CHARGED PARTICLE FLUCTUATION IN HEAVY ION COLLISIONS*

Fritz W. Bopp and Johannes Ranft

Universität Siegen, Fachbereich Physik
57068 Siegen, Germany

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Comparing quantities to analyze charged fluctuations in heavy ion experiments, the dispersion of the charges in a central rapidity box was found to be best suited. Various energies and different nuclear sizes are considered in an explicit Dual-Parton-Model calculation using the DPMJET code and a randomized modification to simulated charge equilibrium. For large enough detection regions charged particle fluctuations can provide a signal of the basic dynamics of heavy ion processes.

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1. Charge fluctuations in fixed target hadron–hadron experiments

Let us look for a moment back to the analysis of purely hadronic multiparticle production. At fixed target experiments it was possible to measure the charges of all forward particles. In this way significant results could be obtained with low energies available at the seventies [1–5]:

- The charge fluctuations found involve mostly a restricted rapidity range.
- Good agreement was obtained with cluster models.

The Quigg–Thomas relation [6,7] (assuming neutral clusters) for fluctuation across a rapidity y boundary

$$\langle \delta Q^2_{>y} \rangle = \langle (Q_{>y} - \langle Q_{>y} \rangle)^2 \rangle = c \, \frac{dN^{\text{non leading}}_{\text{charge}}}{dy} \tag{1}$$

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was found to be roughly satisfied [8–17]. To illustrate, how early this relation was useful we consider *24 GeV proton–proton scattering data.* Taking the rapidity of the forward–backward border as variable, the data could be presented as in figure 1 [2]. The top and bottom lines and points correspond to with and without correcting for the leading charges. The lines correspond to suitably normalized spectrum.

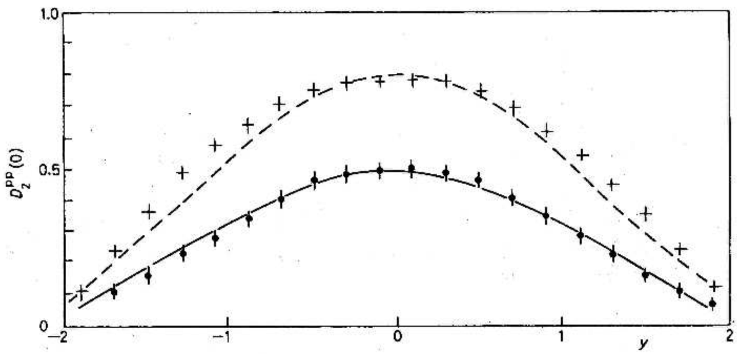

Fig. 1. The measured dispersion (+) and the produced charge dispersion (*), which is corrected for leading charge flow, is compared with the suitably normalized negative (produced) particle spectrum at 24 GeV/c.

The agreement could be improved when string type $q\bar{q}$ -charge exchanges between the clusters were added [8]. Such exchanges appear in a large class of models. Using the Dual Parton Model code DPMJET we re-checked the old results. For pp-scattering at laboratory energies of 205 GeV good agreement was still obtained. The Quigg–Thomas relation is satisfied with $c = 0.70$ comparing to the experimentally preferred value: $c = 0.72$.

2. Charge fluctuations in heavy-ion scattering experiments

In heavy ion scattering it is a central question whether the charges are distributed just randomly or whether there is some of the initial dynamics left influencing the global flow of quantum numbers. The charge flow measurements could again be decisive. It is not an impractical conjecture. In heavy ion experiments the charge distribution of the particle contained in a central box with a given rapidity range $[-y_{\mathrm{max.}}, +y_{\mathrm{max.}}]$ as shown in figure 2 can be measured and the dispersion of this distribution $\langle\, \delta Q^2 \rangle$ can be obtained to sufficient accuracy. For sufficiently large gaps this quantity contains information about long range charge flow. In comparison to the fluctuations in the forward backward charge distributions the charge distribution into a central box (having two borders) can be expected to require roughly twice the rapidity range.

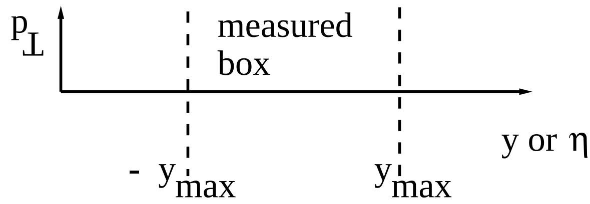

Fig. 2. Kinematic region of the "central box".

3. Equilibrium expectations

Within the framework of equilibrium models it was proposed to use the quantity to distinguish between particles emerging from an equilibrium quark-gluon gas or from an equilibrium hadron gas [18–20]. For a small enough box in a central region at high energies where average charge flow can be ignored, the (essentially) Poisson distributed *hadron gas* yields a simple relation

$$\langle \delta Q^2 \rangle = \langle N_{\rm charged} \rangle \, . \tag{2}$$

for any thermalized particles with charges 0 and ± 1. The inclusion of resonances reduces hadron gas prediction by a significant factor taken [19, 21] to be around 0.7.

It is argued in the cited papers that this relation would change in a *quark gluon* gas to

$$\langle \delta Q^2 \rangle = \sum_i q_i^2 \langle N_i \rangle = 0.19 \langle N_{\rm charged} \rangle \, , \tag{3}$$

where q_i are the charges of the various quark species and where again a central region is considered. The coefficient on the right was calculated [19] with suitable assumptions. A largely empirical final charged multiplicity $N_{\rm charged} = \frac{2}{3}(N_{\rm glue} + 1.2 N_{\rm quark} + 1.2 N_{\rm antiquark})$ was used.

It should be pointed out that the estimate is not without theoretical problems [22, 23]. There is also a number of systematic uncertainties in the above comparison. As explained below in a simple approximation the result strongly depends on what one takes as primordial particles and how the extra quarks needed for hadronization are modeled. Considering these uncertainties we follow the conclusion of Fiałkowski's papers [24] that a clear cut distinction between the hadron- and the quark gluon gas is rather unlikely. This does not eliminate the interest in the dispersion as a *measure of equilibration.*

4. Various measures for charge fluctuations

For the analysis of the charge structure several quantities were discussed in the recent literature. Besides the classic charge dispersion

$$\langle \delta Q^2 \rangle = \langle (Q - \langle Q \rangle)^2 \rangle \tag{4}$$

it was proposed to just measure the mean standard deviation of the ratio R of positive to negative particles

$$\langle \delta R^2 \rangle = \left\langle \left(\frac{N_+}{N_-} - \left\langle \frac{N_+}{N_-} \right\rangle \right)^2 \right\rangle \tag{5}$$

or the quantity F

$$\langle \delta F^2 \rangle = \left\langle \left(\frac{Q}{N_{\text{charged}}} - \left\langle \frac{Q}{N_{\text{charged}}} \right\rangle \right)^2 \right\rangle , \tag{6}$$

where $Q = N_+ - N_-$ is the charge in the box. The motivation for choosing these ratios was to reduce the dependence of multiplicity fluctuations caused by the event structure.

5. Evaluation of the measures

In the region of interest for large nuclei at high energies and *strong centrality* the charge component of the fluctuations dominates. In these region all measures are simply connected by the following relations [19]:

$$\langle N_{\text{charged}} \rangle \langle \delta R^2 \rangle = 4 \, \langle N_{\text{charged}} \rangle \langle \delta F^2 \rangle = 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{\text{charged}} \rangle} . \tag{7}$$

and the question of the optimal quantity is somewhat esoteric. To show this statement all three quantities were calculated in the Dual Parton model implementation DPMJET [25] shown in figure 3.

For the most central 5% Pb–Pb scattering at LHC energies ($\sqrt{s} = 6000$ A GeV) there is indeed a perfect agreement between all three quantities as shown in the figure. This agreement stays true for analogous Pb–Pb data at RHIC energies ($\sqrt{s} = 200$ A GeV) .

Outside the region of interest — *i.e.* in the region of *lower particle densities* — the conventional dispersion, $\langle \delta Q \rangle$, has clear advantages. The alternatives are not suitable for small Δy boxes in less dense events, since

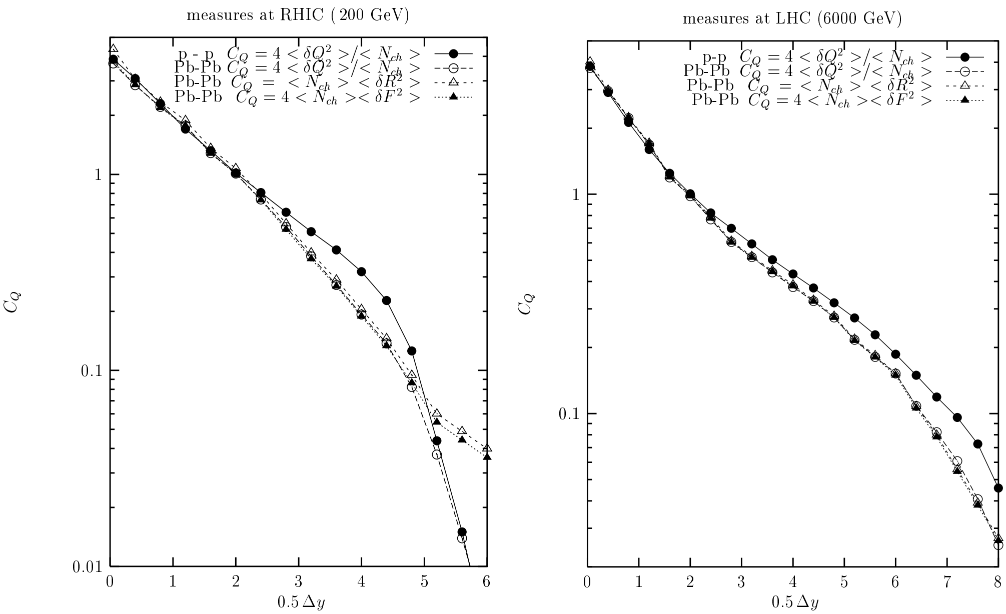

Fig. 3. Charge fluctuations for the most central 5% Pb–Pb scattering at RHIC energies ($\sqrt{s} = 200A$ GeV and at LHC energies ($\sqrt{s} = 6000$ A GeV). Also shown are corresponding data for *pp* scattering.

- if no particle in the corresponding box exists in rare events 0/0 or ∞ is undefined, and
- if one somehow fixes the problem (*e.g.* by not considering problematic events) their mutual relation is destroyed. For the minimum bias S–S scattering at these energies the agreement is lost and the new measures behave rather erratic [26]. The same erratic behavior for the new measures is found for the proton–proton case (figure 4).

As any conclusion will have to depend on a comparison of central processes with minimum bias and proton–proton events, there is a clear advantage to stick to the dispersion of the net charge distribution $\langle \delta Q^2 \rangle$[1].

[1] New RHIC data of the PHENIX collaboration [27] which appeared after the talk resp. paper [28] confirm the problem with $\langle \delta R^2 \rangle$ which does not appear for $\langle \delta Q^2 \rangle$. Equation (7) does not hold in their case as a restricted azimuthal range was considered effectively reducing the density. For the measured very narrow rapidity range the data are essentially consistent with (hadronic) statistical fluctuations.

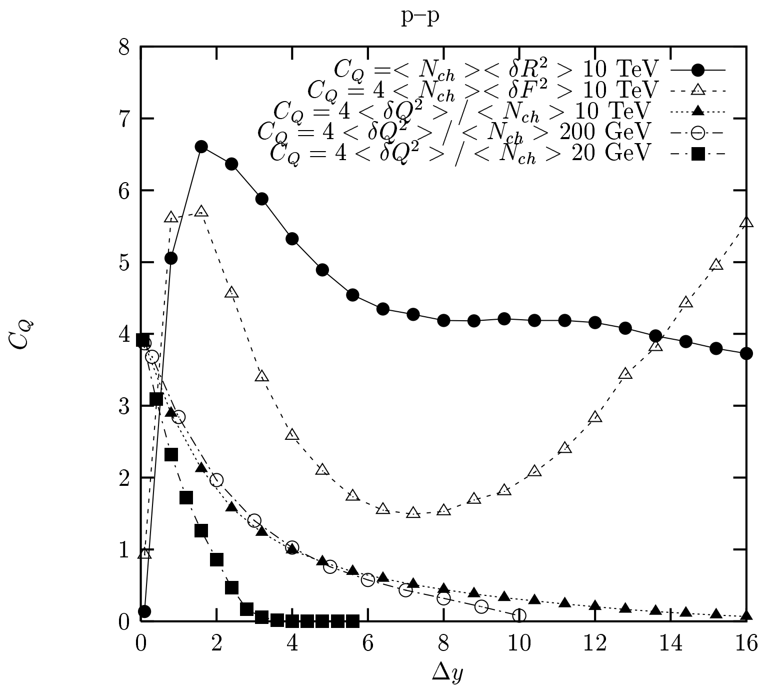

Fig. 4. Charge fluctuations for minimum bias *pp* scattering at SPS, RHIC and LHC energies.

6. A simple relation between the quark line structure and fluctuations in the charge flow

To visualize the meaning of charge flow measurements it is helpful to introduce a general factorization hypothesis. It postulates that the light flavor structure of an arbitrary hadronic amplitude can be described simply by an overall factor, in which the contribution from individual quark lines factorize. It is for most purposes (which consider long range fluctuations) an adequate approximation.

The hypothesis can be used to obtain the following *generalization of the Quigg–Thomas relation* [8,29,30]. It states that the correlation of the charges $Q(y_1)$ and $Q(y_2)$, which are exchanged during the scattering process across two kinematic boundaries, is just

$$\langle \delta Q(y_1) \delta Q(y_2) \rangle = n_{\text{common lines}} \langle \delta q^2 \rangle \,, \tag{8}$$

- where the charges $\delta Q(y_i) = Q(y_i) - \langle Q(y_i) \rangle$ were exchanged across two kinematic boundaries y_1 & y_2 ,

- where q is the charge of the quark on such a line. Values $\langle \delta q^2 \rangle = \langle (q - \langle q \rangle)^2 = 0.22 \ldots 0.25$ are obtained, and
- where $n_{\text{common lines}}$ counts the number of quark lines intersecting both borders, as illustrated in the simple example given in figure 5:

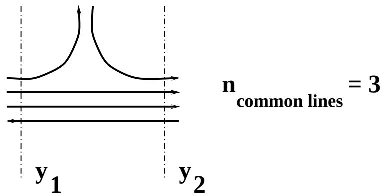

Fig. 5. Example of a quark line graph.

Most observables of charge fluctuations can be expressed using this basic correlation. Our fluctuation of the charges within a $[-y_{\text{max.}}, +y_{\text{max.}}]$ box contains a combination of three such correlations. A simple summation yields:

$$\langle \delta Q[\text{box}]^2 \rangle = n_{\text{lines entering box}} \langle (q - \langle q \rangle)^2 \rangle \,, \tag{9}$$

where $n_{\text{lines entering box}}$ is the number of quark lines entering the box.

7. Charge fluctuations in equilibrium models

Let us use this relation to consider the prediction in more detail. In the *thermalized limit* with an infinite reservoir outside and a finite number of quarks inside, all quark inside will connect with quark lines to the outside as shown in figure 6. The dispersion of the charge transfer is, therefore, proportional to the total number of quarks or particles inside.

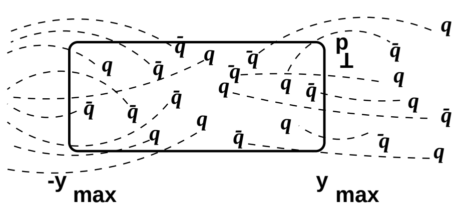

Fig. 6. Quark lines entering the box in the thermodynamic limit.

In an *hadron gas* all particles contain two independent quarks each contributing to the fluctuation with roughly 1/4 yielding the estimate 1/2 as required by equation (2) . In the factorizing limit mesons have a 50% chance to be charged; possible baryon contribution requires only a minor correction.

For the *quark gluon gas* ignoring hadronization one obtains one quark charge fluctuation 1/4 for each charged parton. Equation (2) is drastically changed by a factor of 4. It is, however, not easy for this prediction to survive hadronization. If hadronization would just group initial partons into hadrons, the factorizing hadron gas description would stay completely unchanged. For the reduction it is essential to have only a single quark line contributing to the fluctuation. Only one quark of each hadron has to originate in the primary partonic process and the other quark has to originate in local fluctuations and has not to contribute. The mechanism requires a sufficiently large box so that short range correlation can be avoided.

8. The expanding box

Let us first consider the *limit of a tiny box.* Looking only at the first order in Δy one trivially obtains in any model

$$\frac{\langle \delta Q^2 \rangle}{\langle N_{\text{charged}} \rangle} = 1 \tag{10}$$

which corresponds to the hadron gas value. If the box size increases to one or two units of rapidity on each side this ratio will typically decrease, as most models contain a short range component in the charge fluctuations. The decreasing is not very distinctive. In hadron–hadron scattering processes such short range correlations are known to play a significant role and there is no reason not to expect such correlations for the heavy ion case.

After a box size passed the short range the decisive region starts. In all global equilibrium models [18–20] the ratio will have to reach now a flat value. The only correction comes from overall charge conservation. If the box involves a significant part of the total rapidity, it will force the ratio to drop by a correction factor

$$\text{factor} = \frac{\int_{y_{\text{max.}}}^{Y_{\text{kin.max.}}} \rho_{\text{charge}}^{\text{new}} dy}{\int_{0}^{Y_{\text{kin.max.}}} \rho_{\text{charge}}^{\text{new}} dy} \;\propto\; 1 - \frac{y_{\text{max.}}}{Y_{\text{kin.max}}} \,. \tag{11}$$

9. Charge fluctuations in string models

This flatness is not expected in string models and numerical calculations indicate a manifestly different behavior. Only quark lines which intersect boundaries and which contribute to the charge measure have to be considered. String models contain local compensation of charge. Only contributions of lines originating around the boundaries (as illustrated in the figure 7) will appear. If the distance is larger than the range of charge compensation the dispersion will no longer increase with the box size. The total contribution will now be just proportional to the *density of the particles at the boundaries*:

$$\langle \delta Q^2 \rangle \propto \rho_{\rm charged}(y_{\rm max.})\,. \tag{12}$$

It now just counts the number of strings.

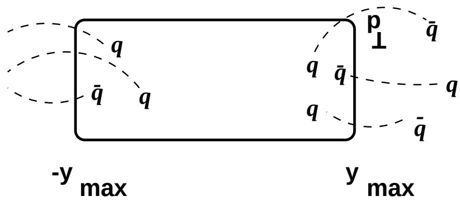

Fig. 7. Quark lines entering the box with local compensation of charge.

This resulting scaling is illustrated in a comparison between both quantities in (12). Shown in figure 8 are the predictions of the Dual Parton Model implementation DPMJET [25] for RHIC and LHC energies. The agreement

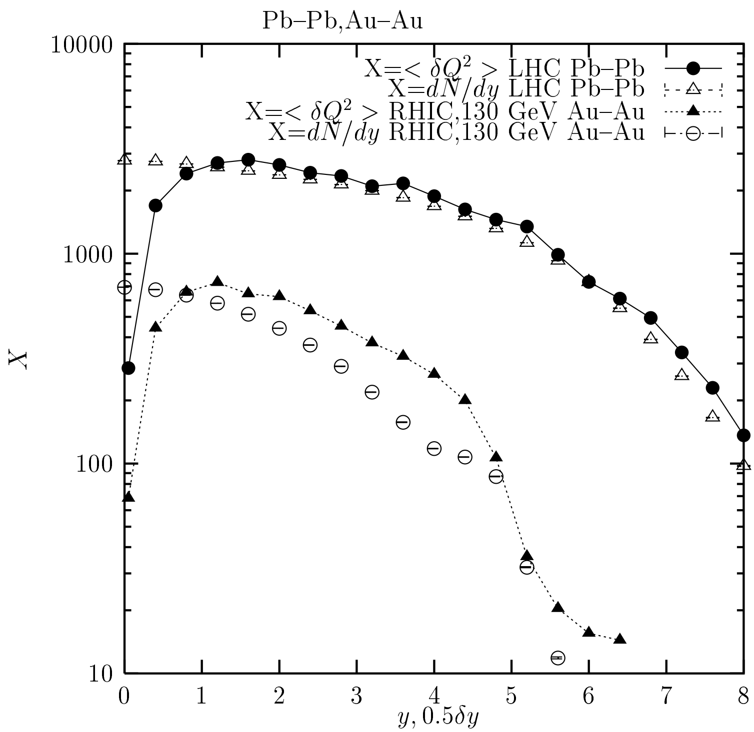

Fig. 8. Comparison of the dispersion of the charge distribution with the density on the boundary of the considered box for central gold–gold resp. lead–lead scattering at RHIC and LHC energies.

is comparable to the proton–proton case shown in figure 9. The proportionality is expected to hold for a gap with roughly $\frac{1}{2}\delta y > 1$ as for smaller boxes some of the quark lines intersect both boundaries. For large rapidity sizes there is a minor increase from the leading charge flow $Q_{\rm L}$ originating in the incoming particles. In a more careful consideration [8] one can subtract this contribution

$$\langle \delta Q^2 \rangle_{\text{leading charge migration}} = \langle Q_{\rm L} \rangle (1 - \langle Q_{\rm L} \rangle) \tag{13}$$

and concentrate truly on the fluctuation.

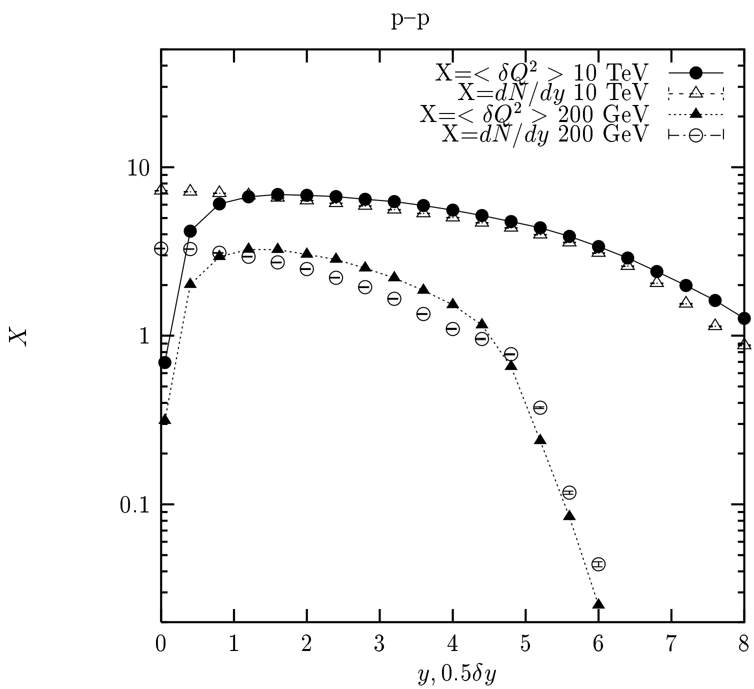

Fig. 9. Comparison of the dispersion of the charge distribution with the density on the boundary of the considered box for proton–proton scattering at RHIC and LHC energies.

A rough estimate of the relative size — with a width of neighboring string break ups and a width from resonance decays — leads to consistent values [28].

10. Bleicher, Jeon, Koch's observation

In a recent publication Bleicher, Jeon, Koch [21] showed:

- The overall charge conservation cannot be ignored at SPS energies.
- It obliviates in this energy range the distinction between even the most extreme models including string models and statistical models with hadronic equilibrium.

They showed that their string model prediction[2] coincides with the expectation of a statistical model of hadrons. Our string model DPMJET supports this conclusion for the SPS energy range as it also obtains fluctuations consistent with the "statistical" expectation.

While forward-backward hemisphere charge fluctuations were meaningful in the FNAL-SPS energy region, the fluctuations of charges into a central box contain two borders and require a correspondingly doubled rapidity range. Unfortunately this means a lot in energies. They are not available at SPS energies.

11. A reference model with statistical fluctuation

It was argued [21] that the experimental results should be "purified" to account for charge conservation. We basically agree with such a correction. Given the uncertainties the correction obviously has to stay on the modeling side.

To obtain such a reference model we *randomize charges a posteriori*. To accurately conserve energy and momentum it was done separately for pions, kaons and nucleons[3]. Using DTMJET events for RHIC and LHC energies for *proton–proton* and *central lead–lead* collisions we obtain the "statistical" prediction shown in figure 10.

To check consistency we employed the correction factor proposed by [21]

$$1 - \frac{\int_0^{y_{\mathrm{max.}}} \rho_{\mathrm{charge}}\, dy}{\int_0^{Y_{\mathrm{kin.max.}}} \rho_{\mathrm{charge}}\, dy}$$

and indeed obtained the flat distribution with the expected "hadron gas" value.

[2] In the energy above $\sqrt{s} = 5$ GeV their UrQMD code is described [31] to be dominated by string fragmentation.

[3] Obviously, the method can also be directly applied to experimental data, at least in a simplified way.

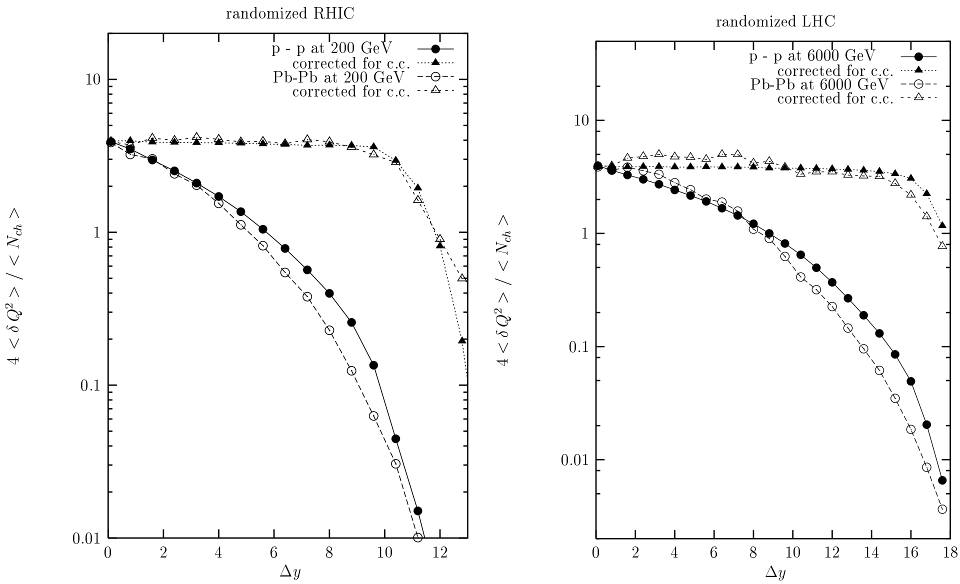

Fig. 10. Charge fluctuations with *a posteriori* randomized charges for p–p scattering and the most central 5% in Pb–Pb scattering at RHIC energies ($\sqrt{s} = 200\ A$ GeV) and at LHC energies ($\sqrt{s} = 6000\ A$ GeV). The results are also shown with a correction factor to account for the overall charge conservation.

12. String model *versus* randomized "hadron gas"

Taking the DPMJET string model and the randomized "hadron gas" version as extreme cases (with the "parton gas" somewhere in between) we can investigate the decisive power of the measure. As shown in figure 11 we find that there is a measurable distinction at RHIC energies and sizable at LHC energies.

The similarity of p–p and Pb–Pb scattering is not surprising. The distinction between both cases is expected from the difference in collective effects. The data for p–p scattering are known to follow the string models, while interaction of comovers, or medium range or complete equilibrium will move the curve upward to a more statistical situation. These effects are presently outside of the model. A measured charge correlation between both extremes will directly reflect the underlying new physics.

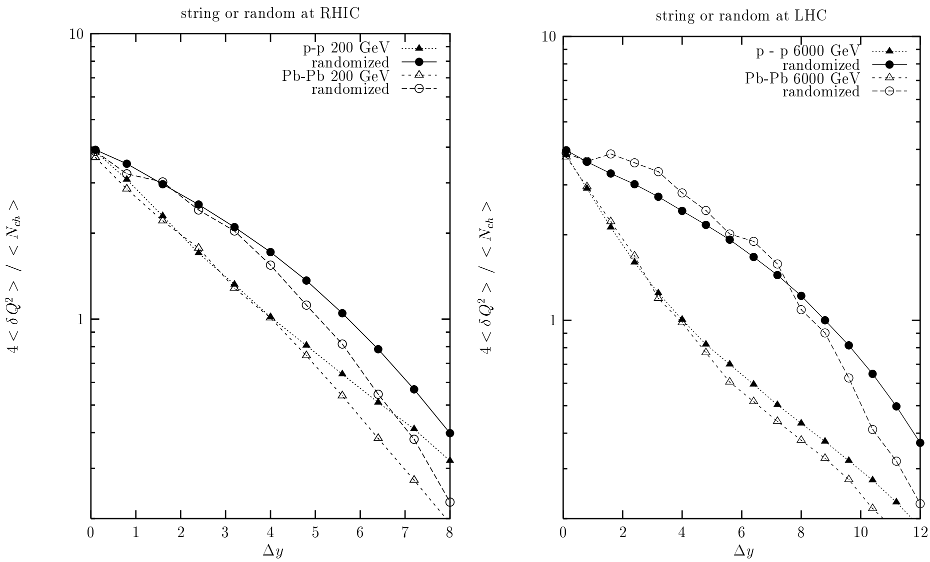

Fig. 11. Comparison of the charge fluctuations obtained in a string model DPMJET with a model using *a posteriori* randomized charges for *p*–*p* scattering and the most central 5% in Pb–Pb scattering at RHIC energies ($\sqrt{s} = 200\ A$ GeV) and at LHC energies ($\sqrt{s} = 6000\ A$ GeV).

13. The *b* dependence of the charge fluctuations

A similar result is obtained when the dependence on the centrality is studied. Without collective effects no such dependence is expected as observed in the model calculation shown in figure 12 (b is the impact parameter). This experimentally measurable centrality dependence allows to directly observe collective effects without reference to model calculations and underlying concepts.

14. Conclusion

In the paper we demonstrated that the dispersion of the charge distribution in a central box of varying size is an extremely powerful measure.

Within the string model calculation the dispersion seen in relation to the spectra shows no difference between simple proton–proton scattering and central lead lead scattering even though both quantities change roughly by a factor of 400.

The dispersion allows to clearly distinguish between conventional string models and hadronic thermal models for a rapidity range which could be available at RHIC energies. In many models the truth is expected to lie somewhere in between and it is a reasonable expectation that the situation can be positioned in a quantitative way.

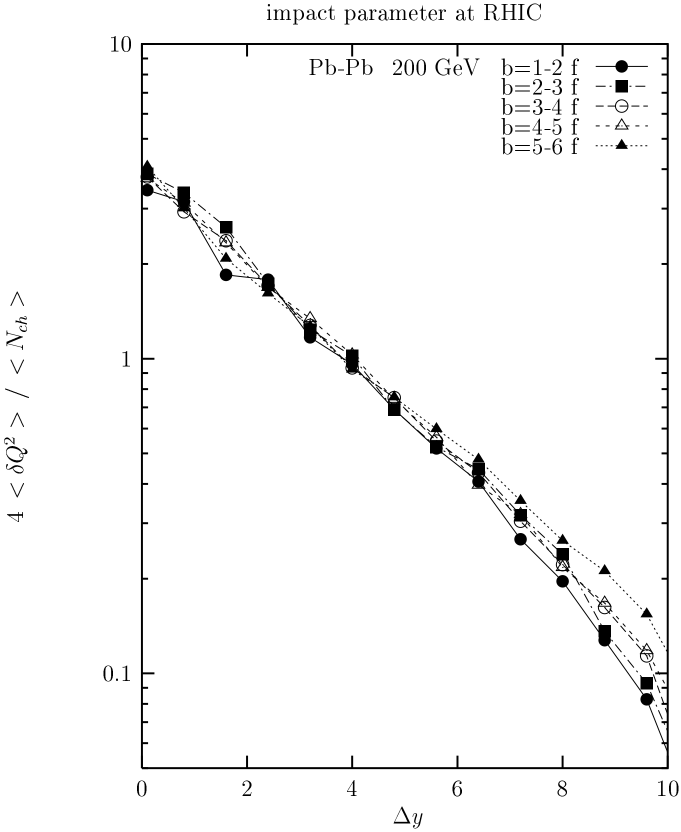

Fig. 12. The b dependence of the charge fluctuations obtained in a string model DPMJET for p–p scattering and the most central 5% in Pb–Pb scattering at RHIC energies ($\sqrt{s} = 200\ A$ GeV).

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