FERMION UNIVERSALITY MANIFESTING ITSELF IN THE DIRAC COMPONENT OF NEUTRINO MASS MATRIX*

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An effective texture is presented for six Majorana conventional neutrinos (three active and three sterile), based on a 6×6 neutrino mixing matrix whose 3×3 active–active component arises from the popular bimaximal mixing matrix of active neutrinos ν_e , ν_μ , ν_τ by three small rotations in the 14, 25, 36 planes of ν_1 , ν_2 , ν_3 and ν_4 , ν_5 , ν_6 neutrino mass states. The Dirac component (*i.e.*, 3×3 active-sterile component) of the resulting 6×6 neutrino mass matrix is conjectured to get a structure *similar* to the charged-lepton and quark 3×3 mass matrices, after the bimaximal mixing, specific for neutrinos, is *transformed out unitarily* from the neutrino mass matrix. The charged-lepton and quark mass matrices are taken in a *universal form* constructed previously by the author with a considerable phenomenological success. Then, for the option of $m_1^2 \simeq m_2^2 \simeq m_3^2 \gg m_4^2 \simeq m_5^2 \simeq m_6^2 \simeq 0$, the proposed texture *predicts* oscillations of solar ν_e's with $\Delta m^2_{\rm sol} \equiv \Delta m^2_{21} \sim$ (1.1 to 1.2) $\times 10^{-5}$ eV^2, not inconsistent with the LMA solar solution, if the SuperKamiokande value $\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} \sim$ (3 to 3.5) $\times 10^{-3}$ eV^2 for oscillations of atmospheric ν_μ's is taken as an input. Here, $\sin^2 2\theta_{\rm sol} \sim 1$ and $\sin^2 2\theta_{\rm atm} \sim 1$. The texture *predicts* also an LSND effect with $\sin^2 2\theta_{\rm LSND} \sim$ (1.4 to 1.9) $\times 10^{-11}\,({\rm eV}/m_1)^4$ and $\Delta m^2_{\rm LSND} \equiv \Delta m^2_{25} \sim m_1^2 +$ (1.1 to 1.2) $\times 10^{-5}$ eV^2. Unfortunately, the Chooz experiment imposes on the LSND effect (in our texture) a very small upper bound $\sin^2 2\theta_{\rm LSND} \lesssim 1.3 \times 10^{-3}$, which corresponds to the lower limit $m_1 \gtrsim$ (1.0 to 1.1) $\times 10^{-2}$ eV.

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1. Introduction

Some time ago we constructed an effective form of fundamental-fermion mass matrix which worked very well for charged leptons e , μ , τ [1], neatly for up and down quarks u , c , t and d , s , b [2] and badly for neutrinos ν_e , ν_μ , ν_τ [3]. Of course, if neutrinos are Majorana particles and/or, beside three active neutrinos carrying Standard Model charges, there exist effectively some sterile neutrinos free of these charges, it is natural to expect the neutrino texture to be different from charged-lepton and quark textures, characteristic for active Dirac particles. However, if neutrinos get a 3×3 Dirac mass matrix $M^{(\mathrm{D})}$ as a component of their overall 6×6 mass matrix

$$M = \left(\begin{array}{cc} M^{(\mathrm{L})} & M^{(\mathrm{D})} \\ M^{(\mathrm{D})\dagger} & M^{(\mathrm{R})} \end{array} \right) , \tag{1}$$

then $M^{(\mathrm{D})}$ may display a similar structure as the charged-lepton and quark 3×3 mass matrices.

Note that the neutrino mass matrix M described in Eq. (1) leads to the neutrino mass term

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} \sum_{\alpha\beta} \left(\overline{\nu^{(a)}_\alpha} \, , \overline{\nu^{(s)}_\alpha} \right) \left(\begin{array}{cc} M^{(\mathrm{L})}_{\alpha\beta} & M^{(\mathrm{D})}_{\alpha\beta} \\ M^{(\mathrm{D})*}_{\beta\alpha} & M^{(\mathrm{R})}_{\alpha\beta} \end{array} \right) \left(\begin{array}{c} \nu^{(a)}_\beta \\ \nu^{(s)}_\beta \end{array} \right) \tag{2}$$

in the Lagrangian, where

$$\nu^{(a)}_\alpha = \nu_{\alpha \mathrm{L}} + (\nu_{\alpha \mathrm{L}})^c \;, \qquad \nu^{(s)}_\alpha = \nu_{\alpha \mathrm{R}} + (\nu_{\alpha \mathrm{R}})^c \;, \tag{3}$$

with $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$ $(\alpha = e, \mu, \tau)$, are the Majorana conventional active and sterile neutrinos, respectively. Further on, for six neutrino flavor fields (states) we will use the notation $\nu^{(a)}_\alpha \equiv \nu_\alpha$ and $\nu^{(s)}_\alpha \equiv \nu_{\alpha_s}$ ($\alpha = e, \mu, \tau$ and $\alpha_s = e_s$, μ_s, τ_s), and then pass to $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau, \nu_{e_s}, \nu_{\mu_s}$, ν_{τ_s} ($\alpha = e$, μ, τ, e_s, μ_s, τ_s). In the last case, six neutrino mass fields (states) will be denoted as $\nu_i = \nu_, \; \nu_2, \; \nu_3, \; \nu_4, \; \nu_5, \; \nu_6$ $(i = 1, 2, 3, 4, 5, 6)$. The relation between both complementary sets of neutrino fields (states) will read

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \left(|\nu_\alpha\rangle = \nu^\dagger_\alpha |0\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \right) , \tag{4}$$

with $U = (U_{\alpha\, i})$ being the unitary neutrino mixing matrix.

In the representation, where the charged-lepton 3×3 mass matrix is diagonal, the neutrino mixing matrix $U = (U_{\alpha\, i})$ is at the same time the unitary matrix diagonalizing the neutrino mass matrix $M = (M_{\alpha\, \beta})$ given in Eq. (1)

$$U^\dagger M U = M_{\mathrm{d}} \equiv \mathrm{diag}(m_1 \, , m_2 \, , m_3 \, , m_4 \, , m_5 \, , m_6) \; . \tag{5}$$

Thus, in this case $M = U M_{\mathrm{d}} U^\dagger$.

The aim of the present paper is a brief discussion on the relevant question, to what extent the Dirac component $M^{(\mathrm{D})}$ of the neutrino mass matrix M may get a *similar* structure to the charged-lepton and quark 3×3 mass matrices constructed previously [1,2]. In fact, in the cases of these active Dirac particles, we constructed with a considerable phenomenological success three 3×3 mass matrices of the *universal form*

$$M^{(\mathrm{D})} = \frac{1}{29} \begin{pmatrix} \mu\varepsilon & 2\alpha & 0 \\ 2\alpha & \frac{4}{9}\,\mu\,(80+\varepsilon) & 8\sqrt{3}\,\alpha \\ 0 & 8\sqrt{3}\,\alpha & \frac{24}{25}\,\mu\,(624+\varepsilon) \end{pmatrix}, \tag{6}$$

where values of the constants $\mu > 0$, $\alpha > 0$ and $\varepsilon > 0$ depended on whether the discussed fermions were charged leptons or up quarks or down quarks (some foundations of our construction are collected in Appendix to the second Ref. [2]). The values for charged leptons are quoted in Eq. (26) later on.

2. A model of neutrino texture

In order to operate with a neutrino texture potentially consistent with oscillation experiments for solar ν_e's and atmospheric ν_μ's as well as with LSND experiment for accelerator ν_μ's, we will consider the 6×6 neutrino mass matrix $M = (M_{\alpha\,\beta})$ $(\alpha\,,\beta = e\,,\,\mu\,,\,\tau\,,\,e_s\,,\,\mu_s\,,\,\tau_s)$ given in Eq. (1). We will assume that its diagonalizing matrix $U = (U_{\alpha\,i})$ $(\alpha = e\,,\mu\,,\tau\,,\,e_s\,,\mu_s\,,\tau_s$ $i =$ 1, 2, 3, 4, 5, 6) can be written as

$$U = \overset{1}{U}\overset{0}{U}\,, \tag{7}$$

where the two factors are unitary matrices

$$\overset{1}{U} = \begin{pmatrix} U^{(3)} & 0^{(3)} \\ 0^{(3)} & 1^{(3)} \end{pmatrix}, \qquad \overset{0}{U} = \begin{pmatrix} C^{(3)} & S^{(3)} \\ -S^{(3)} & C^{(3)} \end{pmatrix}, \tag{8}$$

defined through the 3×3 submatrices

$$U^{(3)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix},\; 1^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\; 0^{(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{9}$$

and

$$C^{(3)} = \begin{pmatrix} c_{14} & 0 & 0 \\ 0 & c_{25} & 0 \\ 0 & 0 & c_{36} \end{pmatrix}, \qquad S^{(3)} = \begin{pmatrix} s_{14} & 0 & 0 \\ 0 & s_{25} & 0 \\ 0 & 0 & s_{36} \end{pmatrix}, \tag{10}$$

while $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ (the possible CP violating phases are ignored). Here, $U^{(3)}$ involving $c_{12} = 1/\sqrt{2} = s_{12}$, $c_{23} = 1/\sqrt{2} = s_{23}$, $c_{13} = 1$, $s_{13} = 0$ and the phase $\delta = 0$ is the popular bimaximal mixing matrix for active neutrinos ν_e , ν_μ , ν_τ , describing in a reasonable approximation oscillations of solar ν_e's and atmospheric ν_μ's as suggested by SNO and SuperKamiokande experiments [4,5]. Using Eqs. (7)–(10) we obtain explicitly

$$U = (U_{\alpha i}) = \begin{pmatrix} \frac{c_{14}}{\sqrt{2}} & \frac{c_{25}}{\sqrt{2}} & 0 & \frac{s_{14}}{\sqrt{2}} & \frac{s_{25}}{\sqrt{2}} & 0 \\ -\frac{c_{14}}{2} & \frac{c_{25}}{2} & \frac{c_{36}}{\sqrt{2}} & -\frac{s_{14}}{2} & \frac{s_{25}}{2} & \frac{s_{36}}{\sqrt{2}} \\ \frac{c_{14}}{2} & -\frac{c_{25}}{2} & \frac{c_{36}}{\sqrt{2}} & \frac{s_{14}}{2} & -\frac{s_{25}}{2} & \frac{s_{36}}{\sqrt{2}} \\ -s_{14} & 0 & 0 & c_{14} & 0 & 0 \\ 0 & -s_{25} & 0 & 0 & c_{25} & 0 \\ 0 & 0 & -s_{36} & 0 & 0 & c_{36} \end{pmatrix}. \tag{11}$$

When s_{14} , s_{25} , $s_{36} \to 0$, then $\overset{0}{U} \to 1$ and so, $U \to \overset{1}{U}$ and $M = UM_{\rm d}U^\dagger \to \overset{1}{U}M_{\rm d}\overset{1}{U}{}^\dagger$. Here, $\overset{1}{U}M_{\rm d}\overset{1}{U}{}^\dagger = M_{\rm d}$ if the neutrino mass spectrum m_i were degenerate for i = 1, 2, 3: $m_1 = m_2 = m_3$. Thus, in the above limit, it would be $M \to M_{\rm d}$ in the case of such degeneracy of m_i.

From Eqs. (5) and (7) we can deduce that

$$\overset{0}{U}{}^\dagger \overset{0}{M}\overset{0}{U} = M_{\rm d} \equiv {\rm diag}(m_1 \, , \, m_2 \, , \, m_3 \, , \, m_4 \, , \, m_5 \, , \, m_6) \, , \tag{12}$$

where $\overset{0}{M}$ is defined by the unitary transformation of M, generated by $\overset{1}{U}$

$$\overset{0}{M} = \overset{1}{U}{}^\dagger M\overset{1}{U} \, . \tag{13}$$

Thus, $\overset{0}{M} = \overset{0}{U}M_{\rm d}\overset{0}{U}{}^\dagger$ since $M = UM_{\rm d}U^\dagger$. Further on, we will conjecture (*cf.* Eq. (20) later on), that the Dirac component of neutrino mass matrix $\overset{0}{M} = \overset{1}{U}{}^\dagger M\overset{1}{U}$ (rather than that of the full neutrino mass matrix M) gets a similar structure to the charged-lepton and quark 3×3 mass matrices constructed previously [1,2] in the universal form (6). If the Dirac component of the full neutrino mass matrix M were similar in structure to the charged-lepton and quark mass matrices of the universal form (6), then in the neutrino case (as is not difficult to show) there would be $\mu = 0$, $\alpha = 0$ and $\varepsilon = 0$ *i.e.*, the Dirac component of M would vanish trivially. In the accepted option, only parameter α must be zero for neutrinos. (Note that, in the spirit of fermion universality, the value $\alpha^{(\nu)} = 0$ when correlated with the electric charge $Q^{(\nu)} = 0$ is consistent with our previous conjecture [2] that for up and down quarks $\alpha^{(u)} : \alpha^{(d)} = Q^{(u)\,2} : Q^{(d)\,2} = 4 : 1$, where $Q^{(u)} = 2/3$ and $Q^{(d)} = -1/3$, and also with the fact that for charged leptons $\alpha^{(e)} \neq 0$, where $Q^{(e)} = -1 \neq 0$.) The appearance of $\overset{0}{M}$ (rather than M) in the correct option for Dirac component of neutrino mass matrix may be understood qualitatively as related to the fact that the unitary transformation $M \to \overset{0}{M}$ (*viz.* $\overset{0}{M} = \overset{1}{U}{}^{\dagger} M \overset{1}{U}$) eliminates from M the specific bimaximal mixing that makes neutrinos differ from other fundamental fermions (charged leptons and quarks). Thus, in our texture, the *fermion universality* manifests itself in the Dirac component of neutrino mass matrix *up to the unitary transformation* (of this matrix) *generated by* $\overset{1}{U}$.

Note from Eq. (7) that $\overset{1}{U} = \left(\overset{1}{U}_{\alpha i}\right)$ and $\overset{0}{U} = \left(\overset{0}{U}_{ij}\right)$, and so, $\overset{0}{M} = \left(\overset{0}{M}_{ij}\right)$ (i, j =1, 2, 3, 4, 5, 6) from Eq. (13). Due to the formula $\overset{0}{M} = \overset{0}{U} M_{\rm d} \overset{0}{U}{}^{\dagger}$ we get explicitly

$$\begin{array}{lll} \overset{0}{M}_{11} = m_1 c^2_{14} + m_4 s^2_{14}, & \overset{0}{M}_{14} = \overset{0}{M}_{41} = (m_4 - m_1) c_{14} s_{14}, & \overset{0}{M}_{44} = m_1 s^2_{14} + m_4 c^2_{14}, \\ \overset{0}{M}_{22} = m_2 c^2_{25} + m_5 s^2_{25}, & \overset{0}{M}_{25} = \overset{0}{M}_{52} = (m_5 - m_2) c_{25} s_{25}, & \overset{0}{M}_{55} = m_2 s^2_{25} + m_5 c^2_{25}, \\ \overset{0}{M}_{33} = m_3 c^2_{36} + m_6 s^2_{36}, & \overset{0}{M}_{36} = \overset{0}{M}_{63} = (m_6 - m_3) c_{36} s_{36}, & \overset{0}{M}_{66} = m_3 s^2_{36} + m_6 c^2_{36}, \end{array} \tag{14}$$

and all other $\overset{0}{M}_{ij} = 0$. Notice the relations $M_{e e_s} = \overset{0}{M}_{14}/\sqrt{2}$, $M_{\mu\mu_s} = \overset{0}{M}_{25}/2$, $M_{\tau\tau_s} = \overset{0}{M}_{36}/\sqrt{2}$ between the diagonal elements of Dirac component $M^{(\rm D)}$ of the full mass matrix $M = (M_{\alpha\beta})$ and the elements of Dirac component $\overset{0}{M}{}^{(\rm D)} = {\rm diag}(\overset{0}{M}_{14}, \overset{0}{M}_{25}, \overset{0}{M}_{36})$ of the mass matrix $\overset{0}{M} = \left(\overset{0}{M}_{ij}\right)$. This is so, since $M^{(\rm D)} = (\overset{1}{U}\overset{0}{M}\overset{1}{U}{}^{\dagger})^{(\rm D)} = U^{(3)} \overset{0}{M}{}^{(\rm D)}$ with $U^{(3)}$ given in Eq. (5) (similarly, $M^{(\rm L)} = U^{(3)} \overset{0}{M}{}^{(\rm L)} U^{(3)\,\dagger}$ and $M^{(\rm R)} = \overset{0}{M}{}^{(\rm R)}$). Of course, $M^{(\rm D)}$ is not diagonal, in contrast to $\overset{0}{M}{}^{(\rm D)}$.

From Eq. (14) we can infer that the neutrino masses m_i can be expressed in the following way:

$$m_{i,j} = \overset{0}{M}_{ii,jj} \mp \frac{s_{ij}}{c_{ij}}\overset{0}{M}_{ij} = \overset{0}{M}_{jj,ii} \mp \frac{c_{ij}}{s_{ij}}\overset{0}{M}_{ij} \;\; (ij = 14\,,\, 25\,,\, 36)\,, \tag{15}$$

if $c_{ij} \neq 0$ and $s_{ij} \neq 0$. These formulae imply the relations

$$\frac{\overset{0}{M}_{ij}}{\overset{0}{M}_{jj} - \overset{0}{M}_{ii}} = \frac{c_{ij}s_{ij}}{c^2_{ij} - s^2_{ij}} \simeq \frac{s_{ij}}{c_{ij}} \;\; (ij = 14\,,\, 25\,,\, 36)\,, \tag{16}$$

where the last approximation is valid for $s^2_{ij} \ll c^2_{ij}$. In this approximation,

$$m_{i,j} \simeq \overset{0}{M}_{ii,jj} \mp \frac{\overset{0}{M}{}^2_{ij}}{\overset{0}{M}_{jj} - \overset{0}{M}_{ii}} \;\; (ij = 14\,,\, 25\,,\, 36)\,. \tag{17}$$

Here, the second term is a perturbation in the small parameter $(s_{ij}/c_{ij})^2$. Henceforth, we will accept this situation.

Now, let us make two conjectures on the neutrino mass matrix $\overset{0}{M} = \overset{0}{M}_{ij}$ given explicitly in Eq. (14). Namely,

(i) its diagonal elements are

$$\overset{0}{M}_{11} = \overset{0}{M}_{22} = \overset{0}{M}_{33} = \overset{0}{m} \gg \overset{0}{M}_{44} = \overset{0}{M}_{55} = \overset{0}{M}_{66} = 0\,, \tag{18a}$$

or

$$\overset{0}{M}_{11} = \overset{0}{M}_{22} = \overset{0}{M}_{33} = 0 \ll \overset{0}{M}_{44} = \overset{0}{M}_{55} = \overset{0}{M}_{66} = \overset{0}{m}\,, \tag{18b}$$

implying through Eq. (17) that

$$m_i \simeq \overset{0}{m} + \frac{\overset{0}{M}{}^2_{ij}}{\overset{0}{m}} \;\;,\;\; m_j \simeq -\frac{\overset{0}{M}{}^2_{ij}}{\overset{0}{m}} \;\; (ij = 14\,,\, 25\,,\, 36)\,, \tag{19a}$$

or

$$m_i \simeq -\frac{\overset{0}{M}{}^2_{ij}}{\overset{0}{m}} \;\;,\;\; m_j \simeq \overset{0}{m} + \frac{\overset{0}{M}{}^2_{ij}}{\overset{0}{m}} \;\; (ij = 14\,,\, 25\,,\, 36)\,, \tag{19b}$$

respectively,

(ii) its off-diagonal elements, forming its Dirac component $\overset{0}{M}{}^{(\mathrm{D})} = \mathrm{diag}\,(\overset{0}{M}_{14}, \overset{0}{M}_{25}, \overset{0}{M}_{36})$, are in their structure similar to the diagonal elements of charged-lepton and quark matrices of the universal form (6), thus

$$\overset{0}{M}_{14} = \frac{\mu}{29}\,\varepsilon, \quad \overset{0}{M}_{25} = \frac{\mu}{29}\,\frac{4}{9}\,(80+\varepsilon), \quad \overset{0}{M}_{36} = \frac{\mu}{29}\,\frac{24}{25}\,(624+\varepsilon), \tag{20}$$

where we put approximately $\varepsilon = 0$ (already for charged leptons ε is small, *cf.* Eq. (26)).

From Eqs. (19) (in the option (a) or (b)) and (20) we calculate (with $\varepsilon = 0$)

$$\begin{aligned} \Delta m^2_{21} &= m^2_2 - m^2_1 = 2\overset{0}{M}{}^2_{25} = 3.00\mu^2, \\ \Delta m^2_{32} &= m^2_3 - m^2_2 = 2\left(\overset{0}{M}{}^2_{36} - \overset{0}{M}{}^2_{25}\right) = 8.50 \times 10^2 \mu^2 \end{aligned} \tag{21a}$$

or

$$\begin{aligned} \Delta m^2_{21} &= m^2_2 - m^2_1 = \frac{\overset{0}{M}{}^4_{25}}{\overset{0}{m}{}^2} = 2.26\frac{\mu^4}{\overset{0}{m}{}^2}, \\ \Delta m^2_{32} &= m^2_3 - m^2_2 = \frac{\overset{0}{M}{}^4_{36} - \overset{0}{M}{}^4_{25}}{\overset{0}{m}{}^2} = 1.82 \times 10^5\,\frac{\mu^4}{\overset{0}{m}{}^2}. \end{aligned} \tag{21b}$$

Thus,

$$\frac{\Delta m^2_{32}}{\Delta m^2_{21}} = 2.83 \times 10^2 \qquad \text{or} \qquad 8.06 \times 10^4\,. \tag{22a or b}$$

Hence, putting

$$\Delta m^2_{32} \equiv \Delta m^2_{\mathrm{atm}} \sim (3.0 \text{ to } 3.5) \times 10^{-3}\mathrm{eV}^2\,, \tag{23}$$

as for atmospheric ν_μ's in the SuperKamiokande experiment, we *predict* for solar ν_e's (from Eq. (22) in the option (a) or (b)) that

$$\begin{aligned} \Delta m^2_{\mathrm{sol}} \equiv \Delta m^2_{21} &\sim (1.1 \text{ to } 1.2) \times 10^{-5}\mathrm{eV}^2, \\ &\text{or } (3.7 \text{ to } 4.3) \times 10^{-8}\mathrm{eV}^2\,. \end{aligned} \tag{24a or b}$$

So, the first option (a) is not inconsistent with the mass-square scale of Large Mixing Angle (LMA) solar solution, while the second option (b) is closer to the LOW solar solution [5]. The SuperKamiokande estimate (23) determines also (from Eq. (21) in the option (a) or (b)) the constant

$$\mu^2 \sim (3.5\,\text{to}\,4.1) \times 10^{-6}\text{eV}^2 \quad \text{or} \quad (1.3\,\text{to}\,1.4) \times 10^{-4}\,\overset{0}{m}\,\text{eV} \qquad \text{(25a or b)}$$

for neutrinos. This gives $\mu \sim (1.9\,\text{to}\,2.0) \times 10^{-3}$ eV or $(1.1\,\text{to}\,1.2) \times 10^{-2}$ $\sqrt{\overset{0}{m}\,\text{eV}}$, respectively.

Note that for charged leptons we got previously [1,2]

$$\mu^{(e)} = 85.9924\,\text{MeV}, \quad \varepsilon^{(e)} = 0.172329, \quad \left(\frac{\alpha^{(e)}}{\mu^{(e)}}\right)^2 = 0.023^{+0.029}_{-0.025}, \qquad (26)$$

when we fitted precisely their three masses m_e, m_μ, m_τ using our mass matrix (6). The error limits for $(\alpha^{(e)}/\mu^{(e)})^2$ came out from the actual error limits for $m_\tau = 1777.03\,^{+0.30}_{-0.26}$ MeV [6]. With $\alpha^{(e)} = 0$ we obtained $m_\tau = 1776.8\,$MeV as a *prediction*, when we used the experimental values of m_e and m_μ as inputs.

More generally, Eqs. (19) and (20) with the estimate (25) (in the option (a) or (b)) lead to the following neutrino mass spectrum

$$\begin{aligned} m_1 &= \overset{0}{m}\,, \\ m_2 &\simeq \overset{0}{m} + 1.50\frac{\mu^2}{\overset{0}{m}} \sim \overset{0}{m} + \frac{(5.3\,\text{to}\,6.2) \times 10^{-6}\,\text{eV}^2}{\overset{0}{m}}\,, \\ m_3 &\simeq \overset{0}{m} + 427\frac{\mu^2}{\overset{0}{m}} \sim \overset{0}{m} + \frac{(1.5\,\text{to}\,1.8) \times 10^{-3}\,\text{eV}^2}{\overset{0}{m}}\,, \end{aligned} \qquad \text{(27a)}$$

or

$$\begin{aligned} m_1 &= 0\,, \\ m_2 &\simeq -1.50\,\frac{\mu^2}{\overset{0}{m}} \sim -(1.9\,\text{to}\,2.1) \times 10^{-4}\,\text{eV}\,, \\ m_3 &\simeq -427\,\frac{\mu^2}{\overset{0}{m}} \sim -(5.5\,\text{to}\,5.9) \times 10^{-2}\,\text{eV}\,, \end{aligned} \qquad \text{(27b)}$$

and

$$\begin{aligned} m_4 &= 0\,, \\ m_5 &\simeq -1.50\,\frac{\mu^2}{\overset{0}{m}} \sim -\frac{(5.3\,\text{to}\,6.2) \times 10^{-6}\,\text{eV}^2}{\overset{0}{m}}\,, \\ m_6 &\simeq -427\,\frac{\mu^2}{\overset{0}{m}} \sim -\frac{(1.5\,\text{to}\,1.8) \times 10^{-3}\,\text{eV}^2}{\overset{0}{m}}\,, \end{aligned} \qquad \text{(28a)}$$

or

$$\begin{aligned} m_4 &= \overset{0}{m}\,, \\ m_5 &\simeq \overset{0}{m} + 1.50\,\frac{\mu^2}{\overset{0}{m}} \sim \overset{0}{m} + (1.9\,\mathrm{to}\,2.1)\times 10^{-4}\,\mathrm{eV}\,, \\ m_6 &\simeq \overset{0}{m} + 427\,\frac{\mu^2}{\overset{0}{m}} \sim \overset{0}{m} + (5.5\,\mathrm{to}\,5.9)\times 10^{-2}\,\mathrm{eV}\,. \end{aligned} \tag{28b}$$

Note that here $m_1, m_2, m_3 \gg |m_4|, |m_5|, |m_6|$ or $|m_1|, |m_2|, |m_3| \ll m_4, m_5, m_6$. Also the second of these two options differs essentially from the familiar seesaw mechanism, where m_4, m_5, m_6 are of a very high mass scale determined by a Grand Unification Theory, while the mass scale $\overset{0}{m}$ may be *e.g.* of the order of 1 eV. Of course, $\sum_i m_i = 3\,\overset{0}{m}$.

Similarly, the mixing tangents (16) become

$$\begin{aligned} \frac{s_{14}}{c_{14}} &= 0, \qquad \frac{s_{25}}{c_{25}} \simeq -1.23\frac{\mu}{\overset{0}{m}} \sim -(2.3\,\mathrm{to}\,2.5)\times 10^{-3}\,\frac{\mathrm{eV}}{\overset{0}{m}}\,, \\ \frac{s_{36}}{c_{36}} &\simeq -20.7\frac{\mu}{\overset{0}{m}} \sim -(3.9\,\mathrm{to}\,4.2)\times 10^{-2}\,\frac{\mathrm{eV}}{\overset{0}{m}}\,, \end{aligned} \tag{29a}$$

or

$$\begin{aligned} \frac{s_{14}}{c_{14}} &= 0, \qquad \frac{s_{25}}{c_{25}} \simeq 1.23\frac{\mu}{\overset{0}{m}} \sim (1.4\,\mathrm{to}\,1.4)\times 10^{-2}\,\sqrt{\frac{\mathrm{eV}}{\overset{0}{m}}}\,, \\ \frac{s_{36}}{c_{36}} &\simeq 20.7\frac{\mu}{\overset{0}{m}} \sim (2.3\,\mathrm{to}\,2.4)\times 10^{-1}\,\sqrt{\frac{\mathrm{eV}}{\overset{0}{m}}}\,. \end{aligned} \tag{29b}$$

3. Neutrino oscillations

We start with the familiar formulae for probabilities of neutrino oscillations $\nu_\alpha \to \nu_\beta$ on the energy shell,

$$P(\nu_\alpha \to \nu_\beta) = \left|\langle \nu_\beta | \mathrm{e}^{iPL} | \nu_\alpha \rangle\right|^2 = \delta_{\beta\alpha} - 4\sum_{j>i} U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i} \sin^2 x_{ji}\ , \tag{30}$$

valid if the quartic products $U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i}$ are real, what is certainly true when a possible CP violation can be ignored (then $U^*_{\alpha i} = U_{\alpha i}$, as in our case of Eq. (11), and $P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$).

In Eq. (30)

$$x_{ji} = 1.27\frac{\Delta m^2_{ji} L}{E}\,, \qquad \Delta m^2_{ji} = m^2_j - m^2_i\,, \tag{31}$$

where Δm^2_{ji}, L and E are measured in eV^2, km and GeV, respectively, (L and E denote the experimental baseline and neutrino energy, while $p_i = \sqrt{E^2 - m^2_i} \simeq E - m^2_i/2E$ are eigenvalues of the neutrino momentum P).

If $m^2_1 \simeq m^2_2 \simeq m^2_3 \simeq \overset{0}{m}{}^2$, where $\Delta m^2_{21} \ll \Delta m^2_{32}$, and $m^2_4 \simeq m^2_5 \simeq m^2_6 \ll \overset{0}{m}{}^2$ as well as $s^2_{ij} \ll c^2_{ij}$ $(ij = 14\,, 25\,, 36)$, as is true in the case of $\mu^2 \ll \overset{0}{m}{}^2$, then the oscillation formulae (30) give in particular

$$\begin{aligned}
P(\nu_e \to \nu_e)_{\rm sol} &\simeq 1 - c^2_{25}\sin^2(x_{21})_{\rm sol} - \tfrac{1}{2}\left(1 + c^2_{25}\right) s^2_{25}\,,\\
P(\nu_\mu \to \nu_\mu)_{\rm atm} &\simeq 1 - \tfrac{1}{2}\left(1 + c^2_{25}\right) c^2_{36}\sin^2(x_{32})_{\rm atm}\\
&\quad - \tfrac{1}{8}\left(1 + c^2_{25} + 2c^2_{36}\right)\left(s^2_{25} + 2s^2_{36}\right)\,,\\
P(\nu_\mu \to \nu_e)_{\rm LSND} &\simeq \tfrac{1}{2}\, s^4_{25}\sin^2(x_{25})_{\rm LSND}\,,\\
P(\bar\nu_e \to \bar\nu_e)_{\rm Chooz} &\simeq 1 - \left(1 + c^2_{25}\right) s^2_{25}\sin^2(x_{25})_{\rm Chooz}
\end{aligned} \tag{32}$$

for solar ν_e's, atmospheric ν_μ's, LSND ν_μ's and Chooz $\bar\nu_e$'s. Here, $\Delta m^2_{21} \equiv \Delta m^2_{\rm sol} \sim 10^{-5}$ eV^2 for LMA solar solution, $\Delta m^2_{32} \equiv \Delta m^2_{\rm atm} \sim (3$ to $3.5) \times 10^{-3}$ eV^2 from the SuperKamiokande atmospheric experiment and $|\Delta m^2_{25}| \equiv \Delta m^2_{\rm LSND} \sim$ *e.g.* 1 eV^2 for the LSND experiment. This is consistent with our previous *identification* (23) of Δm^2_{32} and *prediction* (24) for $\Delta m^2_{\rm sol}$. The first two Eqs. (32) differ, strictly speaking, from the familiar two-flavor oscillation formulae (used in analyzes of solar experiments [5]) by some additive terms that, fortunately, are small enough when $\mu^2 \ll \overset{0}{m}{}^2$. In fact, from Eqs. (29) and the estimate (25) (in the option (a) or (b))

$$\begin{aligned}
s^2_{25} &\simeq 1.50\left(\frac{\mu}{\overset{0}{m}}\right)^2 c^2_{25} \sim (5.3 \text{ to } 6.2)\times 10^{-6}\left(\frac{\rm eV}{\overset{0}{m}}\right)^2,\\
&\quad\text{or}\qquad (1.9 \text{ to } 2.1)\times 10^{-4}\frac{\rm eV}{\overset{0}{m}}\,,
\end{aligned} \tag{33a}$$

$$\begin{aligned}
s^2_{36} &\simeq 427\left(\frac{\mu}{\overset{0}{m}}\right)^2 c^2_{36} \sim (1.5 \text{ to } 1.8)\times 10^{-3}\left(\frac{\rm eV}{\overset{0}{m}}\right)^2,\\
&\quad\text{or}\qquad (5.5 \text{ to } 5.9)\times 10^{-2}\frac{\rm eV}{\overset{0}{m}}\,,
\end{aligned} \tag{33b}$$

where $s_{25}^2 \ll c_{25}^2 \sim 1$ and $s_{36}^2 \ll c_{36}^2 \sim 1$ for $\mu^2 \ll \overset{0}{m}{}^2$. Hence, in our texture, the solar and atmospheric oscillation amplitudes are practically maximal,

$$\sin^2 2\theta_{\rm sol} \equiv c_{25}^2 \sim 1\,, \qquad \sin^2 2\theta_{\rm atm} \equiv \tfrac{1}{2}\left(1 + c_{25}^2\right) c_{36}^2 \sim 1\,, \tag{34}$$

when $\mu^2 \ll \overset{0}{m}{}^2$. If in the matrix $U^{(3)}$ given in Eq. (9) there were $c_{12} \gtrsim 1/\sqrt{2}$ and $s_{12} \lesssim 1/\sqrt{2}$ instead of $c_{12} = 1/\sqrt{2} = s_{12}$, then $\sin^2 2\theta_{\rm sol} \equiv (2c_{12}s_{12})^2 c_{25}^2$ and $\sin^2 2\theta_{\rm atm} \equiv (s_{12}^2 + c_{12}^2\, c_{25}^2) c_{36}^2$ would be suitably smaller.

From the third Eq. (32) we can see that in our texture the *predicted* LSND effect is potentially very small, perhaps unobservable, as having (due to Eq. (33) in the option (a) or(b)) the oscillation amplitude

$$\sin^2 2\theta_{\rm LSND} \simeq \tfrac{1}{2} s_{25}^4 \sim \left\{ \begin{array}{l} (1.4 \text{ to } 1.9) \times 10^{-11} \left(\frac{\rm eV}{\overset{0}{m}}\right)^4 \text{ or} \\ (1.9 \text{ to } 2.2) \times 10^{-8} \left(\frac{\rm eV}{\overset{0}{m}}\right)^2 \end{array} \right. , \tag{35a or b}$$

when $\mu^2 \ll \overset{0}{m}{}^2$. If *e.g.* $\overset{0}{m} = O(1\,{\rm eV}) - O(10^{-2}\,{\rm eV})$, where still $\mu^2 \ll \overset{0}{m}{}^2$, then $\sin^2 2\theta_{\rm LSND} = O(10^{-11}) - O(10^{-3})$ or $O(10^{-8}) - O(10^{-4})$. The corresponding mass-square scale is

$$\Delta m_{\rm LSND}^2 \simeq \left|\Delta m_{25}^2\right| \simeq \left\{ \begin{array}{l} m_2^2 \simeq \overset{0}{m}{}^2 + (1.1 \text{ to } 1.2) \times 10^{-5}\,{\rm eV}^2 \text{ or} \\ m_5^2 \simeq \overset{0}{m}{}^2 + (3.9 \text{ to } 4.2) \times 10^{-4} \overset{0}{m}\,{\rm eV} \end{array} \right. , \tag{36a or b}$$

where $\overset{0}{m} = m_1$ or m_4, respectively.

The fourth Eq. (32) describes the Chooz experiment for reactor $\bar{\nu}_e$'s. Due to its negative result, $P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \sim 1$, there appears the experimental constraint $(1 + c_{25}^2) s_{25}^2 \equiv \sin^2 2\theta_{\rm Chooz} \lesssim 0.1$ for s_{25}^2, if $\Delta m_{25}^2 \equiv \Delta m_{\rm Chooz}^2 \gtrsim 0.1\ {\rm eV}^2$ [7]. This implies for the LSND effect (in our texture) the very small Chooz upper bound

$$\sin^2 2\theta_{\rm LSND} \simeq \tfrac{1}{2}\, s_{25}^4 \lesssim 1.3 \times 10^{-3}\,, \tag{37}$$

if $\Delta m_{25}^2 \gg \Delta m_{32}^2 \sim (3.0 \text{ to } 3.5) \times 10^{-3}{\rm eV}^2$, what is consistent with $\Delta m_{25}^2 \gtrsim 0.1\ {\rm eV}^2$ and gives $(x_{25})_{\rm Chooz} \gg (x_{32})_{\rm Chooz} \simeq (x_{32})_{\rm atm} = O(1)$ as $(x_{ji})_{\rm Chooz} \simeq (x_{ji})_{\rm atm}$ numerically. Then

$$\sin^2 (x_{25})_{\rm Chooz} \simeq \tfrac{1}{2} \tag{38}$$

in the fourth Eq. (32). When combined with the formula (35) (in the option (a) or (b)), the Chooz bound (37) leads to the lower limit

$$\overset{0}{m} \gtrsim (1.0 \text{ to } 1.1) \times 10^{-2}{\rm eV} \quad \text{or} \quad (3.8 \text{ to } 4.1) \times 10^{-3}{\rm eV} \tag{39a or b}$$

for $\overset{0}{m} = m_1$ or m_4, still consistent with our requirement $\mu^2 \ll \overset{0}{m}{}^2$. Thus, Eq. (36) (in the option (a) or (b)) gives the very small lower limit $\Delta m^2_{\rm LSND} \gtrsim (1.1\,{\rm to}\,1.3) \times 10^{-4}{\rm eV}^2$ or $(1.6\,{\rm to}\,1.8) \times 10^{-5}{\rm eV}^2$ for the mass-square scale $\Delta m^2_{\rm LSND}$.

Note that Eqs. (32) imply the sum rule

$$\sin^2 2\theta_{\rm sol} + \sin^2 2\theta_{\rm LSND} + \tfrac{1}{2}\sin^2 2\theta_{\rm Chooz} \equiv c^2_{25} + \tfrac{1}{2}s^4_{25} + \tfrac{1}{2}(1+c^2_{25})s^2_{25} = 1 \quad (40)$$

for the solar, LSND and Chooz oscillation amplitudes. This leaves room for the LSND effect if $\sin^2 2\theta_{\rm sol} \equiv c^2_{25} < 1$ *i.e.*, if oscillations of solar ν_e's are not strictly maximal, what seems to be the case [5].

4. Conclusions

We presented in this paper an effective texture for six Majorana conventional neutrinos (three active and three sterile), based on the 6×6 mixing matrix defined in Eqs. (7)–(10), and on the conjectures (18) (in the option (a) or (b)) and (20) for the resulting mass matrix. The conjecture (20) requires that the Dirac 3×3 component of the unitarily transformed 6×6 mass matrix $\overset{0}{M} = \overset{1}{U}{}^\dagger M \overset{1}{U}$ be *similar* in structure to the charged-lepton and quark 3×3 mass matrices of the *universal form* (6), constructed previously with a considerable phenomenological success [1,2].

Such a texture *predicts* reasonably oscillations of solar ν_e's in the form not inconsistent with the LMA solar solution (for the option (a)), if the SuperKamiokande value of the mass-square scale for atmospheric ν_μ's is taken as an input. In both cases, neutrino oscillations are practically maximal. The proposed texture also *predicts* a potentially very small, perhaps unobservable, LSND effect with the oscillation amplitude of the order $O[10^{-11}(\,{\rm eV}/\overset{0}{m})^4]$ or $O[10^{-8}(\,{\rm eV}/\overset{0}{m})^2]$ and the mass-square scale of the order $O(\overset{0}{m}{}^2) + O(10^{-5}\,{\rm eV}^2)$ or $O(\overset{0}{m}{}^2) + O(10^{-4}\;\overset{0}{m}\;{\rm eV})$ (for the option (a) or (b), respectively). The negative result of Chooz experiment imposes on the oscillation amplitude of LSND effect (in our texture) a very small upper bound of the order of $O(10^{-3})$, implying for $\overset{0}{m}$ a lower limit of the order $O(10^{-2}\,{\rm eV})$ or $O(10^{-3}\,{\rm eV})$. If *e.g.* $\overset{0}{m} = O(1{\rm eV})$, then $\sin^2 2\theta_{\rm LSND} = O(10^{-11})$ or $O(10^{-8})$ and $\Delta m^2_{\rm LSND} = O(1\,{\rm eV}^2)$. In such a case, the LSND effect (in our texture) is, of course, unobservable (though it still exists in principle). Notice that the estimations following from the original LSND experiment [8] are *e.g.* $\sin^2 2\theta_{\rm LSND} = O(10^{-2})$ and $\Delta m^2_{\rm LSND} = O(1\,{\rm eV}^2)$. The new miniBooNE experiment may confirm or revise the original LSND results.

Note finally that, as far as the neutrino mass spectrum is concerned, our model of neutrino texture is of 3 + 3 type in contrast to the models of 3 + 1 or 2 + 2 types [9] discussed in the case when, beside three active neutrinos ν_e , ν_μ , ν_τ, one *extra* sterile neutrino ν_s exists. In these models, three Majorana conventional sterile neutrinos ν_{e_s} , ν_{μ_s} , ν_{τ_s} are decoupled through the familiar seesaw mechanism, as being practically identical with three very heavy mass states ν_4, ν_5, ν_6 (of the GUT mass scale). In our model ν_{e_s}, ν_{μ_s}, ν_{τ_s} are practically identical with three mass states ν_4, ν_5, ν_6 which (in the option (a)) are nearly massless or (in the option (b)) get masses only a little heavier (*e.g.* of the order of 1 eV).

REFERENCES

[1] W. Królikowski, in *Spinors, Twistors, Clifford Algebras and Quantum Deformations*, Proc. 2nd Max Born Symposium 1992, Eds. Z. Oziewicz *et al.*, Kluwer Acad. Press, 1993; *Acta Phys. Pol.* **B27**, 2121 (1996), and references therein.

[2] W. Królikowski, *Acta Phys. Pol.* **B30**, 2631 (1999); `hep-ph/0108157`, and references therein.

[3] W. Królikowski, *Nuovo Cim.* **A111**, 1257 (1998); *Acta Phys. Pol.* **B31**, 1759 (2000).

[4] For a theoretical summary *cf.* J. Ellis, *Nucl. Phys. Proc. Suppl.* **91**, 503 (2001), and references therein.

[5] For a recent analysis *cf.* M.V. Garzelli, C. Giunti, `hep-ph/0108191`; *cf.* also V. Barger, D. Marfatia, K. Whisnant, `hep-ph/0106207`; M.C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, `hep-ph/0108073`.

[6] Review of Particle Physics, *Eur. Phys. J.* **C15**, 1 (2000).

[7] M. Appolonio *et al.*, *Phys. Lett.* **B420**, 397 (1998); M. Appolonio *et al.*, *Phys. Lett.* **B466**, 415 (1999).

[8] G. Mills, *Nucl. Phys. Proc. Suppl.* **91**, 198 (2001); R.L. Imlay, Talk at ICHEP 2000 at Osaka; and references therein.

[9] *Cf. e.g.* V. Barger, B. Kayser, J. Learned, T. Weiler, K. Whisnant, *Phys. Lett.* **B489**, 345 (2000), and references therein; *cf.* also W. Królikowski, *Acta Phys. Pol.* **B32**, 1245 (2001); `hep-ph/0106350`.