

NON-ABELIAN CONSTRAINTS IN MULTIPARTICLE PRODUCTION*

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Internal microscopic symmetry of a many body system leads to global constraints. We obtain explicit forms of the global macroscopic condition assuring that at the microscopic level the evolution respects the overall symmetry.

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1. Introduction

Let us consider a multiparticle interacting system with the internal symmetry taken into account. The internal symmetry leads to conservation laws which put constraints on the evolution of the system. The problem arises: are there only constraints due to the symmetry conservation? In the microscopic formulation with symmetry invariant dynamical equations the answer is given by an analysis of corresponding solutions — assuming that solutions are known.

We are looking here for global conditions to provide consistency with the overall symmetry of the system. These conditions should not depend on the exact analytic form of the solutions. As an example one can take Kepler's laws in the classical mechanics which are related to the orbital momentum conservation and can be proved without knowledge of the analytic solutions of Newton equations. Another simple example is a case of n species of charged particles with individual charges $q_a, q_b, \dots, q_n$. Numbers of particles are given by $N^{(a)}, N^{(b)}, \dots, N^{(n)}$.

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Particle numbers are time-dependent but the global charges must be conserved (exact U(1) symmetry). So there is a condition

$$q_a \frac{dN^{(a)}}{dt} + q_b \frac{dN^{(b)}}{dt} + \cdots + q_n \frac{dN^{(n)}}{dt} = 0, \quad (1.1)$$

valid for any charge conserving interaction.

Our aim [1] is to find a corresponding condition for non-abelian symmetries. In the non-abelian case there is a subsidiary condition besides conditions of the type of Eq. (1.1) due to the charge conservation. This is the demand to preserve the internal symmetry group representations during the evolution of the system.

2. Generalized projection method

Let us consider a system big enough to use methods of statistical physics. When the system reaches the statistical equilibrium then one can extract contributions from particular irreducible representation of the symmetry group [2]. Group projection techniques allowed for a consistent treatment of equilibrium systems and gave tools to obtain canonical partition functions corresponding to the system transforming under a given representation of the symmetry group. This technique can also be used for a more general non-static problem.

Let us consider a system consisting of particles belonging to multiplets α_j of the symmetry group. Particles from the given multiplet α_j are characterized by quantum numbers ν_j related to the symmetry group, and quantum numbers ζ_j characterizing different multiplets of the same irreducible representation α_j .

The number of particles of the specie $\{\alpha, \nu_\alpha; \zeta\}$ is denoted here by $N_{\nu_\alpha; (\zeta)}^{(\alpha)}$. These occupation numbers are time dependent until the system reaches the chemical equilibrium. However, the representation of the symmetry group for the system remains constant in the course of a time evolution. A multiplicity $N^{(\alpha_j)}$ of the representation α_j in this product is equal to a number of particles which transform under this representation

$$N^{(\alpha_j)} = \sum_j \left(\sum_{\zeta_j} N_{\nu_{\alpha_j}; (\zeta_j)}^{(\alpha_j)} \right) = \sum_j N_{\nu_{\alpha_j}}^{(\alpha_j)}. \quad (2.1)$$

We introduce a state vector $\left| N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \right\rangle$ in particle number representation. The probability that $N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}$ particles transforming under

the symmetry group representations $\alpha_1, \dots, \alpha_n$ combine into a state transforming under representation Λ of the symmetry group is given by

$$\overline{P_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}} = \left\langle N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \left| \mathcal{P}^A \right| N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \right\rangle. \quad (2.2)$$

The projection operator $\mathcal{P}^A$ has the form (see *e.g.* [3])

$$\mathcal{P}^A = d(\Lambda) \int_G d\mu(g) \bar{\chi}^{(\Lambda)}(g) U(g). \quad (2.3)$$

Here $\chi^{(\Lambda)}$ is the character of the representation Λ , $d(\Lambda)$ is the dimension of the representation, $d\mu(g)$ is the invariant Haar measure on the group, and $U(g)$ is an operator transforming a state under consideration. In particle number representation the operator $U(g)$ is defined as

$$U(g) \left| N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \right\rangle = \sum_{\nu_1^{(1)}, \dots, \nu_n^{(N_{\nu_n})}} D_{\nu_1^{(1)} \nu_1}^{(\alpha_1)} \dots D_{\nu_1^{(N_{\nu_1})} \nu_1}^{(\alpha_1)} \dots D_{\nu_n^{(1)} \nu_n}^{(\alpha_n)} \dots D_{\nu_n^{(N_{\nu_n})} \nu_n}^{(\alpha_n)} \left| N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \right\rangle. \quad (2.4)$$

$D_{\nu, \nu}^{(\alpha_n)}$ is a matrix elements of the group element g corresponding to the representation α .

One gets finally

$$\overline{P_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}} = \mathcal{A}^{\{\mathcal{N}\}} d(\Lambda) \int_G d\mu(g) \bar{\chi}^{(\Lambda)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}}. \quad (2.5)$$

$D_{\nu, \nu}^{(\alpha_n)}$ is a matrix elements of the group element g corresponding to the representation α and $\mathcal{A}^{\{\mathcal{N}\}}$ is an overall permutation normalization factor

$$\mathcal{A}^{\{\mathcal{N}\}} = \prod_j \prod_{\zeta_j} \mathcal{A}_{(\zeta_j)}^{\alpha_j}, \quad (2.6)$$

where $\mathcal{A}_{(\zeta_j)}^{\alpha_j}$ are partial factors for particles of the kind $\{\alpha, \zeta\}$

$$\mathcal{A}_{(\zeta)}^{\alpha} = \frac{\mathcal{N}_{(\zeta)}^{(\alpha)}!}{d(\alpha)^{\mathcal{N}_{(\zeta)}^{(\alpha)}} \prod_{\nu_{\alpha}} \mathcal{N}_{\nu_{\alpha}; (\zeta)}^{(\alpha)}!}. \quad (2.7)$$

The permutation factor gives a proper normalization of state vectors reflecting indistinguishability of particles

$$\left\langle N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \middle| N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)} \right\rangle = \mathcal{A}^{\{\mathcal{N}\}}. \quad (2.8)$$

Because of the symmetry conservations all weights in Eq. (2.5) should be constant

$$\frac{d}{dt} \overline{P_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}} = 0. \quad (2.9)$$

Introducing here the result of Eq. (2.5) one obtains

$$\begin{aligned} 0 &= \frac{d \log \mathcal{A}^{\{\mathcal{N}\}}}{dt} \int_G d\mu(g) \bar{\chi}^{(\Lambda)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}} \\ &+ \sum_{j=1}^n \sum_{\nu_{\alpha_j}} \frac{d N_{\nu_{\alpha_j}}^{(\alpha_j)}}{dt} \int_G d\mu(g) \bar{\chi}^{(\Lambda)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}} \log \left[D_{\nu_j \nu_j}^{(\alpha_j)} \right]. \end{aligned} \quad (2.10)$$

The integrals which appear in Eq. (2.10) can be expressed explicitly in an analytic form for any compact symmetry group.

To write an expression for the time derivative of the normalization factor $\mathcal{A}^{\{\mathcal{N}\}}$ we perform analytic continuation from integer to continuous values of variables $N_{\nu_{\alpha_n}}^{(\alpha_n)}$. All factorials in Eq. (2.6) are replaced by the Γ -function of corresponding arguments. We encounter here also the digamma function ψ [5]

$$\psi(x) = \frac{d \log \Gamma(x)}{dx}. \quad (2.11)$$

This allows to write

$$\begin{aligned} \frac{d \log \mathcal{A}^{\{\mathcal{N}\}}}{dt} = \\ \sum_j \sum_{\zeta_j} \left[\frac{d \mathcal{N}_{(\zeta_j)}^{(\alpha_j)}}{dt} \psi \left(\mathcal{N}_{(\zeta_j)}^{(\alpha_j)} + 1 \right) - \sum_{\nu_{\alpha_j}} \frac{d \mathcal{N}_{\nu_{\alpha_j}; (\zeta_j)}^{(\alpha_j)}}{dt} \psi \left(\mathcal{N}_{\nu_{\alpha_j}; (\zeta_j)}^{(\alpha_j)} + 1 \right) \right]. \end{aligned} \quad (2.12)$$

Eq. (2.10) can be written in a form

$$\begin{aligned} \sum_{j=1}^n \sum_{\nu_{\alpha_j}} \frac{d N_{\nu_{\alpha_j}}^{(\alpha_j)}}{dt} \frac{d \log \mathcal{P}_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}}}{d N_{\nu_{\alpha_j}}^{(\alpha_j)}} = \\ \sum_j \sum_{\zeta_j} \left(- \frac{d \mathcal{N}_{(\zeta_j)}^{(\alpha_j)}}{dt} \psi \left(\mathcal{N}_{(\zeta_j)}^{(\alpha_j)} + 1 \right) + \sum_{\nu_{\alpha_j}} \frac{d \mathcal{N}_{\nu_{\alpha_j}; (\zeta_j)}^{(\alpha_j)}}{dt} \psi \left(\mathcal{N}_{\nu_{\alpha_j}; (\zeta_j)}^{(\alpha_j)} + 1 \right) \right), \end{aligned} \quad (2.13)$$

where

$$\mathcal{P}_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A} = \int_G d\mu(g) \bar{\chi}^{(A)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}} \quad (2.14)$$

is analytically extended for continuous values of variables $N_{\nu_{\alpha_j}}^{(\alpha_j)}$. This gives

$$\frac{d \log \mathcal{P}_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}}{d N_{\nu_{\alpha_j}}^{(\alpha_j)}} = \frac{\int_G d\mu(g) \bar{\chi}^{(A)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}} \log \left[D_{\nu_j \nu_j}^{(\alpha_j)} \right]}{\int_G d\mu(g) \bar{\chi}^{(A)}(g) \left[D_{\nu_1 \nu_1}^{(\alpha_1)} \right]^{N_{\nu_{\alpha_1}}^{(\alpha_1)}} \dots \left[D_{\nu_n \nu_n}^{(\alpha_n)} \right]^{N_{\nu_{\alpha_n}}^{(\alpha_n)}}}. \quad (2.15)$$

Eq. (2.15) gives a set of conditions related to the internal symmetry of a system. They are meaningful only for nonzero values of coefficients (2.5).

It is easy to see that coefficients $\frac{P_{\{N_{\nu_{\alpha_1}}^{(\alpha_1)}, \dots, N_{\nu_{\alpha_n}}^{(\alpha_n)}\}}^{A, \lambda_A}}{N_{\nu_{\alpha_j}}^{(\alpha_j)}}$ are different from zero only if parameters λ_A are consistent with the conservation of the simultaneously measurable charges related to the symmetry group. A number of such charges is equal to the rank k of the symmetry group. For the isospin SU(2) group that is the third component of the isospin, for the flavor SU(3) that would be the third component of the isospin and the hypercharge. In general case one has k linear relations between variables $N_{\nu_{\alpha_j}}^{(\alpha_j)}$ what reduces correspondingly the number of independent variables.

3. Isotopic hadronic gas

Let us consider a case of isotopic symmetry in a more detailed way. Diagonal matrix elements for the representation (j) in the Euler angles representation have the form [3, 4]

$$D_{mm}^{(j)}(\alpha, \beta, \gamma) = e^{im(\alpha+\gamma)} d_{mm}^{(j)}(\beta), \quad (3.1)$$

where

$$d_{mm}^{(j)}(\cos \beta) = \left(\frac{1 + \cos \beta}{2} \right)^m P_{j-m}^{(0, 2m)}(\cos \beta).$$

and $P_{j-m}^{(0, 2m)}(\cos \beta)$ are Jacobi polynomials

$$P_{j-m}^{(0, 2m)}(x) = \frac{(-1)^{j-m}}{2^m (j-m)!} (1+x)^{-2m} \frac{d^{j-m}}{dx^{j-m}} [(1-x)^{j-m} (1+m)^{j+m}]. \quad (3.2)$$

The measure $d\mu(g)$ for the $SU(2)$ group in this parametrization has the form

$$\int d\mu(g) f[g] = \frac{1}{8\pi^2} \int_0^{2\pi} d\alpha \int_0^{2\pi} d\gamma \int_0^\pi d\beta \sin \beta f[g(\alpha, \beta, \gamma)]. \quad (3.3)$$

There are only three possible nontrivial hadronic isotopic multiplets. These are spinor ($\frac{1}{2}$), vector ($\mathbf{1}$), and ($\frac{3}{2}$) representations. Corresponding d functions are

$$d_{\pm 1/2, \pm 1/2}^{(1/2)}(\beta) = \left(\frac{1 + \cos \beta}{2} \right)^{\frac{1}{2}}, \quad (3.4a)$$

$$d_{\pm 1, \pm 1}^{(1)}(\beta) = \frac{1 + \cos \beta}{2}, \quad (3.4b)$$

$$d_{0,0}^{(1)}(\beta) = \cos \beta, \quad (3.4c)$$

$$d_{\pm 3/2, \pm 3/2}^{(3/2)}(\beta) = \left(\frac{1 + \cos \beta}{2} \right)^{\frac{3}{2}}, \quad (3.4d)$$

$$d_{\pm 1/2, \pm 1/2}^{(3/2)}(\beta) = \frac{1}{2} \left(\frac{1 + \cos \beta}{2} \right)^{\frac{1}{2}} (-1 + 3 \cos \beta). \quad (3.4e)$$

The group theoretic factor (2.14) has the form

$$\begin{aligned} \int_G d\mu(g) \bar{\chi}^{(J)}(g) \left[D_{m_{j_1} m_{j_1}}^{(j_1)} \right]^{N_{m_{j_1}}^{(j_1)}} \dots \left[D_{m_{j_n} m_{j_n}}^{(j_n)} \right]^{N_{m_{j_n}}^{(j_n)}} = \\ \frac{1}{8\pi^2} \sum_{M=-J}^J \int_0^{2\pi} d\alpha \int_0^{2\pi} d\gamma \int_0^\pi d\beta \sin \beta e^{i(N_{m_{j_1}}^{(j_1)} m_{j_1} + \dots + N_{m_{j_n}}^{(j_n)} m_{j_n} - M)(\alpha + \gamma)} \\ \times d_{MM}^{(J)}(\cos \beta) \left[d_{m_{j_1} m_{j_1}}^{(j_1)}(\cos \beta) \right]^{N_{m_{j_1}}^{(j_1)}} \dots \left[d_{m_{j_n} m_{j_n}}^{(j_n)}(\cos \beta) \right]^{N_{m_{j_n}}^{(j_n)}}, \quad (3.5) \end{aligned}$$

where $N_{m_j}^{(j)}$ is a number of particles with the isotopic coordinates $\{j, m_j\}$.

The nonzero values are obtained only when the arguments of the exponent in Eq. (3.5) vanish

$$N_{m_{j_1}}^{(j_1)} m_{j_1} + \dots + N_{m_{j_n}}^{(j_n)} m_{j_n} - M = 0. \quad (3.6)$$

For the hadronic system with the given value $\tilde{M}$ of the third component of the isospin the factor (2.14) is

$$\begin{aligned}
& \int_G d\mu(g) \bar{\chi}^{(J)}(g) \left[D_{m_{j_1} m_{j_1}}^{(j_1)} \right]^{N_{m_{j_1}}^{(j_1)}} \cdots \left[D_{m_{j_n} m_{j_n}}^{(j_n)} \right]^{N_{m_{j_n}}^{(j_n)}} = \\
& \frac{1}{8\pi^2} \delta_{\tilde{M}, N_{m_{j_1}}^{(j_1)} m_{j_1} + \cdots + N_{m_{j_n}}^{(j_n)} m_{j_n}} \int_0^{2\pi} d\alpha \int_0^{2\pi} d\gamma \int_0^\pi d\beta \sin \beta \\
& \times d_{\tilde{M}\tilde{M}}^{(J)}(\cos \beta) \left[d_{m_{j_1} m_{j_1}}^{(j_1)}(\cos \beta) \right]^{N_{m_{j_1}}^{(j_1)}} \cdots \left[d_{m_{j_n} m_{j_n}}^{(j_n)}(\cos \beta) \right]^{N_{m_{j_n}}^{(j_n)}}. \quad (3.7)
\end{aligned}$$

These equations allow to write general forms of the isotopic symmetry factor (2.14) for the hadronic system

$$\begin{aligned}
& \mathcal{P}_{\{N_{-1/2}^{(1/2)}, N_{1/2}^{(1/2)}, N_{-1}^{(1)}, N_0^{(1)}, N_1^{(1)}, N_{-3/2}^{(3/2)}, N_{-1/2}^{(3/2)}, N_{1/2}^{(3/2)}, N_{3/2}^{(3/2)}\}}^{(J, \tilde{M})} = \\
& \frac{2J+1}{2} \int_{-1}^1 dx \left(\frac{1+x}{2} \right)^{\tilde{M}+\mathcal{R}} \left(\frac{-1+3x}{2} \right)^{N_{-1/2}^{(3/2)}+N_{1/2}^{(3/2)}} x^{N_0^{(1)}} P_{J-\tilde{M}}^{(0, 2\tilde{M})}(x), \quad (3.8)
\end{aligned}$$

where

$$\mathcal{R} = N_{m_{j_1}}^{(j_1)} |m_{j_1}| + \cdots + N_{m_{j_n}}^{(j_n)} |m_{j_n}|. \quad (3.9)$$

The definition (3.9) and the constraint (3.6) allow to write

$$\mathcal{R} + \tilde{M} = 2 \sum_{\{j, |m_j|\}} N_{|m_j|}^{(j)} |m_j|. \quad (3.10)$$

This is because

$$\sum_{\{j, m_j\}} N_{m_j}^{(j)} m_j = \sum_{\{j, |m_j|\}} N_{|m_j|}^{(j)} |m_j| - \sum_{\{j, |m_j|\}} N_{-|m_j|}^{(j)} |m_j| \quad (3.11a)$$

and

$$\sum_{\{j, m_j\}} N_{m_j}^{(j)} |m_j| = \sum_{\{j, |m_j|\}} N_{|m_j|}^{(j)} |m_j| + \sum_{\{j, |m_j|\}} N_{-|m_j|}^{(j)} |m_j|. \quad (3.11b)$$

This allows to write Eq. (3.8) in the form

$$\begin{aligned}
& \mathcal{P}_{\{N_{-1/2}^{(1/2)}, N_{1/2}^{(1/2)}, N_{-1}^{(1)}, N_0^{(1)}, N_1^{(1)}, N_{-3/2}^{(3/2)}, N_{-1/2}^{(3/2)}, N_{1/2}^{(3/2)}, N_{3/2}^{(3/2)}\}}^{(J, \tilde{M})} = \\
& \frac{2J+1}{2} \int_{-1}^1 dx \left(\frac{1+x}{2} \right)^{(N_{1/2}^{(1/2)} + 2N_1^{(1)} + N_{1/2}^{(3/2)} + 3N_{3/2}^{(3/2)})} \\
& \times \left(\frac{-1+3x}{2} \right)^{N_{-1/2}^{(3/2)} + N_{1/2}^{(3/2)}} x^{N_0^{(1)}} P_{J-\tilde{M}}^{(0, 2\tilde{M})}(x). \quad (3.12)
\end{aligned}$$

One should notice that this reduction of independent variables is related only to that part of the total weight (2.5) which comes from the group theory. The condition (2.9) is obtained when the expression (3.12) is multiplied by the combinatoric factor (2.6). Number of independent variables in the combinatoric factor can be reduced only by conservation laws of corresponding charges of the system.

To be more specific let us consider as an example an iso-singlet system consisting only of pions and nucleons [1]. We take as independent variables: N total number of particles, B total baryon number, Q total electric charge, n_{π^0} number of neutral pions, n_N number of neutrons. Then

$$\begin{aligned}
& \text{number of negative pions: } n_- = (N - n_N - n_0 - Q)/2, \\
& \text{number of positive pions: } n_+ = -B + n_N + (N - n_N - n_0 + Q)/2, \\
& \text{number of protons: } n_P = B - n_N.
\end{aligned}$$

For iso-singlet state $B = 2Q$ and

$$n_- = \frac{(N - n_N - n_0 - Q)}{2}, \quad (3.13a)$$

$$n_+ = -2Q + n_N + \frac{(N - n_N - n_0 + Q)}{2}, \quad (3.13b)$$

$$n_P = 2Q - n_N. \quad (3.13c)$$

Let us assume [1] that this system reaches the chemical equilibrium in the evolution process governed by Vlasov–Boltzman kinetic equations with the interactions restricted to binary collisions. Then the *total* number of particles remains constant but particles ratios are subjected to constraints (2.9). For the given baryon number B and the given total number of particles N , condition

$$\overline{P_{\{n_N, n_P, n_-, n_0, n_+\}}^{(0,0)}} = \text{const.} \quad (3.14)$$

gives evolution lines in the $n_N - n_0$ plane. The system evolves along these lines which are here the consequences of the isotopic SU(2) symmetry and baryon number conservation. The weights (3.14) expressed by means of variables B , N , n_N and n_0 and calculated according to Eqs. (2.7) and (3.13) have the form

$$\begin{aligned} \overline{P_{\{n_N, n_P, n_-, n_0, n_+\}}^{(0,0)}} &= \frac{B!}{2^B n_N! (B - n_N)!} \\ &\times \frac{(N - B)!}{3^{(N-B)n_0!} \left(\frac{N}{2} - \frac{n_N}{2} - \frac{n_0}{2} - \frac{B}{4}\right)! \left(\frac{N}{2} + \frac{n_N}{2} - \frac{n_0}{2} - 3\frac{B}{4}\right)!} \\ &\times \frac{1}{2} \int_{-1}^1 dx \left(\frac{1+x}{2}\right)^{(N-n_0-B/2)} x^{n_0}. \end{aligned} \quad (3.15)$$

The corresponding evolution lines are shown in Fig. 1.

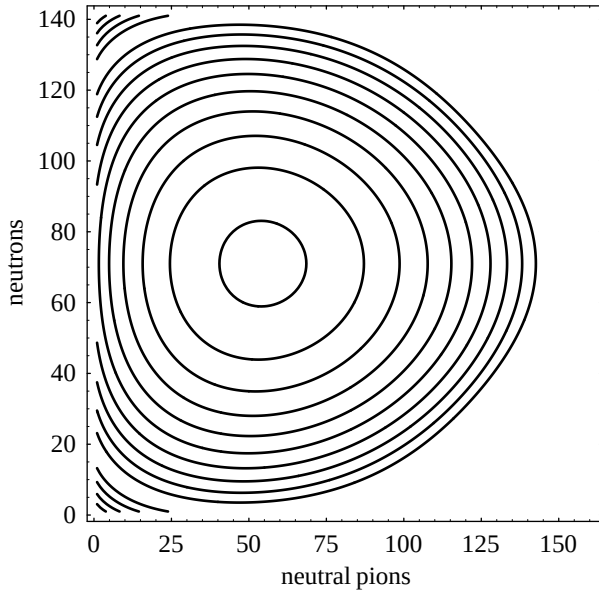


Fig. 1. Evolution curves for the π - N iso-singlet system consisting of 300 particles — pions and nucleons. Baryon number $B = 140$. The chemical equilibrium is reached along equal weights curves (Eq. (3.14)) in the plane n_0 - n_N .

4. Conclusions

We have got relations which are *necessary* global conditions to provide consistency with the overall symmetry of the system. They do not depend on the form of the underlying microscopic interaction. Abelian internal symmetries lead to simple and obvious linear relations as in Eq. (1.1). Non-abelian internal symmetries lead to nonlinear relations as in Eq. (2.10).

If we knew the solutions of symmetry invariant evolution equations then all those constraints would become identities. In other case they give a subsidiary information about the system and can be used as a consistency check for approximative calculations. A case of generalized Vlasov–Boltzman kinetic equations was considered in [1].

New constraints lead to decreasing number of available states for the system during its evolution. New correlations appear and the change in the thermodynamical behavior can be expected. This can be observed in hydrodynamical systems formed in high energy heavy ion collisions [6].

A challenging point is to find structures which would correspond to chemical potentials when system approaches the equilibrium distribution. The equilibrium distribution in the presence of constraints can be constructed by the Lagrange multipliers method. The multipliers related to the “abelian” constraints, such as Eq. (1.1), are well known chemical potentials. Multipliers related to the “non-abelian” constraints (2.10) are more complicated. Because these constraints are nonlinear ones, corresponding multipliers cannot be treated as standard additive thermodynamical potentials.

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