YFS MC APPROACH TO QCD SOFT GLUON EXPONENTIATION* **

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We present two things in this discussion. First, we develop and prove the theory of the extension of the YFS Monte Carlo approach to higher order $SU_{2L} \times U_1$ radiative corrections to the analogous higher order QCD radiative corrections. Contact is made with other pioneering soft gluon resummation theories in the literature. Second, semi-analytical results and preliminary explicit Monte Carlo data are presented for the specific example of the processes $p\bar{p} \to t\bar{t} + n(G) + X$ at FNAL energies, where G is a soft gluon and the respective event generator, ttp1.0, features realistic, event-by-event simulation of multiple, soft, finite $p_{\rm T}$ gluon effects in which the infrared singularities are canceled to all orders in α_s. We comment briefly on the implications of our results on the CDF/D0 observations and on their possible applications to RHIC physics and to LHC physics.

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1. Introduction

The problem of soft gluon resummation is well known [1,2] and some of its many phenomenological applications are also:

- FNAL $t\bar{t}$ production cross section higher order corrections (the current situation [3,4] has the experimental cross section $6.2^{+1.2}_{-1.1}$ to be compared with a theoretical prediction [5] of 5.1 ± 0.5) and the attendant soft gluon uncertainty in the extracted value of $m_t = 0.1743 \pm 0.0051$ TeV, where [6] ~2–3 GeV of the latter error could be due to soft gluon uncertainties.
- RHIC hard scattering polarized pp scattering processes.
- $n(G)$ production in the hard nucleon–nucleon scattering processes which participate in the nucleus–nucleus collisions at RHIC, where G denotes a soft QCD gluon.
- $b\bar{b}$ and J/Ψ production by hard processes at FNAL.
- heavy vector boson production at FNAL and RHIC, *etc.*

For the LHC/TESLA/LC, the requirements on the corresponding theoretical precisions will be even more demanding and the QCD soft $n(G)$ MC exponentiation which we discuss in the following will be an important part of the necessary theory — YFS [7] exponentiated $\mathcal{O}(\alpha_s^2)L$ corrections realized on an event-by-event basis.

The results which we present also will ultimately allow us to investigate from a different perspective some of the outstanding theoretical issues in perturbative QCD:

- Treatment of the $2i(f) + n(G)$ phase space, where we have in mind our usual exact treatment of this phase space to be compared with the somewhat different approach of Catani and Seymour [8], for example — here, f denotes a generic fermion.
- Approaches to resummation itself [1,9–11].
- No-go theorems, such as the result of Ref. [12] , which requires that for calculations beyond $\mathcal{O}(\alpha_s)$, all initial state parton masses must vanish.

Clearly, the results which we present herein have indeed a large arena for further development and application. For definiteness, we will use the process in Fig. 1, $\bar{Q}(p_1)Q(q_1) \to \bar{t}(p_2)t(p_1) + G_1(k_1)\cdots G_n(k_n)$, as prototypical process, where we have written the kinematics as it is illustrated in the figure. This process, which dominates $t\bar{t}$ production at FNAL, contains

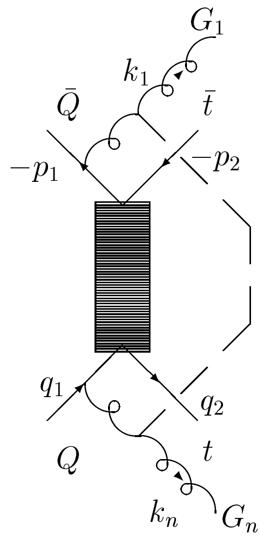

Fig. 1. The process $\bar{Q}Q \to \bar{t} + t + n(G)$. The four-momenta are indicated in the standard manner: q_1 is the four-momentum of the incoming Q, q_2 is the four-momentum of the outgoing t, *etc.*, and $Q = u, d, s, c, b, G$.

all of the theoretical issues that we must face at the parton level to establish YFS QCD soft exponentiation by MC methods — extension to other related processes will be immediate. For reference, let us also note that, in what follows, we use the GPS conventions of Jadach *et al.* [13] for spinors and the attendant photon and gluon polarization vectors that follow therefrom:

$$(\varepsilon^\mu_\sigma(\beta))^* = \frac{\bar{u}_\sigma(k)\gamma^\mu u_\sigma(\beta)}{\sqrt{2}\,\bar{u}_{-\sigma}(k)u_\sigma(\beta)}, \quad (\varepsilon^\mu_\sigma(\zeta))^* = \frac{\bar{u}_\sigma(k)\gamma^\mu \mathfrak{u}_\sigma(\zeta)}{\sqrt{2}\,\bar{u}_{-\sigma}(k)\mathfrak{u}_\sigma(\zeta)}, \tag{1}$$

so that all phase information is strictly known in our amplitudes. This means that, although we shall use the older EEX realization of YFS MC exponentiation as defined in Ref. [14], the realization of our results via the newer CEEX realization of YFS exponentiation in Ref. [14] is also possible and is in progress [15].

We organize our presentation as follows. In the next section, we review the application of YFS MC methods in the EW–QED case. In Sect. 3, we prove that we can extend this application to the QCD theory. In Sect. 4 we illustrate the QCD extension by applying it to $t\bar{t}$ production at FNAL. Sect. 5 contains our summary remarks.

2. Review of YFS theory: an Abelian gauge theory example

In this section, for pedagogical reasons, we review the application of YFS exponentiation to the prototypical Abelian gauge theory example of $e^+e^- \to \bar{f}f + n(\gamma)$ so that we set the stage for our proof of the extension of this theory to the prototypical non-Abelian gauge theory example of $q\bar{q} \to t\bar{t} + n(G)$ in QCD in the next section. Specifically, in Refs. [16–18] we have shown that the process $e^+(p_1)e^-(q_1) \to \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdots, k_n)$, in the renormalization group improved YFS theory [16], is represented by

$$\begin{aligned} d\sigma_{\rm exp} &= \mathrm{e}^{2\alpha\,\mathrm{Re}\,B + 2\alpha\,\tilde{B}} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j^0} \int \frac{d^4y}{(2\pi)^4} \mathrm{e}^{iy(p_1+q_1-p_2-q_2-\sum_j k_j)+D} \\ &\quad \times \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3p_2 d^3q_2}{p_2^0 q_2^0}\,, \end{aligned} \tag{2}$$

where the real infrared function $\tilde{B}$ and the virtual infrared function B are given in Refs. [7, 17, 19–22], and where we note the usual connections

$$\begin{aligned} 2\alpha\,\overline{B} &= \int^{k \le K_{\rm max}} \frac{d^3k}{k_0} \tilde{S}(k) \\ D &= \int d^3k \frac{\tilde{S}(k)}{k^0} \left(\mathrm{e}^{-iy\cdot k} - \theta(K_{\rm max} - k) \right) \end{aligned} \tag{3}$$

for the standard YFS infrared emission factor

$$\tilde{S}(k) = \frac{\alpha}{4\pi^2} \left[Q_f Q_{(\bar{f})'} \left(\frac{p_1}{p_1 \cdot k} - \frac{q_1}{q_1 \cdot k} \right)^2 + (\ldots) \right] \tag{4}$$

if Q_f is the electric charge of f in units of the positron charge. For example, the YFS hard photon residuals $\bar{\beta}_i$ in (2), $i = 0, 1, 2$, are given in Ref. [23] for BHLUMI 4.04 so that this latter event generator calculates the YFS exponentiated exact $\mathcal{O}(\alpha)$ and LL $\mathcal{O}(\alpha^2)$ cross section for Bhabha scattering using a corresponding Monte Carlo realization of (2). In the remainder of this paper, we will develop and apply the analogous theoretical paradigm to the prototypical QCD higher order radiative corrections problem for $q\bar{q} \to t\bar{t} + n(G)$.

3. Extension to non-Abelian gauge theories: proof

Specifically, in Refs. [24], we have analyzed how in the special case of Born level color exchange one applies the YFS theory to QCD by extending the respective YFS IR singularity analysis to QCD to all orders in α_s.

Here, we will take a somewhat different approach. We want to focus on the YFS theory as a general re-arrangement of renormalized perturbation theory based on its IR behavior, just as the renormalization group is a general property of renormalized perturbation theory based on its UV (Ultra-Violet) behavior. Thus, here we will keep our arguments entirely general from the outset, so that it will be immediate that our result applies to any renormalized perturbation theory in which the cross section under study is finite.

Let the amplitude for the emission of n real gluons in our proto-typical subprocess, $q_i^{\alpha} + \bar{q}_i^{\bar{\alpha}} \to t^{\gamma}\bar{t}^{\bar{\gamma}} + n(G)$, where $\alpha, \bar{\alpha}, \gamma$, and $\bar{\gamma}$ are color indices, be represented by

$$\mathcal{M}^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}} = \sum_{\ell} M^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}\ell}, \tag{5}$$

$M_\ell^{(n)}$ is the contribution to $\mathcal{M}^{(n)}$ from Feynman diagrams with ℓ virtual loops. Symmetrization yields

$$M_\ell^{(n)} = \frac{1}{\ell!}\int \prod_{j=1}^{\ell} \frac{d^4 k_j}{(2\pi)^4(k_j^2 - \lambda^2 + i\varepsilon)}\rho_\ell^{(n)}(k_1, \cdots, k_\ell)\,, \tag{6}$$

where this last equation defines $\rho_\ell^{(n)}$ as a symmetric function of its arguments $k_1, \ldots, k_\ell$. λ will be our infrared gluon regulator mass for IR singularities; n-dimensional regularization of the 't Hooft–Veltman [25] type is also possible as we shall see.

We now define the virtual IR emission factor $S_{\rm QCD}(k)$ for a gluon of 4-momentum k, for the $k \to 0$ regime of the respective 4-dimensional loop integration as in (6), such that

$$\lim_{k\to 0} k^2 \left(\rho^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}1}(k) - S_{\rm QCD}(k)\rho^{(n)\alpha\bar{\alpha}}_{\gamma\bar{\gamma}0}\right)|_{\alpha\neq\bar{\alpha}\neq\gamma\neq\alpha} = 0\,, \tag{7}$$

where we have now introduced the Born level color exchange condition as $\alpha \neq \bar{\alpha} \neq \gamma \neq \alpha$ for definiteness. (Henceforth, when we refer to $k \to 0$ gluons we are always referring for virtual gluons to the corresponding regime of the 4-dimensional loop integration in the computation of $M_\ell^{(n)}$.)

In Ref. [24], we have calculated $S_{\rm QCD}(k)$ using the running quark masses to regulate its collinear mass singularities, for example; n-dimensional regularization of the 't Hooft–Veltman type is also possible for these mass singularities and we will also illustrate this presently.

We stress that $S_{\rm QCD}(k)$ has a freedom in it corresponding to the fact that any function $\Delta S_{\rm QCD}(k)$ which has the property that

$$\lim_{k\to 0} k^2 \Delta S_{\rm QCD}(k)\rho_0^{(n)} = 0$$

may be added to it.

Since the virtual gluons in $\rho_\ell^{(n)}$ are all on equal footing by the symmetry of this function, if we look at gluon ℓ, for example, we may write, for $k_\ell \to (0,0,0,0) \equiv O$ while the remaining k_i are fixed away from O, the representation

$$\rho_\ell^{(n)} = S_{\rm QCD}(k_\ell) * \rho_{\ell-1}^{(n)}(k_1, \cdots, k_{\ell-1}) + \beta_\ell^1(k_1, \cdots, k_{\ell-1}; k_\ell)\,, \qquad (8)$$

where the residual amplitude $\beta_\ell^1(k_1, \cdots, k_{\ell-1}; k_\ell)$ will now be taken as defined by this last equation. It has two nice properties:

- it is symmetric in its first $\ell - 1$ arguments,
- the IR singularities for gluon ℓ *that are contained in* $S_{\rm QCD}(k_\ell)$ are no longer contained in it.

We do not at this point discuss whether there are any further remaining IR singularities for gluon ℓ in $\beta_\ell^1(k_1, \cdots, k_{\ell-1}; k_\ell)$. In an Abelian gauge theory like QED, as has been shown by Yennie, Frautschi and Suura in Ref. [7], there would not be any further such singularities; for a non-Abelian gauge theory like QCD, this point requires further discussion and we will come back to this point presently.

We rather now stress that if we apply the representation (8) again we may write

$$\begin{aligned}\rho_\ell^{(n)} &= S_{\rm QCD}(k_\ell) S_{\rm QCD}(k_{\ell-1}) * \rho_{\ell-2}^{(n)}(k_1, \cdots, k_{\ell-2}) \\ &+ S_{\rm QCD}(k_\ell)\beta_{\ell-1}^1(k_1, \cdots, k_{\ell-2}; k_{\ell-1}) \\ &+ S_{\rm QCD}(k_{\ell-1})\beta_{\ell-1}^1(k_1, \cdots, k_{\ell-2}; k_\ell) \\ &+ \beta_\ell^2(k_1, \cdots, k_{\ell-2}; k_{\ell-1}, k_\ell)\,, \end{aligned} \qquad (9)$$

where this last equation serves to define the function $\beta_\ell^2(k_1, \cdots, k_{\ell-2}; k_{\ell-1}, k_\ell)$. It has two nice properties:

- it is symmetric in its first $\ell-2$ arguments and in its last two arguments $k_{\ell-1}, k_\ell$,
- the infrared singularities for gluons $\ell - 1$ and ℓ that are contained in $S_{\rm QCD}(k_{\ell-1})$ and $S_{\rm QCD}(k_\ell)$ are no longer contained in it.

Continuing in this way, with repeated application of (8), we get finally the rigorous, exact rearrangement of the contributions to $\rho_\ell^{(n)}$ as

$$\begin{aligned}\rho_\ell^{(n)} &= S_{\rm QCD}(k_1) \cdots S_{\rm QCD}(k_\ell)\beta_0^0 \\ &+ \sum_{i=1}^{\ell} \prod_{j \neq i} S_{\rm QCD}(k_j)\beta_1^1(k_i) + \cdots + \beta_\ell^\ell(k_1, \cdots, k_\ell)\,, \end{aligned} \qquad (10)$$

where the virtual gluon residuals $\beta_i^i(k_1', \cdots, k_i')$ have two nice properties:

- they are symmetric functions of their arguments,
- they do not contain any of the IR singularities which are contained in the product $S_{\rm QCD}(k_1') \cdots S_{\rm QCD}(k_i')$.

Henceforth, we denote β_i^i as the function β_i for reasons of pedagogy. We can not stress too much that (10) is an *exact* rearrangement of the contributions of the Feynman diagrams which contribute to $\rho_\ell^{(n)}$; it involves no approximations. Here also it is important to note that we do no enter into the question of the absolute convergence of these Feynman diagrams from the standpoint of constructive field theory. Yennie, Frautschi and Suura [7] have already stressed that Feynman diagrammatic perturbation theory is non-rigorous from this standpoint. What we claim is that the relationship between the YFS expansion and the usual perturbative Feynman diagrammatic expansion is itself rigorous even though neither of the two expansions themselves is rigorous.

Introducing (10) into (5) yields a representation similar to that of YFS, and we will call it a "YFS representation",

$$\mathcal{M}^{(n)} = \exp(\alpha_s B_{\rm QCD}) \sum_{j=0}^{\infty} \mathsf{m}_j^{(n)}, \tag{11}$$

where we have defined

$$\alpha_s(Q) B_{\rm QCD} = \int \frac{d^4 k}{(k^2 - \lambda^2 + i\varepsilon)} S_{\rm QCD}(k) \tag{12}$$

and

$$\mathsf{m}_j^{(n)} = \frac{1}{j!} \int \prod_{i=1}^{j} \frac{d^4 k_i}{k_i^2} \beta_j(k_1, \cdots, k_j). \tag{13}$$

We say that (11) is similar to the respective result of Yennie, Frautschi and Suura in Ref. [7] and is not identical to it because we have not proved that the functions $\beta_i(k_1, \ldots, k_i)$ are completely free of virtual IR singularities. What have shown is that they do not contain the IR singularities in the product $S_{\rm QCD}(k_1) \cdots S_{\rm QCD}(k_i)$ so that $\mathsf{m}_j^{(n)}$ does not contain the virtual IR divergences generated by this product when it is integrated over the respective $4j$-dimensional j-virtual gluon phase space. In an Abelian gauge theory, there are no other possible virtual IR divergences; in the non-Abelian gauge theory that we treat here, such additional IR divergences are possible; but, the result (11) does have an improved IR divergence structure over (5) in that all of the IR singularities associated with $S_{\rm QCD}(k)$ are explicitly removed from the sum over the virtual IR improved loop contributions $\mathsf{m}_j^{(n)}$ to all orders in $\alpha_s(Q)$.

Turning now to the analogous rearrangement of the real IR singularities in the differential cross section associated with the $\mathcal{M}^{(n)}$, we first note that we may write this cross section as follows according the standard methods

$$
\begin{aligned}
d\hat{\sigma}^n \ &= \ \frac{{\rm e}^{2\alpha_s {\rm Re}\, B_{\rm QCD}}}{n!}\int\prod_{m=1}^{n}\frac{d^3k_m}{(k_m^2+\lambda^2)^{1/2}}\delta\left(P_1+Q_1-P_2-Q_2-\sum_{i=1}^{n}k_i^0\right)\\
&\times\bar{\rho}^{(n)}(P_1,P_2,Q_1,Q_2,k_1,\cdots,k_n)\frac{d^3P_2 d^3Q_2}{P_2^0Q_2^0}\,, \qquad (14)
\end{aligned}
$$

where we have defined

$$
\bar{\rho}^{(n)}(P_1,P_2,Q_1,Q_2,k_1,\cdots,k_n) = \sum_{\rm color,spin}\left\|\sum_{j=0}^{\infty}\mathsf{m}_j^{(n)}\right\|^2 \qquad (15)
$$

in the incoming $q\bar{q}$ cms system and we have absorbed the remaining kinematical factors for the initial state flux, spin and color averages into the normalization of the amplitudes $\mathcal{M}^{(n)}$ for reasons of pedagogy so that the $\bar{\rho}^{(n)}$ are averaged over initial spins and colors and summed over final spins and colors. We now proceed in complete analogy with the discussion of $\rho_\ell^{(n)}$ above.

Specifically, for the functions

$$
\bar{\rho}^{(n)}(p_1,p_2,q_1,q_2,k_1,\cdots,k_n)\equiv\bar{\rho}^{(n)}(k_1,\cdots,k_n)
$$

which are symmetric functions of their arguments $k_1,\cdots,k_n$, we define first, for $n=1$,

$$
\lim_{|\vec{k}|\to 0}\vec{k}^2\left(\bar{\rho}^{(1)}(k)-\tilde{S}_{\rm QCD}(k)\bar{\rho}^{(0)}\right)=0, \qquad (16)
$$

where the real infrared function $\tilde{S}_{\rm QCD}(k)$ is rigorously defined by this last equation and is explicitly computed in DeLaney *et al.* [24]; like its virtual counterpart $S_{\rm QCD}(k)$ it has a freedom in it in that any function $\Delta\tilde{S}_{\rm QCD}(k)$ with the property that $\lim_{|\vec{k}|\to 0}\vec{k}^2\Delta\tilde{S}_{\rm QCD}(k)=0$ may be added to it without affecting the defining relation (16).

As we show in Ref. [15], we can again repeat the analogous arguments of Ref. [7] to get the "YFS-like" result

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} &= \sum_n d\hat{\sigma}^n \\
&= e^{{\rm SUM}_{\rm IR}({\rm QCD})} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}} \\
&\quad \times \bar{\beta}_n(k_1,\ldots,k_n) \frac{d^3 P_2}{P_2^0} \frac{d^3 Q_2}{Q_2^0}
\end{aligned} \tag{17}
$$

with

$$
\begin{aligned}
{\rm SUM}_{\rm IR}({\rm QCD}) &= 2\alpha_s {\rm Re}\, B_{\rm QCD} + 2\alpha_s \tilde{B}_{\rm QCD}(K_{\rm max}) , \\
2\alpha_s \tilde{B}_{\rm QCD}(K_{\rm max}) &= \int \frac{d^3 k}{k^o} \tilde{S}_{\rm QCD}(k) \theta(K_{\rm max} - k) , \\
D_{\rm QCD} &= \int \frac{d^3 k}{k} \tilde{S}_{\rm QCD}(k) \left[e^{-iy\cdot k} - \theta(K_{\rm max} - k) \right] ,
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{1}{2}\bar{\beta}_0 &= d\sigma^{(1-{\rm loop})} - 2\alpha_s {\rm Re}\, B_{\rm QCD} d\sigma_B , \\
\frac{1}{2}\bar{\beta}_1 &= d\sigma^{B1} - \tilde{S}_{\rm QCD}(k) d\sigma_B , \quad \ldots ,
\end{aligned} \tag{19}
$$

where the $\bar{\beta}_n$ are the QCD hard gluon residuals defined above; they are the non-Abelian analogs of the hard photon residuals defined by YFS. Here, for illustration, we have recorded the relationship between the $\bar{\beta}_n$, $n = 0, 1$ through $\mathcal{O}(\alpha_s)$ and the exact one-loop and single bremsstrahlung cross sections, $d\sigma^{(1-{\rm loop})}$, $d\sigma^{B1}$, respectively, where the latter may be taken from Nason *et al.* [26] and Beenakker *et al.* [27] We stress two things about the right-hand side of (17) :

- It does not depend on the dummy parameter $K_{\rm max}$ which has been introduced for cancellation of the infrared divergences in ${\rm SUM}_{\rm IR}({\rm QCD})$ to all orders in $\alpha_s(Q)$ where Q is the hard scale in the parton scattering process under study here.

- Its analog can also be derived in our new CEEX [14] format.

We now return to the property of (17) that distinguishes it from the Abelian result derived by YFS — namely, the fact that, owing to its non-Abelian gauge theory origins, it may very well be that there are infrared divergences in the $\bar{\beta}_n$ which were not removed into the $S_{\rm QCD}, \tilde{S}_{\rm QCD}$ when these infrared functions were isolated in our derivation of (17).

More precisely, the left-hand side of (17) is the fundamental reduced parton cross section and it should be infrared finite or else the entire QCD parton model has to be abandoned.

There is an observation in the literature [12] that unless we use some collinear mass singularity regulator other than the incoming light quark running masses, the reduced parton cross section on the left-hand side of (17) diverges in the infrared regime at $\mathcal{O}(\alpha_s^2(Q))$. We do not go into this issue here but use n-dimensional methods to regulate such divergences while setting the quark masses to zero as that is an excellent approximation for the light quarks at FNAL energies — we take this issue up elsewhere.

From the infrared finiteness of the left-hand side of (17) and the infrared finiteness of $SUM_{IR}(\mathrm{QCD})$, it follows that the quantity

$$d\bar{\hat{\sigma}}_{\rm exp} \equiv \exp[\mathrm{SUM_{IR}(QCD)}] d\hat{\sigma}_{\rm exp}$$

must also be infrared finite to all orders in α_s.

As we assume the QCD theory makes sense in some neighborhood of the origin for α_s, we conclude that each order in α_s must make an infrared finite contribution to $d\bar{\hat{\sigma}}_{\rm exp}$. At $\mathcal{O}(\alpha_s^0(Q))$, the only contribution to $d\bar{\hat{\sigma}}_{\rm exp}$ is the respective Born cross section given by $\bar{\beta}_0^{(0)}$ in (17) and it is obviously infrared finite, where we use henceforth the notation $\bar{\beta}_n^{(\ell)}$ to denote the $\mathcal{O}(\alpha_s^\ell(Q))$ part of $\bar{\beta}_n$. Thus, we conclude that the lowest hard gluon residual $\bar{\beta}_0^{(0)}$ is infrared finite.

Let us now define the left-over non-Abelian infrared divergence part of each contribution $\bar{\beta}_n^{(\ell)}$ via

$$\bar{\beta}_n^{(\ell)} = \tilde{\bar{\beta}}_n^{(\ell)} + D\bar{\beta}_n^{(\ell)} \,,$$

where the new function $\tilde{\bar{\beta}}_n^{(\ell)}$ is now completely free of any infrared divergences and the function $D\bar{\beta}_n^{(\ell)}$ contains all left-over infrared divergences in $\bar{\beta}_n^{(\ell)}$ which are of non-Abelian origin and is normalized to vanish in the Abelian limit $f_{abc} \to 0$ where f_{abc} are the group structure constants.

Further, we define $D\bar{\beta}_n^{(\ell)}$ by a minimal subtraction of the respective IR divergences in it so that it only contains the actual pole and transcendental constants, $1/\varepsilon - C_{\rm E}$ for $\varepsilon = 2 - d/2$, where d is the dimension of space-time, in dimensional regularization or $\ln\lambda^2$ in the gluon mass regularization. Here, $C_{\rm E}$ is Euler's constant.

For definiteness, we write this out explicitly so that there can be no confusion about what we mean

$$\int dPh\; D\bar{\beta}_n^{(\ell)} \equiv \sum_{i=1}^{n+\ell} d_i^{n,\ell} \ln^i(\lambda^2) \,,$$

where the coefficient functions $d_i^{n,\ell}$ are independent of λ for $\lambda \to 0$ and dPh is the respective n-gluon Lorentz invariant phase space.

At $\mathcal{O}(\alpha_s^n(Q))$, the IR finiteness of the contribution to $d\bar{\hat\sigma}_{\rm exp}$ then requires the contribution

$$d\bar{\hat\sigma}_{\rm exp}^{(n)} \equiv \int \sum_{\ell=0}^{n} \frac{1}{\ell!} \prod_{j=1}^{\ell} \int_{k_j^0 \geq K_{\rm max}} \frac{d^3k_j}{k_j} \tilde{S}_{\rm QCD}(k_j) \sum_{i=0}^{n-\ell} \frac{1}{i!} \prod_{j=\ell+1}^{\ell+i} \times \int \frac{d^3k_j}{k_j} \bar\beta_i^{(n-\ell-i)}(k_{\ell+1},\ldots,k_{\ell+i}) \frac{d^3P_2}{P_2^0} \frac{d^3Q_2}{Q_2^0} \tag{20}$$

to be finite.

From this it follows that

$$Dd\bar{\hat\sigma}_{\rm exp}^{(n)} \equiv \int \sum_{\ell=0}^{n} \frac{1}{\ell!} \prod_{j=1}^{\ell} \int_{k_j^0 \geq K_{\rm max}} \frac{d^3k_j}{k_j} \tilde{S}_{\rm QCD}(k_j) \sum_{i=0}^{n-\ell} \frac{1}{i!} \prod_{j=\ell+1}^{\ell+i} \times \int \frac{d^3k_j}{k_j} D\bar\beta_i^{(n-\ell-i)}(k_{\ell+1},\ldots,k_{\ell+i}) \frac{d^3P_2}{P_2^0} \frac{d^3Q_2}{Q_2^0} \tag{21}$$

is finite. Since the integration region for the final particles is arbitrary, the independent powers of the IR regulator $\ln(\lambda^2)$ in this last equation must give vanishing contributions. This means that we can drop the $D\bar\beta_n^{(\ell)}$ from our result (17) because they do not make a net contribution to the final parton cross section $\hat\sigma_{\rm exp}$. We thus finally arrive at the new rigorous result

$$\begin{aligned} d\hat\sigma_{\rm exp} &= \sum_n d\hat\sigma^n \\ &= \mathrm{e}^{\mathrm{SUM_{IR}(QCD)}} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} \mathrm{e}^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}} \\ &\quad * \tilde{\bar\beta}_n(k_1,\ldots,k_n) \frac{d^3P_2}{P_2^0} \frac{d^3Q_2}{Q_2^0}\,, \end{aligned} \tag{22}$$

where now the hard gluon residuals $\tilde{\bar\beta}_n(k_1,\ldots,k_n)$ defined by

$$\tilde{\bar\beta}_n(k_1,\ldots,k_n = \sum_{\ell=0}^{\infty} \tilde{\bar\beta}_n^{(\ell)}(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$. This is a basic result of this paper. We note here that, contrary to what we claimed in the

Appendix of the first paper in Refs. [24], the arguments in Refs. [24] are not sufficient to derive the respective analog of Eq. (22); for, they did not really expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between $\int dPh\bar{\beta}_n$ and $\int dPh\bar{\beta}_{n+1}$, respectively, that really distinguishes QCD from QED, where no such compensation occurs in the $\bar{\beta}_n$ residuals for QED. Further discussion of the many implications of Eq. (22) is given elsewhere [15].

4. YFS exponentiated QCD corrections to $t\bar{t}$ production at high energies

We shall realize the result above as it is applied to the process in Fig. 1 at high energies by the Monte Carlo event generator methods of Ref. [17,18]. The first application of these methods to QCD processes has already been reported in DeLaney *et al.* [24]. Here, we shall apply the general result in the latter reference to the $t\bar{t}$ production at the parton level and show how to synthesize that parton level result with the realistic parton distributions to arrive at an event generator for $p\bar{p} \to t\bar{t} + X$. Similar results will hold for pp incoming states. Sample MC data will be illustrated. We refer to the respective MC event generator as ttp1.0. It is in the EEX format but a CEEX version is imminent. It will be available from the authors soon.

The starting point in our MC realization of basic result for $t\bar{t}$ production at the hadron level is its realization for the respective parton level processes. In Ref. [24] we have shown how to construct such parton level event generators for the processes such as $q\bar{q} \to t\bar{t} + n(G)$ where q is a light quark. Here, we extend these results to the process $G + G \to t\bar{t} + n(G)$. This is presented in detail elsewhere [15].

Following the procedures in Ref. [24], we then use the Monte Carlo algorithm presented in Jadach *et al.* [17, 19] to realize the result derived in the previous section for both the $q\bar{q} \to t\bar{t} + n(G)$ and $GG \to t\bar{t} + n(G)$ subprocesses on an event-by-event basis in which infrared singularities are now canceled to all orders in α_s.

In order to apply these parton level results to the desired hadron level cross section $\sigma(p\bar{p} \to t\bar{t} + X)$, we use the standard formula

$$\sigma(p\bar{p} \to t\bar{t} + X) = \int \sum_{i,j} F_i(x_i)\bar{F}_j(x_j)d\hat{\sigma}'_{\mathrm{exp},ij}dx_i dx_j \,, \tag{23}$$

where $F_i(\bar{F}_j)$ is the structure function of parton $i(j)$ in $p(\bar{p})$ and where $\hat{\sigma}'_{\mathrm{exp},ij}$ is the result derived above for the $t\bar{t}$ production subprocess with the incoming parton-i, parton-j initial state when the DGLAP synthesization procedure presented in Ref. [28] is applied to it to avoid over-counting resummation effects already included in the structure function DGLAP evolution. In operational terms, the DGLAP synthesized result $\hat{\sigma}'_{\exp,ij}$ is obtained from the result $\hat{\sigma}_{\exp,ij}$ of the previous section by substituting the functions $\tilde{S}^{nls}_{\rm QCD}, B^{nls}_{\rm QCD}$ for $\tilde{S}_{\rm QCD}, B_{\rm QCD}$, respectively, where the former functions are given in Ref. [28] and do not contain big logs already contained in the structure functions, for example.

The formula in (23) we have realized by MC methods by extending the MC realizations of (22) for the $q\bar{q}$ and GG subprocesses to the respective realizations of $d\hat{\sigma}'_{\exp,ij}$, $ij = q\bar{q}$, GG in the presence of the additional two-dimensional structure function distribution in a standard way which is already illustrated for example in Ref. [29]. In this way, we arrive at the MC event generator ttp1.0, which features YFS-style exponentiated multiple soft gluon radiative effects in $p\bar{p} \to t\bar{t} + X$ on an event-by-event basis in which infrared singularities are canceled to all orders in α_s. We now illustrate its application with both semi-analytical results and with some preliminary sample MC data at FNAL energies.

As a first check of the approach, we have realized the formula in (22) by semi-analytical methods. This is useful for a number of reasons. For example, it allows us to check the normalization of our work with that of other resumed calculations in the literature. An important outcome is an estimate of the size of the $\mathcal{O}(\alpha_s^n), n \geq 2$ contribution to $\sigma(t\bar{t})$ at FNAL [5].

Specifically, as we show in Ref. [30], our semi-analytical result for the ratio of the YFS exponentiated cross section to the corresponding Born cross section is given by

$$\begin{aligned} r^{nls}_{\exp} &= \exp\left\{\frac{\alpha_s}{\pi}\left[\left(2C_F - \frac{1}{2}C_A\right)\frac{\pi^2}{3} - \frac{1}{2}C_F\right]\right\} \\ &= \begin{cases} 1.086, & \alpha_s = \alpha_s(\sqrt{s})\,, \\ 1.103, & \alpha_s = \alpha_s(2m_t)\,, \\ 1.110, & \alpha_s = \alpha_s(m_t)\,. \end{cases} \end{aligned} \tag{24}$$

This implies that the prediction for the contribution to $\sigma(p\bar{p} \to t\bar{t})$ from $(\mathcal{O}(\alpha_s^n),\, n \geq 2)$ is

$$\begin{aligned} &\delta\sigma(p\bar{p} \to t\bar{t})^{\exp} = \int \sum_{i,j} D_i(x_i)\bar{D}_j(x_j) \\ &\times \left(r^{nls}_{\exp} - 1 - \frac{\alpha_s}{\pi}\left[\left(2C_{ij} - \frac{1}{2}C_A\right)\frac{\pi^2}{3} - \frac{1}{2}C_{ij}\right]\right) d\hat{\sigma}_B(x_i x_j s)dx_i dx_j\,, \end{aligned} \tag{25}$$

where

$$C_{ij} = \begin{cases} C_F, & ij = q\bar{q},\ \bar{q}q\,, \\ C_A, & ij = GG\,. \end{cases} \tag{26}$$

From this we get that $(\mathcal{O}(\alpha_s^n), n \geq 2)$ contributes 0.006–0.008 of the NLO cross section, in agreement with Catani *et al.* in Ref. [9].

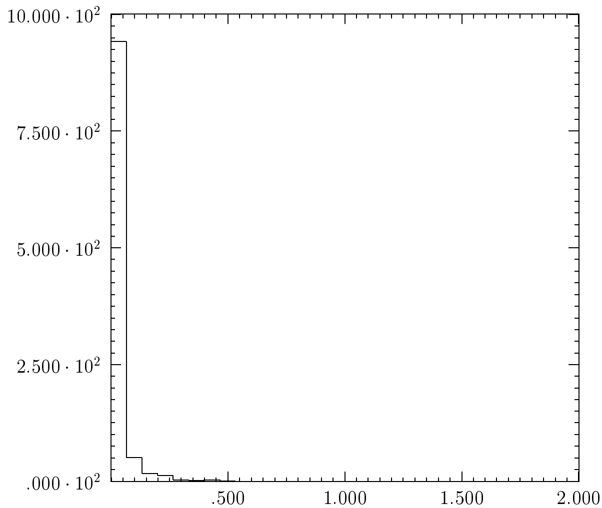

Fig. 2. $n(G)$ transverse momentum distribution for in $p\bar{p} \to t\bar{t} + X + n(G)$. The result is a preliminary.

We have also generated preliminary MC data using the proto-typical version of ttp1.0. A detail presentation of such data will appear elsewhere [15]. Here, in view of its importance, we show preliminary data for the $p_{\rm T}$ spectrum from ttp1.0. This is shown in Fig. 2. What we see is that, indeed, the soft $n(G)$ effects do modulate significantly the $p_{\rm T}$ spectrum on the scale of a few GeV — they must be taken into account properly to understand observables derived from them on this level.

5. Conclusions

We conclude that YFS theory, both the EEX and CEEX formulations, extends to QCD. In our work, a full MC event generator realization of this extension is near. Our early semi-analytical results agree with the literature on $t\bar{t}$ production at FNAL energies. Preliminary MC data show that the $n(G)$ $p_{\rm T}$ is significant in the latter process. We are currently pursuing the attendant ramifications for LHC and RHIC as well.

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