x$. Although it is not clear at what scale μ the counting rules are valid, recent investigations based on quark–hadron duality confirm the exponent $n \sim 1$ for $F(x)$ at $\mu \sim 2{\rm GeV}$ [38].

4.2. Spectator model and quark loop calculations

The spectator model is a very simple model where one can illustrate the source of the problems in this kind of calculations and the relation of gauge invariance and normalization. Actually, in the case of the pion such a model corresponds to a unregularized NJL calculation. The spectator model consists of replacing the full sum of intermediate states in the soft piece of the Compton amplitude by a spectator particle, which for the pion has quark quantum numbers [39]. All what is required to do the calculation is to know the $\pi q\bar{q}$-coupling. We take a pseudoscalar coupling as follows

$$\Gamma^a_{\pi qq} = -\gamma_5 \tau^a g_{\pi qq} \, . \tag{73}$$

At the level of the Compton scattering amplitude it corresponds to the well-known hand-bag diagram. Straightforward calculation of the unregularized structure function yields in the Bjorken limit

$$\begin{aligned} F_1(x,Q^2) \;\to\; & \frac{1}{2}\left(e_u^2+e_d^2\right)\frac{4N_c g_{\pi qq}^2}{(4\pi)^2} \\ & \times\left\{-\log\left[\frac{M^2-x(1-x)m_\pi^2}{Q^2}\right]+\frac{m_\pi^2 x(1-x)}{M^2-x(1-x)m_\pi^2}\right\}, \end{aligned} \tag{74}$$

which is a scaling violation quite different from those found in QCD. Actually, we get $F_2(x,\infty)=\infty$ instead of Eq. (71). Scaling can be restored by attaching a form factor to the πqq vertex or putting a O(4) cut-off in the loop. No solution is really free of problems, since either gauge invariance is violated or extra singularities are introduced.

At the level of the quark–target scattering amplitude the spectator picture corresponds to a single constituent quark in an intermediate state. The result for a pseudoscalar coupling is

$$F_1(x) = \frac{1}{2}\left(e_u^2+e_d^2\right)\frac{4N_c g_{\pi qq}^2}{4\pi}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{k_\perp^2+M^2}{\left[k_\perp^2+M^2-x(1-x)m_\pi^2\right]^2}\,. \tag{75}$$

However, we see that the integral in $k_\perp$ is actually logarithmically divergent. This only reflects the fact that for the unregularized quark loop the separation between soft and hard processes involved in the Bjorken limit takes place at $k_\perp^2 \sim Q^2$. Obviously a form factor for the vertex or a transverse cut-off, $|k_\perp| \le \Lambda$ (both Q^2 independent), may be introduced yielding a finite result.

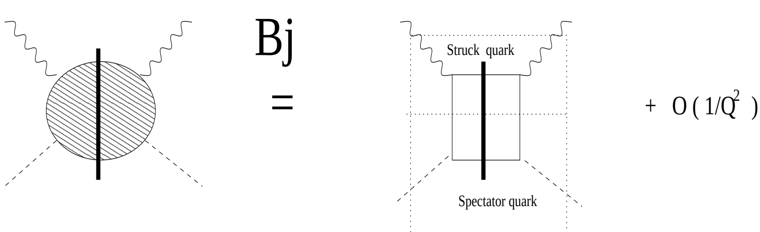

Fig. 7. Spectator model. The sum over intermediate states in the soft hadronic contribution is replaced by a single quark state.

Thus, we learn in this particular model that the connection between the Compton amplitude and the quark–target scattering formulas is only justified provided everything is finite. So, some high energy suppresion should be applied, with the help of a regularization. The problem is that we expect to do it in a gauge invariant manner, since after all we are dealing with an electromagnetic process.

4.3. PDF calculation from the Compton amplitude

In the PV regularized NJL model, the gauge invariant forward virtual Compton scattering amplitude is given the sum of the hand-bag diagram, the crossed contribution and the $\gamma\pi \to \pi* \to \pi\gamma$ involving the off-shell electromagnetic pion form factor (see Fig. (8)). Actually, it can be shown that in the Bjorken limit only the hand-bag contribution survives, since the crossed and pion off-shell contributions are higher twist. This result has been obtained in Refs. [25, 27] and turns out to coincide with applying PV regularization, Eq. (31), to Eq. (74). The result is

$$F_1(x) = \frac{1}{2}\Big(e_u^2 + e_d^2\Big) 4N_c g_{\pi qq}^2 \frac{d}{dm_\pi^2}\Big[m_\pi^2 F(m_\pi^2, x)\Big] , \tag{76}$$

where the function $F(p^2, x)$ is given by Eq. (44). Taking π^+ for definiteness, one gets in the chiral limit

$$u_{\pi^+}(x) = \bar{d}_{\pi^+}(x) = 1 , \qquad 0 < x < 1 . \tag{77}$$

This gives the following result for the valence-quark distribution function

$$V(x) = 2 , \qquad 0 < x < 1 . \tag{78}$$

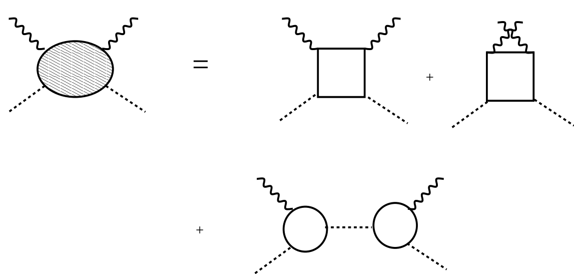

Fig. 8. Gauge invariant Compton amplitude in the NJL model. The closed loops are Pauli–Villars regularized. Wavy lines are photons, solid lines are constituent quarks and dashed lines composite pions.

As we see the result is independent of the PV regulators and has proper support, normalization and fulfills the momentum sum rule. Moreover, Eq. (78) is consistent with chiral symmetry, in the sense that it has been obtained taking explicitly into account the boson Goldstone nature of the pion. The result has also been obtained within the NJL model imposing a transverse cut-off [40] upon the quark–target amplitude (75).

Eq. (78) disagrees with other quark loop calculations. If Eq. (64) is used with a four-dimensional cut-off [41] or Eq. (68) is used with Lepage–Brodsky regularization [40] in the NJL model, different shapes for the quark distributions are obtained. The null-plane [42], NJL model [41], spectator model [39] and the recent quark loop [48] calculations also produce different results. In all cases, the use of momentum dependent form factors or non-gauge invariant regularization make the connection between Eq. (64) and Eq. (68) doubtful and, furthermore, spoil normalization. The results based on a quark loop with momentum dependent quark masses [49, 50] seem to produce a non-constant distribution.

4.4. PDF calculation in light cone coordinates

Perhaps the simplest way to obtain PDF for the pion in the NJL model is by using the pion Bethe–Salpeter propagator. The probability distribution is defined through the pion expectation value of the number operator corresponding to a quark of a given species. Taking into account the proper normalization factors one gets at the pion pole the following expression for the probability distribution of finding a valence quark with momentum fraction x and transverse momentum $\vec{k}_\perp$

$$\frac{1}{2}V_\pi(x,k_\perp) = \int dk^- D(p,k)\Big|_{k^+=m_\pi x, p^2=m_\pi}\,, \tag{79}$$

where the Bethe–Salpeter normalization in the Pauli–Villars regularization scheme is

$$D(p,k) = 4N_c i g^2_{\pi qq} \times \frac{d}{dp^2}\left\{p^2 \sum_i c_i \frac{1}{k^2 - M^2 - \Lambda_i^2 + i\varepsilon}\,\frac{1}{(k-p)^2 - M^2 v - \Lambda_i^2 + i\varepsilon}\right\}\Bigg|_{p^2=m_\pi^2}. \tag{80}$$

Introducing light cone variables

$$k^+ = k^0 + k^3\,, \qquad k^- = k^0 - k^3\,, \qquad d^4k = \frac{1}{2}dk^+ dk^- d^2k_\perp\,. \tag{81}$$

Employing the definition (79) one finds

$$\frac{1}{2}V_\pi(x,\vec{k}_\perp) = -\frac{2iN_c g_{\pi qq}^2}{f_\pi}\int \frac{dk^+dk^-}{(2\pi)^4}\frac{1}{x(1-x)}$$
$$\times \frac{d}{dm_\pi^2}\left\{\sum_j c_j \frac{\delta\left(k^+ - xp^+\right)}{k^- - m_\pi - \frac{\vec{k}_\perp^2+M^2+\Lambda_j^2+i0^+}{m_\pi(1-x)}}\,\frac{1}{k^- - \frac{\vec{k}_\perp^2+M^2+\Lambda_j^2+i0^+}{m_\pi x}}\right\},$$

where the location of poles in the k^- variable has been explicitly displayed. For $x>1$ or $x<0$ both poles are above and below the real axis, respectively, and hence the integral vanishes in either case. For $0<x<1$ the integral yields

$$\frac{1}{2}V_\pi(x,k_\perp) = \frac{4N_c g_{\pi qq}^2}{16\pi^3}\frac{d}{dm_\pi^2}\left\{m_\pi^2 \sum_i c_i \frac{1}{k_\perp^2+m^2+\Lambda_i^2-m_\pi^2 x(1-x)}\right\}. \quad (82)$$

Notice that, due to the Pauli–Villars subtractions, we have the asymptotic behavior in the transverse momentum $k_\perp$,

$$\frac{1}{2}V_\pi(x,k_\perp) \sim \frac{4N_c g_{\pi qq}^2}{16\pi^3}\frac{\sum_i c_i \Lambda_i^4}{k_\perp^6}. \quad (83)$$

This guarantees the convergence of the $k_\perp$ integral without introducing a transverse cut-off. Thus, the Pauli–Villars regulators automatically provide a (gauge-invariant) form of a transverse cut-off. Integrating the transverse momentum we get the PDF

$$\frac{1}{2}V_\pi(x) = \int d^2k_\perp \frac{1}{2}V_\pi(x,\vec{k}_\perp) \quad (84)$$
$$= 4N_c g_{\pi qq}^2 \frac{d}{dm_\pi^2}\left[m_\pi^2 F(m_\pi^2,x)\right], \quad (85)$$

which is the result found in Ref. [25] (see also Eq. (75) above). In the chiral limit, $m_\pi=0$, one can use the Goldberger–Treiman relation for the constituent quarks, $g_{\pi qq} f_\pi = M$. Then $f_\pi^2 = 4N_c M^2 F(0)$, which gives the very simple formulas

$$\frac{1}{2}V_\pi(x,\vec{k}_\perp) = \frac{4N_c M^2}{16\pi^3 f_\pi^2}\sum_i c_i \frac{1}{k_\perp^2+\Lambda_i^2+M^2}, \quad (86)$$
$$\frac{1}{2}V_\pi(x) = 1. \quad (87)$$

In the chiral limit $V_\pi(x,\vec{k}_\perp)$ becomes trivially factorizable, since it is independent of x. A remarkable feature is that the last relation, Eq. (87), is independent of the PV regulators.

The light cone interpretation has been pursued (see *e.g.* [42] and references therein) and more recently [40,67] within a LC quantization. In these cases transverse cut-off's were introduced, *a posteriori*. As we have shown above this is not necessary in the PV regularization.

4.5. QCD evolution

Due to radiative corrections, parton distribution functions evolve logarithmically with scale through the DGLAP equations [3]. Non-singlet (or valence) distribution functions are easily evolved in terms of their Mellin moments. For $\pi^+ = u\bar{d}$ one defines

$$V(x) = u(x) - \bar{u}(x) + \bar{d}(x) - d(x) \tag{88}$$

and takes (we assume chiral limit for simplicity)[5]

$$\frac{1}{2} V_\pi(x,\mu_0) = 1\,. \tag{89}$$

At leading order, evolution reads

$$V_n(\mu) \equiv \int\limits_0^1 dx x^n V(x,\mu) = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_n^{\rm NS}/2\beta_0} \int\limits_0^1 dx x^n V(x,\mu_0)\,, \tag{90}$$

where the anomalous dimension is

$$\gamma_n^{\rm NS} \;=\; -2C_{\rm F}\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{k=1}^{n+1}\frac{1}{k}\right], \tag{91}$$

with $C_{\rm F} = 4/3$. Taking n as a complex number, which also requires an analytical continuation of both $V_n(\mu_0)$ and $\gamma_n^{\rm NS}$, Eq. (90) can be inverted using

$$V(x,\mu) = \int\limits_{c-i\infty}^{c+i\infty} \frac{dn}{2\pi i} x^{-n} V_n(\mu)\,, \tag{92}$$

[5] For non experts it may sound unnatural to take the high energy, $Q^2 \to \infty$, limit in a model and use it as initial condition for QCD evolution at a low scale μ_0. The point is that if the quark model scales, the asymptotic behavior can be separated according to increasing power corrections in $1/Q^2$ in a twist expansion. The anomalous dimensions relevant for QCD evolution of structure functions are only known for the lowest orders of such an expansion, but not for the full structure function. Using Eq. (89) as initial condition automatically complies with the asymptotic result of Eq. (71).

where c has to be chosen as to leave all the singularities on the left hand side of the contour. The result for the pion valence PDF found in Ref. [25] at LO and [28] at NLO is displayed in Fig. 9 at $\mu^2 = Q^4 = 4\mathrm{GeV}^2$ compared with phenomenological analysis for the pion [13]. As one can see, the agreement is quite impressive. Despite the fact that $\alpha(\mu_0)/(2\pi) \sim 0.3$ the differences between LO and NLO turn out to be small.

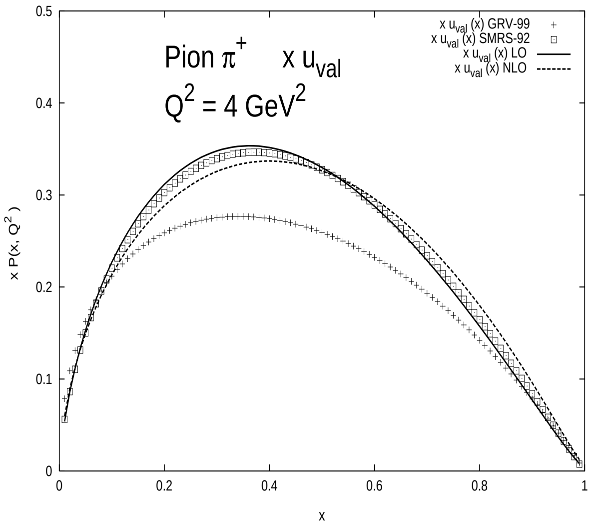

Fig. 9. Valence distributions in the pion at $Q^4 = 4\mathrm{GeV}^2$ at LO [25] and NLO [28] compared with phenomenological analysis for the pion SMRS92 [13], GRV99 [53]. We take $\langle xV\rangle_\pi = 0.47$ at $Q^2 = 4\mathrm{GeV}^2$ [13].

It is interesting to analyze these results in the light of the counting rules, Eq. (72), of Ref. [37], as was already done in Ref. [55]. Using the inverse Mellin formula, Eq. (92), it can be shown that, if $V(x,\mu_0) \to C(1-x)^N$ then

$$V(x,\mu) \to C(1-x)^{N-\frac{4C_{\mathrm{F}}}{\beta_0}\ln\frac{\alpha(\mu)}{\alpha(\mu_0)}} \qquad x \to 1\,. \tag{93}$$

The value of the additional contribution to the exponent is a weakly depending function for $\mu > 1\mathrm{GeV}$. For the pion we find, using Eq. (15),

$$V(x,2\mathrm{GeV}) \to 2(1-x)^{1.1\pm 0.1} \qquad x \to 1\,, \tag{94}$$

which is consistent with the counting rules and the phenomenological analysis [13], *after evolution*. A more detailed account of the sea and gluon distribution functions at LO and NLO and also for K and η mesons can be found in Ref. [28].

5. Pion distribution amplitude

5.1. Pion transition form factor

The matrix element for the $\gamma^* \to \pi^0\gamma$ transition form factor is defined

$$\Gamma^{\mu\nu}_{\pi^0,\gamma,\gamma}(p,q_1,q_2) = \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta F_{\pi\gamma\gamma}(p,q_1,q_2)\,, \tag{95}$$

where $F_{\pi\gamma\gamma}(p,q_1,q_2)$ is the π^0 electromagnetic transition form factor [4]. Two QCD results apply for this transition form factor. In the chiral limit and for on-shell pions, $p^2=0$, and photons, $q_1^2=q_2^2=0$, corresponding to the $\pi^0 \to 2\gamma$ decay one gets the normalization condition

$$F_{\pi\gamma\gamma}(0,0,0) = \frac{1}{4\pi^2 f_\pi}\,, \tag{96}$$

which is the standard result expected from the chiral anomaly. The definition of the leading twist contribution to the Pion Distribution Amplitude (PDA), $\varphi_\pi(x)$, from the transition form factor is

$$\begin{aligned}\lim_{Q^2\to\infty} Q^2 F_{\pi\gamma\gamma}(p,q_1,q_2)\Big|_{q_1^2=-\frac{(1+\omega)}{2}Q^2,\, q_2^2=-\frac{(1-\omega)}{2}Q^2} \\ = \frac{4f_\pi}{N_c}\int_0^1 dx \frac{\varphi_\pi(x)}{1-\omega(2x-1)}\,,\end{aligned} \tag{97}$$

where $\omega \equiv (q_1^2-q_2^2)/(q_1^2+q_2^2)$ is the photon asymmetry, $-1<\omega<+1$, which is kept finite when taking the limit $Q^2\to\infty$. It can be shown that in the parton model the PDA can be computed by

$$\varphi_\pi(x) = \int d^2k_\perp \Psi_\pi(x,k_\perp)\,, \tag{98}$$

where the pion light-cone wave function (pseudoscalar component) is defined as the low-energy matrix element [43–45]

$$\begin{aligned}\Psi_\pi(x,\vec{k}_\perp) &= -\frac{i\sqrt{2}}{4\pi f_\pi}\int d\xi^- d^2\xi_\perp e^{i(2x-1)\xi^- p^+ - \xi_\perp\cdot k_\perp} \\ &\quad\times \langle\pi^+(p)|\bar{u}(\xi^-,\xi_\perp)\gamma^+\gamma_5 d(0)|0\rangle\,,\end{aligned} \tag{99}$$

where $p^\pm = m_\pi$ and $\vec{p}_\perp = \vec{0}_\perp$. An important relation found in Ref. [4] reads

$$f_\pi \int_0^1 dx \Psi_\pi(x,0_\perp) = F_{\pi\gamma\gamma}(0,0,0)\frac{N_c}{\pi}\,. \tag{100}$$

Radiative logarithmic corrections to the pion distribution amplitude can be easily implemented through the QCD evolution equations [4], which yield for $Q^2 \to \infty$ the asymptotic wave function of the form

$$\varphi_\pi(x,\infty) = 6x(1-x)\,. \tag{101}$$

Moreover, the pion transition form factor has been recently measured by the CLEO collaboration [56] and a theoretical analysis of PDA based on these data and light-cone sum rules has been undertaken [57], showing that at $Q = 2.4\,\mathrm{GeV}$ PDA is neither asymptotic, nor possesses the structure proposed in early works [58].

The pion distribution amplitude has been evaluated with QCD sum rules [59], in standard [60] (only the second ξ-moment) and transverse lattice approaches [61–63], and in chiral quark models [64–72]. In chiral quark models the results are not always compatible to each other, and even their interpretation has not always been the same. While in some cases there are problems with chiral symmetry and proper normalization [64,65,69], in other cases [66–72] it is not clear how to associate the scale at which the model is defined, necessary to define the starting point for the QCD evolution. The fact that several calculations [64, 65, 67–71] produce a PDA strongly resembling the asymptotic form suggests that their working scale is already large, and the subsequent QCD evolution becomes unnecessary, or numerically insignificant. This also tacitly assumes that these models already incorporate the QCD radiative corrections.

5.2. PDA calculation for the unregularized quark loop

Since $\gamma^* \to \pi^0\gamma$ is an abnormal parity process, the standard procedure in the NJL model is not to regularize it because this is the only way to preserve the anomaly (see also Ref. [33, 73])). Straightforward calculation yields the result

$$F_{\pi\gamma\gamma}(p,q_1,q_2) = 8g_{\pi qq}M\frac{1}{i}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-M^2}\frac{1}{(k-q_1)^2-M^2}\frac{1}{(k-q_2)^2-M^2}\,. \tag{102}$$

In the chiral limit $g_{\pi qq}f_\pi \to M$ and for on-shell pions, $p^2=0$, and photons, $q_1^2 = q_2^2 = 0$, corresponding to the $\pi^0 \to 2\gamma$ decay one gets

$$F_{\pi\gamma\gamma}(0,0,0) = \frac{8M^2}{f_\pi}\frac{1}{i}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2)^3} = \frac{1}{4\pi^2 f_\pi}\,, \tag{103}$$

which agrees with the chiral anomaly expectations, Eq. (96). This result is the main motivation for not introducing an explicit cut-off in the abnormal parity processes. In order to compute the the high momentum behavior, we use the Feynman trick in the two propagators containing q_1 and q_2, and shift the integrating variable. For on-shell massless π^0, $p^2 = 0$, we get

$$\begin{aligned} F_{\gamma^*\gamma^*\pi} &= \frac{8M^2}{f_\pi} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2} \\ &\times \frac{1}{i} \int_0^1 dx \frac{1}{(k + xq_1 + (1-x)q_2)^2 - M^2} \, . \end{aligned} \tag{104}$$

The $Q^2 \to \infty$ limit can be taken yielding (see Eq. (97))

$$\varphi_\pi(x, Q^2) \to -\frac{4N_c g_{\pi qq} M}{(4\pi)^2} \log \left[\frac{M^2 - x(1-x)m_\pi^2}{Q^2} \right] , \tag{105}$$

where a finite pion mass has been reinserted. Thus, the prescription not to regularize the abnormal parity vertex does not agree with the factorization result found in a parton model approach. Instead, there appear scaling violations, which do not correspond to those expected from QCD evolution, contradicting the asymptotic result, Eq. (101). This problem is of a similar nature as the one found in the discussion of the spectator model after Eq. (74), and similar remarks concerning gauge invariance apply here.

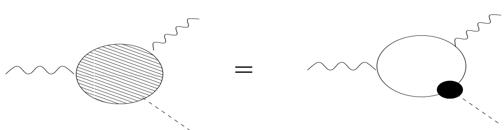

Fig. 10. Transition form factor in the NJL model. The closed loop is Pauli–Villars regularized. Wavy lines are photons, solid lines are constituent quarks and dashed lines composite pions.

5.3. PDA calculation from the transition form factor

To achieve factorization, it is necessary to effectively cut-off the transverse momentum, *i.e.*, we have to be able to make Q^2 larger than any scale in the loop. This is not consistent with integrating in k up to infinity[6].

[6] If we formally take the limit we get an expression looking like Eq. (97) with, $\varphi_\pi(x) = 1$ but with the unregularized form of f_π^2 (see Eq. (49). To achieve Eq. (97) with the regularized definition of f_π^2 some regularization must be introduced.

If we introduce PV regulators[7], we get

$$Q^2 F_{\gamma^*\gamma^*\pi}(p,q_1,q_2)|_{\rm reg.} \to \frac{16M^2}{f_\pi}\sum_i c_i \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2-\Lambda_i^2)^2} \times \int_0^1 dx \frac{1}{1-\omega(2x-1)}\,, \tag{106}$$

which, using Eq. (49) and (97) corresponds to (after inclusion of a finite m_π)

$$\varphi_\pi(x) = 4N_c M \frac{g_{\pi qq}}{f_\pi} F(m_\pi^2, x)\,, \tag{107}$$

where $F(p^2,x)$ is given in Eq. (44). $\varphi_\pi(x)$ depends only on x and is properly normalized (see Eq. (49)). In the chiral limit one gets

$$\varphi_\pi(x) = 1 \qquad 0 < x < 1\,, \tag{108}$$

independently of the regulators. However, if we PV regularize the vertex there is a violation of the anomaly, since

$$F_{\pi\gamma\gamma}(0,0,0)|_{\rm reg} = \frac{1}{4\pi^2 f_\pi}\sum_i c_i \frac{M^2}{M^2+\Lambda_i}\,. \tag{109}$$

For typical values of the parameters one finds a 25% reduction in the amplitude which means a 40% reduction in the width for $\pi^0 \to \gamma\gamma$. Such a result was also found in Ref. [33] using a Euclidean O(4) cut-off. Thus, in the present framework we have to choose between preserving the anomaly and not obtaining factorization or *vice versa*, *i.e.* violating the anomaly and reproducing factorization.

There is a subtlety in the previous reasoning, because the prescription of not regularizing the abnormal parity contribution to the action really means that the result is conditionally convergent and can be given unambiguously if one insists on maintaining vector gauge invariance [73]. A practical way to implement this fact is to introduce a gauge invariant regulator and remove it at the end of the calculation. The triangle graph is linearly divergent, and thus a regularization must be introduced. If one insists on preserving vector gauge invariance the regulator must preserve that symmetry, but then the axial current is not conserved generating the standard chiral anomaly.

[7] This regularization guarantees that the momentum routing is irrelevant. The Euclidean O(4) cut-off of Ref. [66, 71] requires a very special momentum routing *after* regularization; a different momentum routing with the same cut-off would produce finite cut-off contributions to their results.

The obvious question arises whether the limit $Q^2 \to \infty$ must be taken before or after removing the cut-off by sending it to infinity. If one takes the sequence $Q^2 > \Lambda^2 \to \infty$, the results of Ref. [71] produce a constant PDA, in agreement with our low energy calculation.

5.4. PDA calculation in light-cone coordinates

The pion distribution amplitude is defined through

$$f_\pi p^\mu \varphi_\pi(x) = \int \frac{dk^+ dk^- d^2 k_\perp}{2(2\pi)^4} \, \delta\left(k^+ - xp^+\right) \mathrm{Tr}\left[\chi_P(k)\gamma^\mu \gamma_5\right], \qquad (110)$$

where the PV regularized form the pion BS wave function, Eq. (52), is understood. Formally, in momentum space, Eq. (99) corresponds to integration over the quark momenta in the loop integral used in the evaluation of f_π Eq. (49), but with $k^+ = p^+ x = m_\pi x$ and $k_\perp$ fixed. Thus, with the PV method and after working out the Dirac traces, we have to compute

$$\begin{aligned} \Psi_\pi(x, \vec{k}_\perp) = & -\frac{2iN_c M g_{\pi qq}}{f_\pi} \int \frac{dk^+ dk^-}{(2\pi)^4} \frac{\delta\left(k^+ - xp^+\right)}{m_\pi^2 x(1-x)} \\ & \times \sum_j c_j \frac{1}{k^- - m_\pi - \frac{\vec{k}_\perp^2 + M^2 + \Lambda_j^2 + i0^+}{m_\pi(1-x)}} \, \frac{1}{k^- - \frac{\vec{k}_\perp^2 + M^2 + \Lambda_j^2 + i0^+}{m_\pi x}}, \end{aligned} \qquad (111)$$

where, again, the location of poles in the k^- variable has been explicitly displayed. This integral coincides with that found when computing the PDF (see Eq. (82)). Evaluating the k^- integral gives the pion LC wave function in the NJL model with the PV regularization:

$$\Psi_\pi(x, k_\perp) = \frac{4N_c M g_{\pi qq}}{16\pi^3 f_\pi} \sum_j c_j \frac{1}{k_\perp^2 + \Lambda_j^2 + M^2 - x(1-x)m_\pi^2}. \qquad (112)$$

For $m_\pi \neq 0$ it is non-factorizable in the $k_\perp$ and x variables. As a consequence of the PV condition with two subtractions one has, for large $k_\perp$

$$\Psi_\pi(x, k_\perp) \;\to\; \frac{4N_c M g_{\pi qq}}{16\pi^3 f_\pi} \frac{\sum_i c_i \Lambda_i^4}{k_\perp^6}, \qquad (113)$$

which gives a finite normalization and a finite second transverse moment

$$\langle k_\perp^2 \rangle \;=\; \int d^2 k_\perp \int_0^1 dx \, \Psi_\pi(x, k_\perp) k_\perp^2 \,. \qquad (114)$$

Integrating with respect to $k_\perp$ reproduces the pion distribution amplitude of Eq. (107). In the chiral limit, $m_\pi = 0$, one can use the Goldberger–Treiman relation for the constituent quarks, $g_{\pi qq} f_\pi = M$, yielding in addition to Eq. (108) the very simple formulas

$$\Psi_\pi(x, k_\perp) = \frac{1}{2} V_\pi(x, \vec{k}_\perp)\,, \tag{115}$$

$$\langle \vec{k}_\perp^2 \rangle_\pi = -\frac{M\langle \bar{q}q \rangle}{2f_\pi^2}\,. \tag{116}$$

where $V_\pi(x, \vec{k}_\perp)$ is given by Eq. (86). Note that these relations are independent of the PV regulators.

One aspect of regularization should be analyzed here. According to previous studies [4], the value of the LC wave function in the chiral limit is fixed at $\vec{k}_\perp = 0_\perp$ by the chiral anomaly, Eq. (100). In the PV regularized NJL model this is not the case, since

$$f_\pi \int_0^1 dx \Psi_\pi(x, 0_\perp) = F(0,0,0)\frac{N_c}{\pi} = \frac{4N_c M^2}{16\pi^3 f_\pi} \sum_i c_i \frac{1}{\Lambda_i^2 + M^2}\,. \tag{117}$$

The first two terms in this equation indicate the consistency of our calculations between the transition form factor and the low energy matrix element, but confirms the anomaly violation we have referred to above.

This is a clear deficiency of the NJL model and its regularization procedure, and we do not know of any convincing way of avoiding this problem in this model[8]. Nevertheless, it can be shown that it is possible to write down a chiral quark model based on the concept of spectral regularization [51] where the proper anomaly is reproduced and the results $\varphi_\pi(x, \mu_0) = V_\pi(x, \mu_0)/2 = 1$ still hold [46].

5.5. QCD evolution

The leading-twist PDA requires the inclusion of radiative logarithmic corrections through the QCD evolution [4]. For the pion distribution amplitude this is done in terms of the Gegenbauer polynomials, by interpreting our low-energy model result as the initial condition. In the chiral limit

$$\varphi_\pi(x, \mu_0) = 1\,. \tag{118}$$

[8] In the Euclidean version of the model the accepted regularization prescription is to regularize the real part of the action (normal parity processes) and not to regularize the imaginary part of the action (abnormal parity processes) [73]. Such a prescription agrees with the anomaly but *does not* agree with factorization and produces instead Eq. (105).

Then, the LO-evolved distribution amplitude reads [4]

$$\varphi_\pi(x,\mu) \;=\; 6x(1-x){\sum_{n=0}^{\infty}}' C_n^{3/2}(2x-1)a_n(\mu)\,, \tag{119}$$

where the prime indicates summation over even values of n only. The matrix elements, $a_n(\mu)$, are the Gegenbauer moments given by

$$a_n(\mu) \;=\; \frac{2}{3}\frac{2n+3}{(n+1)(n+2)}\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_n^{\rm NS}/(2\beta_0)} \times \int_0^1 dx C_n^{3/2}(2x-1)\varphi_\pi(x,\mu_0)\,, \tag{120}$$

with $C_n^{3/2}$ denoting the Gegenbauer polynomials, and $\gamma_n^{\rm NS} > 0$ is given by Eq. (91). From Eq. (118) one gets immediately

$$\int_0^1 dx C_n^{3/2}(2x-1)\varphi_\pi(x,Q_0) = 1\,. \tag{121}$$

Thus, for a given value of μ we may predict PDA. To determine the initial scale μ_0, or, equivalently, the evolution ratio $r=\alpha(Q)/\alpha(Q_0)$ Ref. [29] uses the result of Ref. [57], where it is found $a_2(2.4\mathrm{GeV}) = 0.12 \pm 0.03$. Using this input, one gets

$$\mu_0 = 313^{+60}_{-30}\mathrm{MeV}\,. \tag{122}$$

Within uncertainties, this result is compatible with the values obtained from the momentum fraction analysis, Eq. (15) and the pion electromagnetic form factor, Eq. (61). The result for the pion distribution amplitude obtained in Ref. [29] is shown in Fig. 11 for $Q = 2.4$GeV and reflecting the uncertainties from Ref. [57]. After evolution the results closely resemble those found in the transverse lattice [61–63]. The analysis of the end point behavior yields, *after* evolution, the estimate

$$\varphi_\pi(x,\mu) \to 8x\zeta\left(\frac{4C_{\rm F}}{\beta_0}\ln\frac{\alpha(\mu)}{\alpha(\mu_0)}+1\right) \sim\, 12.5\,x\,, \tag{123}$$

for $x \to 0$. Here $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ is the Riemann zeta function and the numerical value corresponds to take $\mu = 2.4$GeV. The value obtained in Ref. [29] for the second ξ-moment ($\xi = 2x-1$) is:

$$\langle \xi^2 \rangle = \int_0^1 dx\, \varphi_\pi(x, Q = 2.4\text{GeV})(2x-1)^2 = 0.040 \pm 0.005, \tag{124}$$

to be compared with $\langle \xi^2 \rangle = 0.06 \pm 0.02$ obtained in the standard lattice QCD for $Q = 1/a = 2.6 \pm 0.1$ GeV [60].

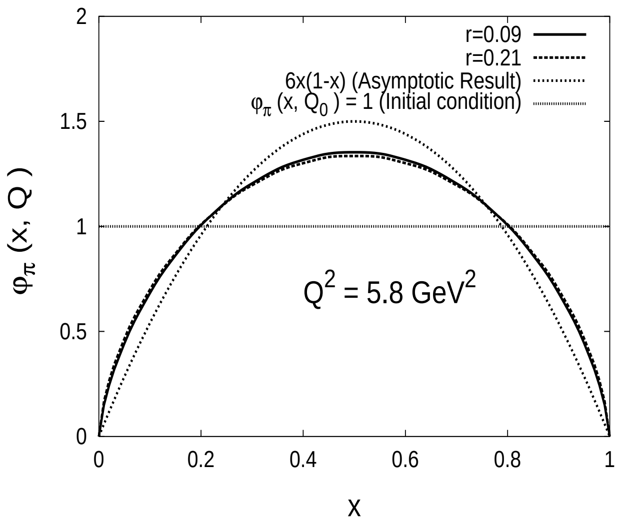

Fig. 11. The pion distribution amplitude in the chiral limit evolved to the scale $Q^2 = (2.4\text{GeV})^2$. The two values for the evolution ratio $r = \alpha(Q)/\alpha(Q_0)$ reflect the uncertainties in the values of Ref. [57] based on an analysis of the CLEO data. We also show the unevolved PDA, $\varphi_\pi(x, Q_0) = 1$, and the asymptotic PDA, $\varphi_\pi(x, \infty) = 6x(1-x)$. We use, as suggested by the analysis of the DIS, $\alpha(Q_0) = 2.14$ of Ref. [25], which corresponds to $Q_0 = 313$ MeV.

Based on the identities $\varphi_\pi(x, \mu_0) = V_\pi(x, \mu_0)/2 = 1$ between the PDF and the PDA in the chiral limit at the quark model scale a remarkable integral relation between $\varphi_\pi(x, \mu)$ and $V_\pi(x, \mu)/2$ has recently been derived using LO evolution equations [29],

$$\frac{\varphi_\pi(x, \mu)}{6x(1-x)} - 1 = \int_0^1 dy K(x, y) V_\pi(y, \mu)\,, \tag{125}$$

where the explicit expression for the scale independent kernel, $K(x, y)$ is given in Ref. [29]. Using this equation one can regard Fig. 11 as a prediction of the PDA in terms of the PDF parameterizations of Ref. [13] (see Fig. 9). Using Eq. (125) one gets the following estimate for the leading twist contribution to the pion form factor at LO

$$\frac{Q^2 F_{\gamma^*,\pi\gamma}(Q)}{2f_\pi}\bigg|_{\rm Twist-2} = \int_0^1 dx \frac{\varphi_\pi(x,Q)}{6x(1-x)} = 1.25 \pm 0.10\,. \qquad (126)$$

The experimental value obtained in CLEO [56] for the full form factor is $Q^2 F_{\gamma^*,\pi\gamma}(Q)/2f_\pi = 0.83 \pm 0.12$ at $Q^2 = 5.8\mathrm{GeV}^2$. Taking into account that we have not included neither NLO effects nor an estimate of higher twist contributions, the result is rather encouraging. Finally, using this estimate for $\varphi_\pi(x,Q)$ in the electromagnetic form factor, Eq. (60), one deduces an enhacement

$$-Q^2 F_\pi^{\rm em}(-Q^2)\Big|_{\rm Twist-2} = 0.20 \pm 0.03\,, \qquad Q^2 = 5.8\mathrm{GeV}^2\,, \qquad (127)$$

which also brings the number closer to experimental number [31], although it only accounts for half of its value $\sim 0.38 \pm 0.04\mathrm{GeV}^2$, perhaps due to the lack of NLO or higher twists. At present this point seems not to be understood and deserves further investigation.

6. Conclusions

Chiral quark models incorporate two essential features of QCD at low energies: spontaneous chiral symmetry breaking and quark degrees of freedom. In addition, it can be made compatible with large N_c counting rules, and calculations in the leading order approximation have been undertaken in the past. In these lectures we have focused and reviewed particular applications in a prototype chiral quark model, the Nambu–Jona-Lasinio model, but many results extend trivially to other models. We have also restricted to the pion because we do not expect a better theoretical understanding of a hadron, including the fact that confinement is hoped not to play an essential role.

A very important issue concerning the treatment of low energy models is the existence of a high momentum suppression in the interaction. In the NJL model this is done via a regularization method, which has to comply with several properties, like gauge invariance and scaling in the high energy limit, *i.e.* the absence of spurious logarithmic corrections. This allows to clearly identify power corrections of pion observables.

These models make sense at a given low renormalization scale μ_0. Any result for a given observable may be used to compute that observable at a higher scale μ through QCD evolution. In this way the correct behavior for

QCD radiative corrections may be incorporated using the chiral quark model as an initial condition for the evolution. Although radiative corrections are only known perturbatively, there is a chance that the matching scale, μ_0, makes perturbation theory meaningful. This is a weak point which can only be addressed by computing higher order corrections to the model and higher order corrections to the QCD evolution. Nevertheless, it is encouraging that three different determinations of the scale based on matching to LO perturbative QCD evolution and available experimental data yield within uncertainties the scale $\mu_0 = 320\text{MeV}$ for $\Lambda_{\text{QCD}} = 225\text{MeV}$. Actually, using this low scale the description of the distribution functions and distribution amplitudes agrees remarkably well with phenomenological analysis.

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