MONOPOLE–ANTI-MONOPOLE BOUNDED PAIRS

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(Received November 11, 2001; revised version received January 13, 2002)

We show that in the dual version of the generalized Dick model monopole-anti-monopole pairs have finite energy. It is possible to use the potential between monopole and anti-monopole to find the mass spectrum of the glueballs. The results are discussed in connection with the Faddeev–Niemi model and toroidal soliton solutions. Some other finite energy configurations are found, both in the magnetic and electric sector.

PACS numbers: 11.15.Kc, 11.10.Ef, 12.39.Mk

1. The model

Recently it was shown that the Dick model, in the version discussed in [1, 2], can be used to model confinement of quarks (this version is slightly different from the original Dick model [3] but there is a sector of the parameters for which both the models give the same results [1]). Moreover, the confining potential becomes in agreement with the phenomenological data [4, 5] for the particular value of the parameter of the model. In that sense the modified Dick model is a good candidate for the effective model for the low energy QCD. However, such an effective model should describe not only quark–antiquark states but also glueballs. In the present paper a possible way of appearance of the glueballs in the framework of the modified Dick model is considered. It will occur that it is possible to look at the glueball states as monopole–anti-monopole bounded pairs. However, these pairs are found not in the original modified Dick model but in its "dual" version *i.e.* in the model where confinement of the quark sources in the original theory is interchanged with confinement of magnetic monopoles. Due to that the glueballs and the scalar mesons appear to be connected by kind of "dual" transformation. Because of the fact that the interaction of the (non)-Abelian

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magnetic monopoles is not understood sufficiently one can find our model interesting also from the mathematical point of view.

Let us consider the following action

$$S = \int d^4x \left[-\frac{1}{4} \left(\frac{\phi}{\Lambda} \right)^{-8\delta} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right], \tag{1}$$

where $\delta \geq \frac{1}{4}$ and Λ is a dimensional constant. Indeed, one can recognize in this action the dual version of the modified Dick model [1], with the following generalized "dual" transformation

$$F^a_{\mu\nu} \to \phi^{8\delta} \, {}^*F^a_{\mu\nu} \,. \tag{2}$$

The dual field tensor is defined in the standard way as ${}^*F^a_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. The "duality" (2) is equivalent to $\delta \longrightarrow -\delta$ that means the interchanging of the strong coupling with weak coupling sector.

The field equations for (1) read

$$D_\mu \left[\left(\frac{\phi}{\Lambda} \right)^{-8\delta} F^{a\mu\nu} \right] = j^{a\nu} \,, \tag{3}$$

$$\partial_\mu \partial^\mu \phi = 2\delta F^a_{\mu\nu} F^{a\mu\nu} \frac{\phi^{-8\delta-1}}{\Lambda^{-8\delta}} \,, \tag{4}$$

where $j^{a\mu}$ is the external color current density.

2. Magnetic solutions

Let us consider more detailed the Abelian magnetic sector. For example one can choose

$$A^a_\mu = \delta^a_3 A_\mu \,, \tag{5}$$

where $\mathrm{A}_0 = 0$, $A_i = A_i(x,y,z)$. Static Abelian monopole solutions can be obtained by means of the Bogomolny equations. Let us rewrite the energy of a static configuration as

$$E_N = \int d^3x \left[\frac{1}{4} \left(\frac{\phi}{\Lambda} \right)^{-8\delta} F_{ij} F_{ij} + \frac{1}{2} (\partial_i \phi)^2 \right]. \tag{6}$$

The corresponding Bogomolny equation is

$$F_{ij} = \left(\frac{\phi}{\Lambda} \right)^{4\delta} \epsilon_{ijk} \partial_k \phi \,. \tag{7}$$

It is easy to show that this field tensor fulfills the Gauss law automatically. Moreover, the equation of motion of the scalar field takes the same form as the Bianchi identity, namely

$$\frac{1}{r^2}\left(r^2\phi'\right)' + 4\delta\phi^{-1}\left(\phi'\right)^2 = 0\,. \tag{8}$$

One can find the singular solutions in the following form

$$\phi(r) = \mathcal{A}\Lambda\left(\frac{1}{\Lambda r}\right)^{1/(1+4\delta)}, \tag{9}$$

where $\mathcal{A} = [g(1+4\delta)]^{1/(1+4\delta)}$ and g is the charge of the monopole. Then the field tensor reads

$$F_{ij} = -g\epsilon_{ijk}\frac{x^k}{r^3}\,. \tag{10}$$

Of course, these Abelian monopoles are the well known Dirac monopoles with the string attached to them. For these configurations of the fields we get the energy density

$$\varepsilon = \mathcal{A}^{-8\delta}\frac{g^2}{r^4}\left(\frac{1}{\Lambda r}\right)^{-(8\delta)/(1+4\delta)}\,. \tag{11}$$

Indeed, the energy is divergent at large distances that is at $r \to \infty$. One can observe that the energy density has singularity also at $r = 0$ but this singularity is integrable. We conclude that single monopoles disappear from the physical spectrum of the theory.

The dual Dick model contains also nonsingular magnetic monopoles labeled by positive parameter β_0

$$\phi = \mathcal{A}\Lambda\left(\frac{1}{\Lambda r} + \frac{1}{\beta_0}\right)^{1/(1+4\delta)}\,. \tag{12}$$

The energy density reads

$$\varepsilon = \mathcal{A}^{-8\delta}\frac{g^2}{r^4}\left(\frac{1}{\Lambda r} + \frac{1}{\beta_0}\right)^{-(8\delta)/(1+4\delta)}\,. \tag{13}$$

These configurations give finite energy

$$E_N = \int \varepsilon r^2 dr = \Lambda\frac{4\delta+1}{4\delta-1}\mathcal{A}^{-8\delta}g^2\beta_0^{(4\delta-1)/(4\delta+1)}. \tag{14}$$

However, as it was shown in [2] the finite energy monopole sector can be removed by adding a potential term for the scalar field. It is easy to check that this potential can have the following form

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{\Lambda}\right)^{4+8\delta} . \tag{15}$$

Vanishing of the monopoles from the phisical spectrum is not sufficient to claim that confinement appears. It has to be checked weather a dipole-like state has a finite energy. We assume that monopole and anti-monopole lay on the z-axis in the distance $R/2$ from the origin. Then the equations of motion take the form:

$$\nabla \cdot \vec{B} = g \left[\delta \left(z - \frac{R}{2} \right) - \delta \left(z + \frac{R}{2} \right) \right] , \tag{16}$$

$$\nabla \times \left[\left(\frac{\phi}{\Lambda}\right)^{-8\delta} \vec{B} \right] = 0 \, , \tag{17}$$

$$-\frac{1}{r^2} \left(r^2 \phi' \right)' - 4\delta \frac{\phi^{-8\delta - 1}}{\Lambda^{-8\delta}} \vec{B}^2 = 0 \, . \tag{18}$$

We applied the Ansatz (5) and the magnetic field is given by

$$\vec{B} = \nabla \times \vec{A} \, . \tag{19}$$

Of course, due to the r.h.s. of (16) the field $\vec{A}$ cannot be regular everywhere *i.e.* the Dirac string is still present. In order to solve the remaining two equations we adopt the procedure presented in the papers [1, 6, 13]. Firstly, we introduce the cylindrical coordinates (ρ, ϕ, z), which are natural in our problem. Secondly, we express the vector gauge potential by means of a scalar flux field $\Phi(\rho, z)$ in the following way

$$\vec{A} = \frac{\hat{\phi}}{2\pi\rho} \Phi \, . \tag{20}$$

Using this expression we obtain from (17) and (18) the equations for the scalar fields

$$\nabla \left[\frac{1}{\rho} \left(\frac{\phi}{\Lambda}\right)^{-8\delta} \nabla \Phi \right] = 0 \, , \tag{21}$$

$$\nabla^2 \phi + 4\delta \left(\frac{\phi}{\Lambda}\right)^{-8\delta} \frac{\phi}{\rho} \left| \nabla \Phi \right|^2 = 0 \, . \tag{22}$$

To solve these equations one should find the boundary conditions. This can be done if we realize that in the monopole–anti-monopole case, the Dirac string has finite size and connects the monopoles. It is equivalent to

$$\begin{aligned} \Phi &= 0 && \text{for } \rho = 0,\ |z| > \frac{R}{2}\,, \\ \Phi &= g && \text{for } \rho = 0,\ |z| < \frac{R}{2}\,, \\ \Phi &\longrightarrow 0 && \text{for } \rho^2 + z^2 \longrightarrow \infty\,, \end{aligned} \tag{23}$$

where the last condition emerges due to the expectation that energy density has to fall down to zero at the spatial infinity. It is obvious that the equations (21), (22) are too complicated to find analytical solutions. However, it is possible to construct an approximate solution which obeys the boundary conditions and has finite energy:

$$\Phi = g\left(\frac{z+\frac{R}{2}}{\sqrt{\rho^2+\left(z+\frac{R}{2}\right)^2}} - \frac{z-\frac{R}{2}}{\sqrt{\rho^2+\left(z-\frac{R}{2}\right)^2}}\right), \tag{24}$$

$$\phi = \mathcal{A}\Lambda\left(\frac{1}{\Lambda\sqrt{\rho^2+\left(z+\frac{R}{2}\right)^2}}\right)^{1/(1+4\delta)} - \mathcal{A}\Lambda\left(\frac{1}{\Lambda\sqrt{\rho^2-\left(z-\frac{R}{2}\right)^2}}\right)^{1/(1+4\delta)}. \tag{25}$$

Although these functions do not obey the field equations one can use them to find the upper bound for the total field energy [2]. We get

$$E_{\text{pair}} = \beta\, g^{2/(1+4\delta)}\Lambda(\Lambda R)^{(4\delta-1)/(4\delta+1)}\,, \tag{26}$$

where β is a finite numerical constant. As it was said before, the quark–anti-quark potential derived from the modified Dick model with $\delta = 3/4$ can be applied to obtain the spectrum of masses in the quarkonium system. The masses, for many different effective potentials, were obtained using the Klein–Gordon equation [7]. It seems reasonable to expect that the masses of glueballs can be also found in the similar "effective potential" model. Now, the effective potential is a potential between monopole and anti-monopole in the dual version of the original model (26). So one can apply the Klein–Gordon equation and find the masses M_n of the glueball states. Actually, the effective potential idea in the glueball physics has been recently very successfully exploited [8]. For example, it was shown by West [9] that the lightest glueball 0^{++} can be understood as a bound state of the massless gluons with the potential:

$$U_{\text{West}} = \frac{9}{4}\sigma R - \frac{\alpha}{R}\,, \tag{27}$$

where σ is the string tension and α is the strong coupling constant. One can immediately see that our model admits the confining part of the West potential in the limit $\delta \longrightarrow \infty$. This limit is equivalent, in the original modified Dick model, to the well known string picture of the quarks confinement [1,2]. Very similar results were also obtained for the dual Ginzburg–Landau model [10,11].

The non-Abelian magnetic monopoles can be obtained if we take into account all non-Abelian degrees of freedom in the magnetic sector. In fact, there is the Wu-Yang non-Abalian monopole

$$A_i^a = \epsilon_{\mathrm{aik}} \frac{x^k}{r^2}, \qquad A_0^a = 0, \tag{28}$$

$$\phi = \mathcal{C}\Lambda \left(\frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{1/(1+4\delta)}, \tag{29}$$

where $\mathcal{C} = (1+4\delta)^{1/(1+4\delta)}$. The Dirac string is no longer present. The point-like singularity which we observe in the gauge potential is integrable at the energy density level. Moreover, identically as in the Abelian case, the total energy, for $\beta_0 = \infty$, is infinite due to the fact that the energy density falls too slowly at the spatial infinity. One can check that the energy of the non-Abelian dipole is finite. So, the confining behavior is visible also in the non-Abelian theory. The approximate monopole–anti-monopole solution reads

$$A_i^a = \epsilon_{\mathrm{aik}} \left(\frac{x^k + x_0^k}{x^2 + y^2 + \left(z + \frac{R}{2}\right)^2} - \frac{x^k - x_0^k}{x^2 + y^2 + \left(z - \frac{R}{2}\right)^2} \right). \tag{30}$$

Here $x_0 = (0, 0, R/2)$ is the position of one of the monopoles. The scalar function has the form (25).

3. Electric solutions

In order to have the general picture of the physics in the dual modified Dick model we discuss the Coulomb problem. For simplicity we restrict ourselves to the static non-Abelian source

$$j^{a\mu} = 4\pi\, q\delta(r)\, \delta^{3a} \delta^{\mu 0}. \tag{31}$$

The field equations take the form

$$\left[r^2 \left(\frac{\phi}{\Lambda} \right)^{-8\delta} E \right]' = q\delta(r). \tag{32}$$

$$\nabla_r^2 \phi = 4\delta E^2 \, \frac{\phi^{-8\delta - 1}}{\Lambda^{-8\delta}} \, . \tag{33}$$

Here $E^{ai} = -F^{a0i}$ and $\vec{E}^a = E\delta^{3a}\hat{r}$. The solutions of these equations form the family parameterized by β_0

$$\phi(r) = \mathcal{B}\Lambda \left(\frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{1/(1-4\delta)} , \tag{34}$$

$$E(r) = \mathcal{B}^{8\delta} \frac{q}{r^2} \left(\frac{1}{\Lambda r} + \frac{1}{\beta_0} \right)^{8/(1-4\delta)} , \tag{35}$$

where $\mathcal{B} = [q(1-4\delta)]^{1/(1-4\delta)}$. The energy for the family is finite and has the form

$$E_N = \Lambda \frac{4\delta - 1}{4\delta + 1} q^2 \mathcal{B}^{8\delta} \beta_0^{(4\delta+1)/(4\delta-1)} \, . \tag{36}$$

Similar to the magnetic sector there is a singular solution of the Coulomb problem which is divergent at the spatial infinity

$$\phi(r) = \mathcal{B}\Lambda \left(\frac{1}{\Lambda r} \right)^{1/(1-4\delta)} , \tag{37}$$

$$E(r) = \mathcal{B}^{8\delta} q \Lambda^2 \left(\frac{1}{\Lambda r} \right)^{2/(1-4\delta)} . \tag{38}$$

However, in that case the singularity is strong enough to remove also quark–anti-quark solution from physical spectrum *i.e.* the energy of such configurations is still infinite.

4. Conclusions

In our work we have pointed out the model which admits the bounded monopole–anti-monopole states, whereas the single monopole solution has infinite energy. Such objects we call magnetic mesons. We assume that these mesons can be interpreted as glueballs. The potential between monopole and anti-monopole can be used to find the mass spectrum of the glueballs. In the limit $\delta \longrightarrow \infty$ we reconstruct the confining part of the famous West potential. This result is in agreement with the standard superconductor picture where the monopole potential grows linearly [11]. Moreover, in our picture, glueballs appear to be "dual" objects to scalar mesons. That is quark–anti-quark states and glueballs can be described by means of actions which are connected by a very simple "dual" transformation (2). We believe that this property is not unique and should be observed in other models which are used to model quarks confinement. For example, it should be possible to find the transformation which interchanges the quarks confining sector with the magnetic monopoles confining sector in the Pagels–Tomboulis effective model [12, 13].

Recently it was observed by Faddeev and Niemi [14], inspired by Cho [15], that at low energies the appropriate order parameter is a unit length vector field n^a, $a = 1, 2, 3$. For the pure SU(2) Yang–Mills theory they have proposed the effective action

$$S_{\mathrm{FN}} = \int d^4x \left[m^2 \left(\partial_\mu \vec{n}\right)^2 + \frac{1}{e^2} \left(\vec{n}, \partial_\mu \vec{n} \times \partial_\nu \vec{n}\right)^2 \right], \tag{39}$$

where m is a mass parameter and e is a coupling constant. This action has nontrivial topology. Namely, localized static solutions where $\vec{n} \longrightarrow \vec{n}_0$ for $r \to \infty$ can be understood as a map from S^3 to S^2. These maps are divided into homotopy classes $\pi_3(S^2) \simeq Z$ numbered by the Hopf invariant. Such knotted solutions were found for several topological numbers [16]. One can identify them with the so called magnetic glueballs [17] which are supposed to form physical spectrum of the gauge theory in the low energy limit. Moreover, as it was mentioned in [17], the knotted solitons have neither baryonic nor monopole charges. Following that, we expect that it is possible to interpret knots as bound states consisting of monopole–anti-monopole pairs. One can suppose, that the Faddeev–Niemi model and the dual version of the modified Dick model refer to the same physics seen from non-topological or topological point of view, respectively. Then the magnetic glueball is a topological object in the Faddeev–Niemi theory, with appropriate Hopf number or a non-topological soliton in the dual modified Dick model. Unfortunately, we do not know how the models correspond to each other. However, as it was shown in [10] it is possible to obtain the glueball 0^{++} in the toroidal as well as in the effective potential framework in the dual Ginzburg–Landau model. Because of that one can try to find the potential representation of the Faddeev–Niemi model and fit our parameter δ to it.

Both the problems *i.e.* the glueballs spectrum as well as the connection between the Faddeev–Niemi and our model will be considered in the next papers.

We would like to thank Professor H. Arodź for many helpful comments and suggestions.

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