Vol. 33 (2002) *ACTA PHYSICA POLONICA B* No 4

STOCHASTIC VARIATIONS OF GALACTIC COSMIC RAYS*

Michael V. Alania

Institute of Mathematics and Physics of Podlasie University
3 Maja 54, 08-110 Siedlce, Poland
Institute of Geophysics Georgian Academy of Sciences, Tbilisi, Georgia
e-mail: alania@ap.siedlce.pl

(Received February 6, 2002)

The generalized anisotropic diffusion tensor, streams and drift velocities of Galactic Cosmic Rays (GCR) for the three dimensional Interplanetary Magnetic Field (IMF) have been analysed. Stochastic and regular changes of GCR, especially 11-year and 27-day variations have been studied. It is stressed that in seventies the generalized anisotropic diffusion tensor has been rarely used due to lack of the direct evidence of the latitudinal component of the IMF. However, now this tensor must be largely used as far as the experimental data and theoretical investigations show the existence of the latitudinal component of the IMF, *i.e.* heliospheric magnetic field is three-dimensional. The nature of the 11-year variation of GCR is critically considered. It is concluded that the general mechanism of the 11-year variation of GCR must be the change of the structure of the stochastic IMF. Particularly the effective size of the irregularities of the IMF responsible for the diffusion of GCR increases in the minima epochs of solar activity with respect to the maxima epochs. Thus, the different character of the diffusion of GCR in different epochs of solar activity is the general mechanism of 11-year variation of GCR. The temporal changes of the energy spectrum of the 11-year variations of GCR *versus* the solar activity, namely soft energy spectrum in the maxima epochs and hard one in the minima epochs, conform this conclusion. The modelling and experimental investigations show that the amplitude of the 27-day variations of GCR is greater about 1.5 times in the period of the $qA>0$ solar magnetic cycle than in the period of the solar magnetic cycle $qA<0$, which is not yet well explained according to the modern theory of GCR modulation.

PACS numbers: 96.40.Cd, 05.40.Ca

* Presented at the XIV Marian Smoluchowski Symposium on Statistical Physics, Zakopane, Poland, September 9–14, 2001.

1. Introduction

An expansion of the solar wind plasma (the corona of the Sun) into the interplanetary space caused an extension of the lines of the force of the general solar magnetic field and there is created the Interplanetary Magnetic Field (IMF) (Parker, (1958), (1963)). According to Parker(1958) a magnetic field that is convected radially outward with the solar wind and is strongly connected with the rigidly rotating Sun will execute the spiral pattern. The strength $H(r,t)$ of the IMF and the solar wind velocity $U(r,t)$ were considered as the sums of two components. The regular parts, H_0 and U_0 and the stochastic components, $H_1(r,t)$ and $U_1(r,t)$. Thus, the averages of the strength $\langle H(r,t)\rangle$ of the IMF, and the solar wind velocity $\langle U(r,t)\rangle$ can be represented as $\langle H(r,t)\rangle = \langle H_0\rangle + \langle H_1(r,t)\rangle$ and $\langle U(r,t)\rangle = \langle U_0\rangle + \langle U_1(r,t)\rangle$. The angle brackets denote averaging over the statistical ensembles of the strength of the IMF and the solar wind velocity. The average values of the $\langle H_1(r,t)\rangle$ and $\langle U_1(r,t)\rangle$ equal zero, $\langle H_0\rangle = H_0, \langle U_0\rangle = U_0$, so $\langle H(r,t)\rangle = H_0$ and $\langle U(r,t)\rangle = U_0$. Stochastic parts of the solar wind velocity and of the IMF strength are the source of the short and large scale inhomogeneities (irregularities) in the interplanetary space responsible for the scattering of Galactic Cosmic Rays (GCR), while H_0 is responsible for the drift and anisotropic diffusion and U_0 — for the convection and energy change of GCR. Regular, quasiregular (with different periods) and stochastic changes of GCR intensity in the large energy range are connected with similar changes of the solar activity and solar wind parameters. An important parameter for the modulation of GCR is a tensor of diffusion containing uncertainties due to the vaguely known stochastic structure of the IMF. In spite of the availibity of the direct measurements of the parameters of the interplanetary medium (of course in different points in space), the parameters characterizing not only the stochastic parts of the IMF, but also the regular part of the IMF, are not well known. Particularly, it is of interest how the regular latitudinal component of the IMF influence on the different classes of GCR variations in whole of the inner and outer heliosphere. In this paper we review our recent investigations concerning the problem of the tensor of anisotropic diffusion of GCR involving the latitudinal component of the IMF, *i.e.* for the three dimensional IMF and study peculiarities of 11-yar and 27-day variations of GCR intensity.

2. Anisotropic diffusion tensor of GCR for one, two and three dimensional IMF

In order to obtain the anisotropic diffusion tensor of GCR for the three-dimensional IMF the Maxwell–Boltzmann equation for plasma state in the phase space (*e.g.* Huxley and Crompton, (1977)) has been adjusted to the distribution of GCR particles (protons) as in Alania (1980),

$$\frac{\partial F}{\partial t} + V\frac{\partial F}{\partial r} + q\left[E + \frac{V \times B}{c}\right]\frac{\partial F}{\partial P} + F_{\mathrm{st}} = 0\,, \tag{1}$$

where, $F = nf$ and V, p, q, n are velocity, momentum, charge and density of GCR particles, respectively, c is speed of light, F_{st} represents the change of the distribution function due to collisions of GCR particles with the magnetic irregularities of solar wind, f is a distribution function in the velocity space, t is time, E is electric field and B is the induction of the IMF. Taking into account that B is considered as a "frozen" in the high conductivity solar wind plasma, one can assume, that $E = -\frac{1}{c}(U_{\mathrm{sw}} \times B)$, where U_{sw} is a velocity of solar wind, equation (1) can be written:

$$\frac{\partial F}{\partial t} + V\frac{\partial F}{\partial r} + q\left[(V - U_{\mathrm{sw}}) \times B\right]\frac{\partial F}{\partial P} + F_{\mathrm{st}} = 0\,. \tag{2}$$

Equation (2) can be simplified assuming that:

(a) $U_{\mathrm{sw}} = 0$ (a motion of particles is considered in a frame of reference rigidly connected with the lines of the IMF);

(b) a distribution function f is axially symmetrical with respect to the azimuthal angle φ in the velocity space, $f(r,V,\theta,\varphi,t) \equiv f(r,V,\theta,t)$, and can be represented as the converging series:

$$f\,(r,V,\theta,t) = f_0\,(r,V,t) + \sum_{k=1}^{\infty} f_k\,(r,V,t)\,P_k(\cos\theta)\,, \tag{3}$$

where $P_k(\cos\theta)$ are Legendre polynomials;

(c) for the case of isotropic scattering, $F_{\mathrm{st}} \approx \nu f_1$, where ν is the frequency of scattering of GCR particles on the magnetic irregularities of solar wind, and

(d) there exists a stream of GCR particles with the average velocity U due to the weak disturbances of the distribution function of the particles and drift in the regular IMF equalling to $\nu f_1/3f_0$ ($U = \nu f_1/3f_0$).

Taking into account all above-mentioned assumptions the equation (2) can be written (Huxley and Crompton, (1977)) for GCR protons (Alania, (1980)) in the form:

$$\nu F_0 + \omega \times F_0 U = -\frac{V^2}{3}\mathrm{grad}\,F_0\,, \tag{4}$$

where $F_0 = nf_0$, $\omega = qB/mc$, (m is mass of GCR particles), B is the induction of the three dimensional IMF (B_x, B_y, B_z). It is clear that the equation (4) is simplified, but good enough (as it will be shown below) to obtain the anisotropic diffusion tensor of GCR in the frame of reference connected with the lines of the IMF and in the heliocentric frame of reference for the two and three dimensional IMF. The system of the scalar equations obtained from the vector equation (4) has the form:

$$\begin{aligned} F_0\left(\nu U_x - \omega_z U_y + \omega_y U_z\right) &= -\frac{V^2}{3}\frac{\partial F_0}{\partial x}\,, \\ F_0\left(\omega_z U_x + \nu U_y - \omega_x U_z\right) &= -\frac{V^2}{3}\frac{\partial F_0}{\partial y}\,, \\ F_0\left(-\omega_y U_x + \omega_x U_y + \nu U_z\right) &= -\frac{V^2}{3}\frac{\partial F_0}{\partial z}\,, \end{aligned} \tag{5}$$

where $\omega^2 = \omega_x^2 + \omega_y^2 + \omega_z^2 \quad (\omega_x = qB_x/mc;\ \omega_y = qB_y/mc;\ \omega_z = qB_z/mc)$

System of the equations (5) can be rewritten in matrix form:

$$\|M\|\, F_0 U = -\frac{V^2}{3}\mathrm{grad}\, F_0\,, \tag{6}$$

where

$$\|M\| = \begin{pmatrix} \nu & -\omega_z & \omega_y \\ \omega_z & \nu & -\omega_x \\ -\omega_y & \omega_x & \nu \end{pmatrix}.$$

A stream of GCR due to the weak disturbances of the distribution function f and drift of GCR particles in the regular IMF can be represented as:

$$I = -F_0 U = -\|M\|^{-1}\frac{V^2}{3}\mathrm{grad}\, F_0\,, \tag{7}$$

where

$$\|M\|^{-1} = \frac{1}{\nu\left(\omega^2 + \nu^2\right)}\left[\delta_{ij}\nu^2 + \varepsilon_{ijk}\omega_k\nu + \omega_i\omega_j\right] \tag{8}$$

and ε_{ijk} is the unit skew symmetric tensor.

The expression

$$K_{I,J} = \|M\|^{-1}\frac{V^2}{3} \qquad (I, J = x, y, z) \tag{9}$$

can be considered as the anisotropic diffusion tensor of GCR in the interplanetary space. The expressions

$$\frac{V^2}{3\nu\left(\omega^2 + \nu^2\right)}\left[\delta_{ij}\nu^2 + \omega_i\omega_j\right] \tag{10}$$

and

$$\frac{V^2}{3\nu\left(\omega^2+\nu^2\right)}\varepsilon_{ijk}\omega_k\nu \tag{11}$$

are the symmetric and antisymmetric components of the diffusion tensor (9) for the three dimensional IMF in Cartesian coordinate system, respectively.

For the frame of reference connected with the regular magnetic field $B(1,0,0)$, *i.e.* $B_x = B$, $B_y = B_z = 0$, and $\omega_x = \omega$, one can obtain from (8):

$$\|M\|^{-1} = \frac{1}{\nu\left(\nu^2+\omega^2\right)}\begin{pmatrix} \nu^2+\omega^2 & 0 & 0 \\ 0 & \nu^2 & \nu\omega \\ 0 & -\nu\omega & \nu^2 \end{pmatrix}. \tag{12}$$

The expression (9) can be rewritten:

$$K_{I,J} = \begin{pmatrix} K_{\|} & 0 & 0 \\ 0 & K_{\perp} & K_d \\ 0 & -K_d & K_{\perp} \end{pmatrix} \tag{13}$$

where $K_{\|} = \frac{V^2}{3\nu}$, $K_{\perp}=\frac{V^2\nu}{3\left(\omega^2+\nu^2\right)}$ and $K_{\mathrm{d}} = \frac{V^2\omega}{3\left(\omega^2+\nu^2\right)}$ are parallel, perpendicular and drift diffusion coefficients of GCR in the regular IMF, respectively.

The expression (13) can be represented as:

$$K_{I,J} = K_{\|}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & \alpha_1 \\ 0 & -\alpha_1 & \alpha \end{pmatrix} \qquad I,J = 1,2,3\,, \tag{14}$$

$K_{\|} = \nu\lambda/3$, $\alpha = K_{\perp}/K_{\|} = \frac{1}{1+\omega^2\tau^2}$, $\alpha_1 = K_d/K_{\|} = \frac{\omega\tau}{1+\omega^2\tau^2}$, where according to relationships: $V{=}\lambda/\tau$ and $\nu{=}\ 1/\tau$ (λ is the free path and τ — the time between two sequence collisions of GCR particles with the solar wind irregularities).

The expression (14) is the anisotropic diffusion tensor for GCR in the frame of reference of the regular IMF. This type of tensor was obtained by Chapman and Cowling (1960) for the description of the ionized gas behavior in the regular magnetic field.

For long time the IMF (heliospheric magnetic field) has been considered as an Archimedes spiral, having only the radial and heliolongitudinal components. The anisotropic diffusion tensor for the two dimensional B_r and B_φ of the IMF in spherical coordinate system (r,θ,φ) was obtained, *e.g.*

by Dorman (1968) as a transformation of type $(AK_{I,J}A^{\mathrm{T}})$, where A is the rotational matrix and A^{T} is the transposed one of the matrix A

$$A = \begin{pmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{pmatrix}. \tag{15}$$

The anisotropic diffusion tensor for the two dimensional (B_r and B_θ) IMF has the form:

$$K_{ij} = K_{\|} \begin{pmatrix} \cos^2\psi + \alpha\sin^2\psi & -\alpha_1\sin\psi & (\alpha-1)\cos\psi\sin\psi \\ \alpha_1\sin\psi & \alpha & \alpha_1\cos\psi \\ (\alpha-1)\cos\psi\sin\psi & -\alpha_1\cos\psi & \alpha\cos^2\psi + \sin^2\psi \end{pmatrix} \quad (i,j = r,\theta,\varphi)\,, \tag{16}$$

where ψ is the angle between the lines of the IMF and radial direction from the Sun.

In seventies there was not yet any explicit indications about the existence of the regular latitudinal component of the IMF except the data of the IMF, King (1977) and some papers, *e.g.* Slavin and Smith (1983) showing that there exists a sign alternating nonregular latitudinal component, B_θ. However, in our papers Alania (1978), (1980) the existence of the regular latitudinal component B_θ of the IMF was assumed and the generalized anisotropic diffusion tensor, expressions for the components of the drift velocity and stream of GCR for the three-dimensional IMF have been obtained. The generalized anisotropic diffusion tensor of GCR for the three-dimensional IMF was obtained as a transformation of type $(CK_{ij}C^{\mathrm{T}})$, where K_{ij} is the anisotropic diffusion tensor of GCR for the two-dimensional IMF; C is the rotational matrix and C^{T} is the transposed one of the matrix C.

$$C = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{17}$$

γ is the angle between the lines of the IMF and radial direction from the Sun in the meridian plane.

The anisotropic diffusion tensor of GCR for the three dimensional IMF in the Cartesian coordinate system B (B_x, B_y, B_z) has the following form:

$$\begin{aligned}
\varkappa_{11} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\nu^2+\omega_x^2\right), & \varkappa_{21} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_x\omega_y-\nu\omega_z\right), \\
\varkappa_{12} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_x\omega_y+\nu\omega_z\right), & \varkappa_{22} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\nu^2+\omega_y^2\right), \\
\varkappa_{13} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_x\omega_z+\nu\omega_y\right), & \varkappa_{23} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_y\omega_z+\nu\omega_x\right), \\
\varkappa_{31} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_x\omega_z+\nu\omega_y\right), \\
\varkappa_{32} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\omega_y\omega_z-\nu\omega_x\right), \\
\varkappa_{33} &= \tfrac{V^2}{3\nu(\nu^2+\omega^2)}\left(\nu^2+\omega_z^2\right). && (18)
\end{aligned}$$

The anisotropic diffusion tensor of GCR for the three dimensional IMF ($\varkappa_{ij} = K_{i,j}\ i,j = 1,2,3$) in the spherical coordinate system B (B_r, B_θ, B_φ) corresponding to the anisotropic diffusion tensor of GCR in Cartesian coordinate system (18) has the following form:

$$\begin{aligned}
\varkappa_{11} &= K_{||}\left[\cos^2\gamma\cos^2\psi+\alpha\left(\cos^2\gamma\sin^2\psi+\sin^2\gamma\right)\right], \\
\varkappa_{12} &= K_{||}\left[\sin\gamma\cos\psi\cos^2\psi\,(1-\alpha)-\alpha_1\sin\psi\right], \\
\varkappa_{13} &= K_{||}\left[\sin\psi\cos\gamma\cos\psi\,(\alpha-1)-\alpha_1\sin\gamma\cos\psi\right], \\
\varkappa_{21} &= K_{||}\left[\sin\gamma\cos\gamma\cos^2\psi\,(1-\alpha)+\alpha_1\sin\psi\right], \\
\varkappa_{22} &= K_{||}\left[\sin^2\gamma\cos^2\psi+\alpha_1\left(\sin^2\gamma\sin^2\psi+\cos^2\gamma\right)\right], \\
\varkappa_{23} &= K_{||}\left[\sin\gamma\sin\psi\cos\psi\,(\alpha-1)+\alpha_1\cos\gamma\cos\psi\right], \\
\varkappa_{31} &= K_{||}\left[\cos\gamma\sin\psi\cos\psi\,(\alpha-1)+\alpha_1\sin\gamma\cos\psi\right], \\
\varkappa_{32} &= K_{||}\left[\sin\gamma\sin\psi\cos\psi\,(\alpha-1)-\alpha_1\cos\gamma\cos\psi\right], \\
\varkappa_{33} &= K_{||}\left[\sin^2\psi+\alpha\cos^2\psi\right], \qquad (19)
\end{aligned}$$

where $\gamma = \arctan(B_\theta/B_r)$ and $\psi = \arctan(-B_\varphi/B_r)$ in the spherical coordinate system for $qA>0$ solar magnetic cycle.

The importance of the anisotropic diffusion tensor for the three dimensional IMF was first mentioned at the ICRC in Kyoto (1979) (*e.g.* Alania and Dorman, (1979); Alania and Japiashivli, (1979)) and has been used for the calculation of the role of the heliolatitudinal component B_θ of the IMF in GCR modulation by Alania *et al.* (1982).

At the beginning of the eighties the observations of the IMF by space probes in the heliosphere confirm, on average, that the IMF closely follows the predicted Parker spiral. However, it has been found that the IMF has a significant latitudinal component and the Archimedean spiral character of the IMF can be violated in high heliolatitudes, Slavin and Smith (1983), Smith *et al.* (1995). First attempts to modify the Parker's spiral field have been done by Nagashima *et al.* (1986). Then Jokipii and Kota (1989) reexamined the structure of the IMF in the polar regions and found that the magnetic field may depart significantly from the structure inferred from applications of the Archimedean spiral. Fisk (1996) stressed that a significant correction needs to be made to the Parker spiral pattern, for the simple reason that the Sun does not rotate rigidly, but rather rotates differentially, with the solar poles rotating on the order of 20% slower than the solar equator. According to the experimental data, *e.g.* Smith *et al.* (1995) and the theoretical investigation (*e.g.* Fisk (1996), (2001); Giacalone, (2001)) it can be stated that there exists the polar (heliolatitudinal) component of the heliospheric magnetic field, and more, the Archimedean spiral character of the IMF is significantly violated in the high heliolatitude regions (Fisk, (2001)). Thus, the generalized anisotropic diffusion tensor for three-dimensional IMF (19) must be used in modelling of GCR propagation in the interplanetary space.

3. Streams and drift velocities of GCR in the interplanetary space

The expected three-dimensional streams induced weak disturbances of the distribution function of GCR in interplanetary space and the GCR particle's drift in the regular IMF ($\operatorname{grad} F_0 \neq 0$) can be represented as:

$$I = -\frac{V^2}{3\nu\left(\nu^2+\omega^2\right)}\left[\delta_{ij}\nu^2+\varepsilon_{ijk}\omega_k\nu+\omega_i\omega_j\right]\operatorname{grad} F_0 \tag{20}$$

or

$$I = -\frac{V^2\tau}{3\nu\left(1+\omega^2\tau^2\right)}\left[\delta_{ij}+\omega_i\omega_j\tau^2\right]\operatorname{grad} F_0 - \frac{V^2\tau}{3\nu\left(1+\omega^2\tau^2\right)}\varepsilon_{ijk}\omega_k\tau\operatorname{grad} F_0 \,. \tag{21}$$

The expression (21) can be represented as the sum of two different streams $I = I_1 + I_2$, where:

$$I_1 = -\frac{V^2\tau}{3\left(1+\omega^2\tau^2\right)}\left[\delta_{ij}+\omega_i\omega_j\tau^2\right]\operatorname{grad} F_0 \,, \tag{22}$$

$$I_2 = -\frac{V^2\tau}{3\left(1+\omega^2\tau^2\right)}\varepsilon_{ijk}\omega_k\tau\operatorname{grad} F_0 \,. \tag{23}$$

Diffusion stream I_1 of GCR exists due to the symmetric part of the diffusion tensor and the drift stream I_2 due to the antisymmetric part of the anisotropic diffusion tensor in the three dimensional regular IMF. Using the expressions of GCR streams (22) and (23) it is possible to calculate the expected three components of the anisotropy of GCR.

The radial ($U_{\mathrm{D},r}$), latitudinal ($U_{\mathrm{D},\theta}$) and azimuthal ($U_{\mathrm{D},\varphi}$) components of drift velocity of GCR for the three dimensional IMF B (B_r,B_θ,B_φ) in the spherical coordinate system can be expressed as:

$$\begin{aligned}
\langle U_{\mathrm{D},r}\rangle &= \frac{\varkappa_0}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\alpha_1\sin\theta\sin\psi\right) + \frac{\varkappa_0}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(-\alpha_1\cos\psi\sin\gamma\right) , \\
\langle U_{\mathrm{D},\theta}\rangle &= \frac{\varkappa_0}{r}\frac{\partial}{\partial r}\left(r\alpha_1\sin\psi\right) + \frac{\varkappa_0}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\alpha_1\cos\psi\cos\gamma\right) , \\
\langle U_{\mathrm{D},\varphi}\rangle &= \frac{\varkappa_0}{r}\frac{\partial}{\partial r}\left(r\alpha_1\sin\gamma\cos\psi\right) + \frac{\varkappa_0}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\alpha_1\sin\theta\cos\gamma\cos\psi\right) . \quad (24)
\end{aligned}$$

These generalized components of drift velocity can be used for modelling of GCR propagation in three-dimensional magnetic field. All spatial components of the stream of GCR in the three dimensional IMF only due to drift effect are (Alania, (1978)):

$$\begin{aligned}
I_r &= \mp\alpha_1\sin\psi\,\nabla_\theta^n \mp \alpha_1\sin\gamma\cos\psi\nabla_\varphi^n , \\
I_\theta &= \pm\alpha_1\sin\psi\,\nabla_r^n \pm \alpha_1\cos\gamma\cos\psi\nabla_\varphi^n , \\
I_\varphi &= \pm\alpha_1\sin\gamma\cos\psi\,\nabla_r^n \mp \alpha_1\cos\gamma\cos\psi\nabla_\theta^n , \quad (25)
\end{aligned}$$

where ∇_r^n, ∇_θ^n, ∇_φ^n are spatial gradients of GCR density n. Thus, taking advantage of the assumptions (3) and others accepted above, the equation (2) was reduced to the equation (4), which is simple but at the same time good enough to obtain the correct expressions for the tensor of anisotropic diffusion for the one, two and three dimensional IMF. This generalized anisotropic diffusion tensor can be largely used in solving of the Parker's transport equation for different types of three-dimensional IMF (Parker, (1958); Jokipi and Kota, (1989); Fisk, (1996), (2001))

4. 11-year and 27-day variations of GCR

To study stochastic and regular character of the different classes of GCR variations Parker's transport equation obtained based on the equation (2) and on the solar wind theory (Parker, (1958), (1963), (1965)) including the cooling of GCR in the expended solar wind (Singer *et al.*, (1963)) has been used

$$\frac{\partial N}{\partial t} = \nabla_i\left(\varkappa_{i,j}\nabla_j N\right) - \nabla_i\left(U_i N\right) + \frac{1}{3R^2}\frac{\partial\left(R^3 N\right)}{\partial R}\left(\nabla_i U_i\right) , \quad (26)$$

where N, and R are density (in interplanetary space) and rigidity of GCR particles, respectively; in the right side of the equation (26) the first term describes diffusion due to the symmetric part and drift due to the antisymmetric part of the anisotropic diffusion tensor $\varkappa_{ij}$; the second term describes convection and third one a change of the energy of GCR particles due to interaction with solar wind. U_i is the solar wind velocity and t is time. Krymski (1964), Dorman (1965), Axword (1965), Gleeson and Axford (1968), Dolginov and Toptigin (1968) have contributed significantly to the development of the theory of GCR modulation. Papers of Jokipii (1971), Jokipii *et al.* (1977), Isenberg and Jokipii (1979), Jokipii and Kopriva (1979), Jokipii and Thomas (1981) and Jokipii and Kota (1983), Potgieter and Moraal (1985) and late Burger and Potgieter (1989) were the significant contributions in the new understanding of the role of drift in the regular heliospheric magnetic field for the modulation of GCR. It is obvious that in the drift theory of GCR modulation the spatial structure of the regular interplanetary magnetic field (IMF) must play very important role, *i.e.* the using of the generalized tensor for the three dimensional IMF is vital one in solving Parker's transport equation including drift.

4.1. 11-year variations of GCR

For a long time much effort has been put to explain features of GCR modulation in the large energy range, including anomalous component. Convection, diffusion, drift and energy changes of GCR due to the interaction with the solar wind (adiabatic cooling or acceleration of cosmic rays depending on the character of the solar wind velocity changes) are the general reasons of GCR modulation. Earlier, Parker (1958), (1963), (1965), Dorman (1963), Krymsky (1969), Jokipii (1971), then Fisk (1976), Levy (1978), Jokipii *et al.* (1977), Jokipii and Kopriva (1979), Wibberenz (1979), Jokipii and Kota (1983), Palmer (1982) and relatively late, Jokipii (1986), Burger and Potgieter (1989), Kota and Jokipii (1989), (1998), Bieber *et al.* (1994), Potgieter and Roux (1992), Fisk (1996), Bieber, Wanner, Matthaeus (1996), Bieber and Matthaeus (1997) have made much contribution in the understanding of GCR modulation. Among the problems of GCR modulation the fundamental one is the 11-year variation. The great part of GCR intensity changes is falling to the 11-year variation which generally is related with the similar variation of solar activity, Dorman (1963), Nagashima and Morishita (1980), Alania and Dorman (1981), Le Roux and Potgieter (1991), Bazilevskaya, *et al.* (1993), Belov, *et al.* (1993), Bazilevskaya and Svirzhevskaya (1998), McKibben (1998).

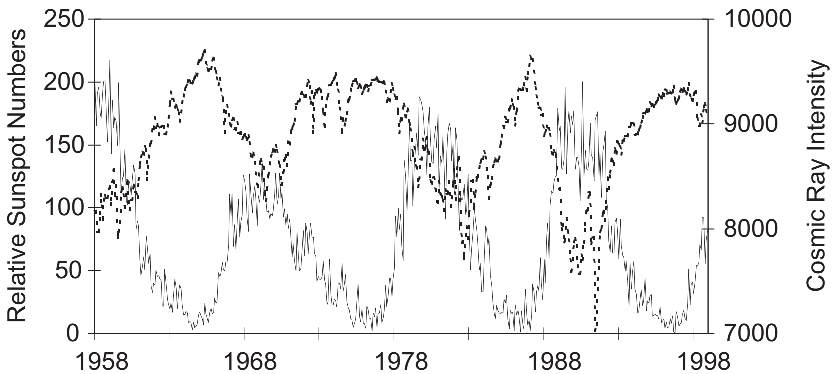

Fig. 1. Changes of the relative sunspot numbers (solid curve, scale on the left ordinate axis) and the intensity of galactic cosmic rays by Moscow's neutron monitor station (dotted curve, scale on the right ordinate axis) for the period 1958–1998

The changes of the relative numbers of sunspots and observations of GCR intensity on the Earth by Mocsow and neutron monitors for the period of 1958–1998 are plotted in figure 1. The question is which of the parameters or the group of parameters of solar activity and of solar wind are responsible for the 11-year variation of GCR. In order to answer to this question it is necessary to estimate separately the roles of all these processes: convection, diffusion, drift and energy changes of GCR due to the interaction with the solar wind causing a modulation of GCR. In connection with this it is necessary to make some important remarks:

1. To ascribe the mechanism of 11-year variation of GCR, for instance, to the solar wind velocity as to a source of the convection is not justified. Direct observations show that in the minima epochs of solar activity the solar wind velocity increases almost twice at the middle and high heliolatitudes with the weak tendencies to increase at the low heliolatitudes (Simpson, (1998); Lazarus *et al.*, (1998), Suess *et al.*, (1998)). So, if the solar wind velocity had been considered as an important source of the observed 11-year variation of GCR due to convection, there would be observed some decrease of GCR intensity in minima epochs of solar activity, contrary to the significant increase which in fact is observed. It is true that in connection with the changes of the average solar wind velocity *versus* the solar activity one can state that these changes are not significant near the helioequator region. Thus, the role of convection of GCR intensity during the 11-year cycle of solar activity in other equal conditions is weakly changeable.

2. Under the condition that the solar wind velocity is practically constant at the low latitudes *versus* the solar activity, there must not be a noticeable distinction between the energy changes of GCR due to their interaction with the solar wind in different epochs of solar activity, *i.e.* a divergence of the solar wind velocity practically is constant in different epochs of solar activity.

3. Drift effects of GCR which exist due to the gradient and curvature of the IMF and the Heliospheric Neutral Sheet (HNS) should play a specific role in the changes of the character of the profiles (rounded or levelled) of 11-year variation of GCR for the different magnetic halves cycles ($qA>0$, $qA<0$) of the Sun in the minima epochs of solar activity (Fig. 1). Due to the drift effect an amplitude of 11-year variation of GCR is greater for the period of 11-year cycle when $qA<0$, than for the period of $qA>0$ (Fig. 1). Nevertheless, in the changes of the amplitude of 11-year variation of GCR in the different 11-year periods ($qA>0$, $qA<0$) a drift effect is not a decisive one. In equal other conditions, as it is seen from the figure 1, the role of a drift can be estimated as 15–20% of the whole amplitude of 11-year variation of GCR.

4. Thus it is clear that the change of the character of the diffusion *versus* the level of the 11-year cycle of solar activity must play an important role in the creation of the 11-year variation of GCR. However, if it is the case, it is necessary to find what factor is responsible for the changes of the character of the diffusion *versus* the solar activity, or in other words, what is the mechanism of 11-year variation of GCR for the energy larger than 1 GeV? We think that neither the sunspots numbers presented in figure 1, nor other parameters of solar activity and solar wind widely used for the characterizing of the solar activity are directly responsible for the 11-year variation of GCR, Alania and Iskra (1995); Alania, *et al.* (1997).

In fact, a modulation of GCR takes place in regions of inner and outer heliosphere where temporal fluctuations of the spectral density of the IMF strength responsible for the diffusion of GCR are significantly different, Burlaga, (1995), Burlaga and Ness, (1998). The 11-year modulation of GCR observed at the Earth's orbit is the result of the continuous action of the whole heliosphere ('filled' with stochastic and regular magnetic and electric fields) on the GCR particles in the course of theirs motion from the boundary of the heliosphere to the point of observation. So, it is obvious that it is hardly possible to find one to one quantitative correspondence between the temporal changes of the power spectrum exponent, n of the IMF strength fluctuations near the Earth's orbit and the observed amplitude of 11-year variation of GCR. The formation of the energy spectrum of 11-year variation of GCR which is observed at Earth orbit (soft one in the maxima epochs and hard one in the minima epochs of solar activity) takes place in the large volume of interplanetary space with the radius much greater than a few Astronomical Unites (AU). The above-mentioned statement, of course, does not exclude the existence of the certain direct relationship between the observed the IMF strength fluctuations and 11-year modulation of GCR at the Earth's orbit. One can conclude that in the interplanetary space there must be a significant difference in the structure of the irregularities of solar wind in the minima and maxima epochs of solar activity (*e.g.* Burlaga, (1995)), *i.e.* that the effective sizes of the inhomogeneities of solar wind, on which GCR particles with the energy about 1 GeV are scattering, are greater in the minima epochs than in maxima epochs of solar activity (Alania and Iskra, (1995)).

Thus, the structural rearrangement of the large scale fluctuation of the IMF strength from the minima to the maxima epochs of solar activity, *i.e.* the radical changes of the effective sizes of the inhomogeneities of solar wind which cause the various diffusion (change of the diffusion coefficient) of galactic cosmic rays in the minima and maxima epochs of solar activity, is the general mechanism of the 11-year variation of GCR for the energy more than 1 GeV.

4.2. 27-day variations of GCR

The 27-day variation of GCR is generally stochastic phenomenon. However, not even mentioning the minima epochs, during the maxima epochs of solar activity there are observed numerous cases when the amplitude of the 27-day variation of GCR is relatively constant for the period of 4–5 rotations of the Sun. Thus, in order to show a dependence of the amplitude of the 27-day variation of GCR on the distances from the Sun in different solar magnetic cycles of the $qA>0$ and the $qA<0$, a steady-state case can be considered. Neglecting the term $\partial N/\partial t$ and taking into account that diffusion coefficients $K_{r\theta} = -K_{\theta r}$ and $K_{\theta\varphi} = -K_{\varphi\theta}$, the equation (26) in spherical coordinate system r, θ, φ, can be written:

$$A_1 \frac{\partial^2 n}{\partial r^2} + A_2 \frac{\partial^2 n}{\partial \theta^2} + A_3 \frac{\partial^2 n}{\partial \varphi^2} + A_4 \frac{\partial^2 n}{\partial r \partial \varphi} + A_5 \frac{\partial n}{\partial r} + A_6 \frac{\partial n}{\partial \theta} + A_7 \frac{\partial n}{\partial \varphi} + A_8 \frac{\partial n}{\partial R} + A_9 n = 0\,. \tag{27}$$

In the equation (27) the relative density, $n = N/N_0$ (where N_0 is density of GCR in the interstellar medium accepted as, $N_0 \propto R^{-4.5}$ for the rigidities to which neutron monitors are sensitive; the dimensionless distance $r = \rho/r_0$, where r_0 is the size of the modulation region and ρ is the distance from the Sun; A_1, A_2,…,A_9, are the function of r, θ, φ, and R. Parameters being responsible for the 27-day variation of GCR have the following expressions:

1. the heliolongitudinal asymmetry of the solar wind velocity changes as,

$$U = U(1 + 0.2\sin(\varphi))\,. \tag{28}$$

The IMF lines corresponding to the solar wind velocity $1.2U_0$ reaches to the IMF lines corresponding to the solar wind velocity U_0 at the radial distance of 7–8 AU. So, in order to exclude an intersection of the IMF lines in space the dependence of U on the heliolongitudinal angle φ takes place only up to the distance of 7 AU on the Sun's equatorial plane. This distance changes with the heliolatitudes according to the Parker's IMF's spiral rule.

2. The parallel diffusion coefficient, $K_{||}$ is represented in the following way:

$$K_{||} = K_0 K(r) K(r,\theta,\varphi) K(R)\,, \tag{29}$$

where $K(r) = 1 + \alpha_0 r^{\beta}$, $K(R) = R^{\gamma}$, and
$K(r,\theta,\varphi) = 1 + 0.5\sin\varphi\sin(3\theta)\exp(-\alpha_2 r)$
K_0 is equal to the 2×10^{22} $\mathrm{cm^2 S^{-1}}$ for the energy of 10 GeV.

The existence of the heliolongitudinal asymmetries (HA) of the diffusion coefficient and of the solar wind velocity in the range of the heliolatitudes $60^\circ \le \theta \le 120^\circ (\pm 30^\circ$ with respect to the solar equatorial plane) are determined by the $\sin(3\theta)$, which equals zero at the $\theta = 60^\circ$ and at the $\theta = 120^\circ$. For heliolatitudes of the range of $0^\circ \le \theta < 60^\circ$ and $120^\circ < \theta \le 180^\circ$, $K(r,\theta,\varphi) = \exp(-\alpha_2\rho)$, $K(r) = 1 + \alpha_0 r^{\beta}$, and $K(R) = R$ (in units of GV); the radius of the modulation region is 100 AU and the solar wind velocity U equals 4×10^7cm/s. The ratio α of the perpendicular $K_\perp$ and parallel $K_{||}$ diffusion coefficients ($\alpha = K_\perp / K_{||}$) is assumed as:

(1) $\alpha = (1 + \omega^2 t^2)^{-1}$. It is assumed that for the energy of 10 GeV $\omega\tau = 3$ and then it changes depending on the spatial coordinates according to the Parker's spiral magnetic field. At the boundary of the modulation region α tends to 1.

(2) $\alpha = 0.1$ and it is a constant for the whole heliosphere and

(3) $\alpha = 0.1$ near the helioequatorial region and enhances in the solar polar direction. The equation (2) has been solved numerically using the difference grid method taking into account drift due to the gradient and curvature of the IMF and the HNS drift.

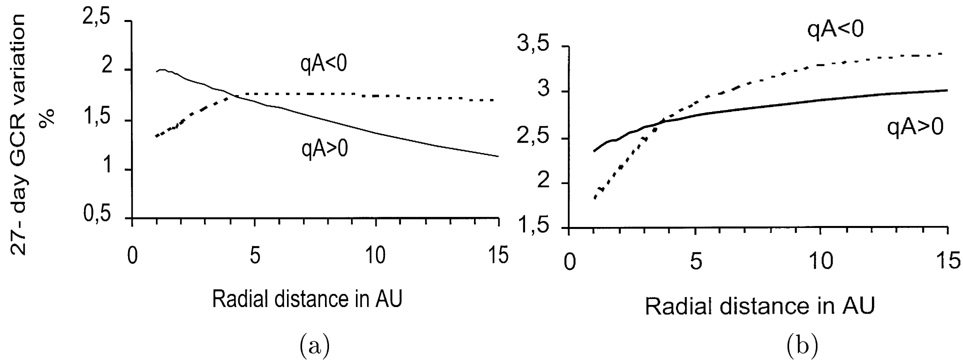

Fig. 2. Radial changes of the amplitudes of the 27-day variation of GCR (a) in the existence and (b) in absence of the radial decay of the HA for constant α; solid line (qA>0), dashed line (qA <0).

The solutions of the equation (27) for the rigidity of 10 GV of GCR are presented in figures 2(a) (b) for the solar magnetic cycles, $qA>0$ (solid line) and $qA<0$ (dashed line) taking into account the above-mentioned expression of the solar wind velocity (28), diffusion coefficient (29), different ratios of α of the perpendicular $K_\perp$and parallel $K_{||}$ diffusion coefficients ($\alpha = K_\perp/K_{||}$), for the constant values of the coefficients, $\alpha_0 = 100$, $\beta = 1$, $\gamma = 1$, and for different values of $\alpha_1 = 0$ (an absence of the radial decay of the HA, Fig. 2(b)) and $\alpha_1 = 0.07$ (an existence of the radial decay of the HA, Fig. 2(a)).

In the both figures 2(a) (b), $\alpha = 0.1$ near the helioequatorial region and then enhances in the solar polar directions as,

$$\alpha = 0.2 - 0.06\theta, \quad \text{for } 0 \le \theta < \pi/2$$

and

$$\alpha = 0.1 + 0.06(\theta - \pi/2), \quad \text{for } \pi/2 \le \theta \le \pi\,.$$

It is seen from the figure 2(a), (b) that the amplitudes of the 27-day variations of GCR for the both solar magnetic cycles of the $qA>0$ and $qA<0$ are greater when the radial decay of the HA is absent. The amplitudes of the 27-day variations of GCR near the Earth's orbit is greater in the $qA>0$ solar magnetic cycle than in the $qA<0$ cycle for the both cases when the radial decay of the HA is absent, and when the radial decay of the HA exists. For distances less than 5 AU the amplitudes of the 27-day variations of GCR for the case of $qA>0$ are greater than in the $qA<0$ case. This results qualitatively coincide with the experimental data observed by neutron monitors, Alania *et al.* (2001).

5. Conclusion

1. The generalized anisotropic diffusion tensor, streams and drift velocities of GCR for the three dimensional IMF obtained before by the present author have been reconstructed and comprehensively analysed. It is stressed that the generalized anisotropic diffusion tensor should be largely used in solving of the Parker's transport equation as far according to the experimental data and theoretical investigations the heliospheric magnetic field must be considered as the three-dimensional.

2. Stochastic and regular changes of GCR, especially 11-year and 27-day variations have been studied. It is concluded that the general mechanism of the 11-year variation of GCR must be the change of the structure of the stochastic IMF causing the different character of the diffusion of GCR in different epochs of solar activity.

3. The amplitude of the 27-day variations of GCR is greater at the Earth's orbit (about 1.5 times) in the period of the $qA>0$ solar magnetic cycle than in the period of the $qA<0$ solar magnetic cycle, which is not yet well explained according to the existed modulation theory of GCR.

Author cordially thanks Ms A. Wawrzynczak for her help in preparing of this paper.

REFERENCES

Alania, M.V., Modulation of Cosmic Rays, Proceedings of the Institute of Geophys. Georgian Academy of Sciences, p. 5, Tbilisi, 1978, in Russian.

Alania, M.V., Japiashvili T.V., XVI ICRC, 3, p. 19, Kyoto, Japan 1979.

Alania, M.V., Dorman, I.L., XVI ICRC, 3, p. 57, Kyoto, Japan 1979.

Alania, M.V., Variation of Cosmic Rays, Georgian Academy of Science, Mec niereba, Tbilisi, p. 11, 1980, in Russian.

Alania, M.V., Dorman, I.L., The Spatial Distribution of the Density and Stream of Galactic Cosmic Rays, Tbilisi, Mecniereba 1981, in Russian.

Alania, M.V., *et al.*, VIII, ECRS, A.3.7, Rome 1982.

Alania, M.V., Bochorishvili, T.B., Iskra K., *Adv. Space Res.* **19**, 925 (1997).

Alania, M.V., Iskra, K., *Adv. Space Res.* **16**, 241 (1995).

Alania, M.V., *et al.*, *Adv. Space Res.* **27**, 619 (2001).

Axford, W.I., *Planet. Space Sci.* **13**, 115 (1965).

Bazilevskaya, G.A., Svirzhevskaya, A. K., *Space Sci. Rev.* **85**, 431 (1998).

Belov, A., *et al.*, Proc. of 23rd ICRC, Calgary, 3, 605, (1993).
Bieber, J.W., *et al.*, *Astrophys. J.* **420**, 294 (1994).
Bieber, J.W., Wanner, W., Matthaeus, W.H., *Geophys. Res.* **101**, 2511 (1996).
Bieber, J.W., Matthaeus, W.H., *Astrophys. J.* **485** 655 (1997).
Burger, R.A., Podgieter, M.S., *Astrophys. J.* **339**, 501 (1989).
Burlaga, L.F., Ness, N. F., *Space Sci. Rev.* **83**, 105 (1998).
Burlaga, L.F., *Interplanetary Magneto Hydrodynamics*, Oxford University Press, 1995.
Chapman, S., Cowling, T.G., *Matematicheskaja Teoria Neodnorodnix Gazov*, IL, Moscow 1960, in Russian.
Dolginov, A.Z., Toptigin, I.N., ICARUS, 1968, pp. 54.
Dorman, L.I., *Cosmic Rays Variations and Space Exploration*, Nauka, Moscow 1963, in Russian.
Dorman, L.I., Proc. IX ICRC, London, 1, 1965, p. 505.
Fisk, L.A., *J. Geophys. Res.* **81**, 4646 (1976).
Fisk, L.A., *J. Geophys. Res.* **101**, 15.547 (1996).
Fisk, L.A., *J. Geophys. Res.* **106**, 15.849 (2001).
Giacalone J., *J. Geophys. Res.* **106**, 15881 (2001).
Gil A., Alania M.V., Proceedings of ICRC 2001, Hamburg, 2001, p. 3725.
Gleeson, L.J., Axford, W.I., *Astrophys. J. Lett.* **149**, (1967).
Isenberg, P.A., Jokipii, J.R., *Astrophys. J.* **234**, 746 (1979).
Jokipii, J.R., *Rev. Geophys. Space Phys.* **9**, 1 (1971).
Jokipii, J.R., Levy, E.H., Hubbard, W.B., *Astrophys. J.* **213**, 861 (1977).
Jokipii, J.R., Kopriva, D.A., *Astrophys. J.* **234**, 384 (1979).
Jokipii, J.R., Thomas, B.T., *Astrophys. J.* **243**, 115 (1981).
Jokipii, J.R., Kota, J., *Astrophys. J.* **265**, 573 (1983).
Jokipii, J.R.,*J. Geophys. Res.* **91**, 2929 (1986).
Jokipii, J.R., Kota, J., *Geophys. Res. Lett.* **16**, 1 (1989).
Kota, J., Jokipii, J.R., *Space Sci. Rev.* 137 (1998).
Krymsky, G.F. *Geomagn. Aeron.* **4**, 763 (1964).
Krymsky, G.F., *Modulation of Cosmic Rays in Interplanetary Space*, Nauka, Moscow 1969, in Russian.
Lazarus, A.J., *Space Sci. Rev.* **83**, 87 (1998).
Le Roux, J.A., Podgieter, M.S., *Astron. Astrophys.* **243** 531 (1991).
Levy, E.H., *Geophys. Res. Lett.* **5**, 969 (1978).
McKibben, R.B., *Space Sci. Rev.* **83**, 21 (1998).
Nagashima, K., *et al.*, *Planet. Space Sci.* **34**, 1299 (1986).
Nagashima, K., Morishita, J., *Planet. Space Sci.* **28**, 177 (1980).
Palmer, I.D., *Rev. Geophys.* **20**, 335 (1982).

Parker, E.N., *Phys. Rev.* **109**, 1328 (1958).

Parker, E.N., *Interplanetary Dynamical Processes*, Interscience Publishers, New York 1963.

Parker, E.N., *Planet. Space Sci.* **13**, 9 (1965).

Potgieter, M.S., le Roux, *Astrophys. J.* **386**, 336 (1992).

Potgieter, M.S., Moraal, M., *Astrophys. J.* **294**, 425 (1985).

Simpson, J.A., *Space Sci. Rev.* **83**, 7 (1998).

Suess, S.T., *et al.*, *Space Sci. Rev.* **83**, 75 (1998).

Singer, S.F., Laster, H., Lencheck, A.M., *J. Phys. Soc. Jpn., Suppl.* **17**, A-11, 583 (1962).

Slavin, J.A., Smith, E.J., *Solar Wind Five*, NASA, 1983, p. 323.

Smith, E.J., *et al.*, *Space Sci. Rev.* **72**, 165 (1995).

Wibberenz, G., 16th ICRC, Rapp. Rep., 234, (1979).