

QUARK SCATTERING AT QGP/2SC INTERFACE*

M. SADZIKOWSKI[†]Center for Theoretical Physics, Massachusetts Institute of Technology
Cambridge, MA 02139, USA

and

Henryk Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland*(Received February 28, 2002)*

We considered the phenomenon of the Andreev reflection of quarks at the interface between the cold quark–gluon plasma phase and 2SC color superconductor.

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1. Introduction

The very interesting phenomenon arises at the junction between a normal metal and a superconductor. The reflection of a conduction electron from the superconducting barrier, first elaborated in the paper [1], is known in the literature as the Andreev reflection. It was shown that the excitation in the normal metal outgoing from the reflection point has the sign of the momentum, the sign of the charge, and the sign of the effective mass opposite to that of the incoming electron (hole quantum numbers). This peculiar prediction was checked by tunneling experiments and ballistic (direct) observations [2].

In this letter we raise the interesting question of the interaction between the phase containing the free quarks (QGP) and the Color Superconductor (CS). We shall show that the free quarks from the QGP phase falling at the CS barrier are reflected in the similar fashion as was described by Andreev in [1].

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[†] Foundation for Polish Science Fellow.

In the next section we develop the Bogolubov–de Gennes equations for superconducting phase. The third section contains the detailed discussion of the Andreev reflection. In the last chapter we make some comments about the influence of the Andreev reflection on the phenomena that can take place inside the Neutron Stars.

2. Bogolubov–de Gennes equations for 2SC phase

Our interest is the description of the phenomena that happens at the QGP/CS junction at moderate densities expected in the Neutron Stars. In this work we do not address the question of the gluon interaction with the CS phase thus we just concentrate on the fermionic degrees of freedom. The model of QGP is the gas of the free quarks. The quarks in QGP built the Fermi ladder, with the Fermi energy μ (of the order of 400–500 MeV in the Neutron Stars), where μ is a quark chemical potential. In the case of the two light quarks, which we consider here, the CS phase creates the 2SC superconductor (for the review of the subject see [3]). The essential physics of 2SC phase at moderate densities, is described by the effective four-fermion point interaction with the attractive pseudoscalar channel [4]. This model leads to the Cooper instability of the Fermi sea which creates the new vacuum of condensed Cooper pairs. The physical excitations are fermionic quasi-particles separated from the vacuum by the energy gap $|\Delta|$. The energy gap was calculated in the mean-field approximation in the variety of models giving the values 40–140 MeV which is about 10–30% of the Fermi energy.

The effective Hamiltonian at the mean-field level takes the form:

$$H = \int d^3x \left\{ \psi_\alpha^{j\dagger} \left(-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu \right) \psi_\alpha^j + \frac{\Delta}{2} \psi_\alpha^{j\dagger} (\tau_2)_{jk} t_{\alpha\beta} C \gamma_5 \psi_\beta^{k\dagger} - \frac{\Delta^*}{2} \psi_\alpha^j (\tau_2)_{jk} t_{\alpha\beta} C \gamma_5 \psi_\beta^k + \mathcal{H}(\Delta, m) \right\}, \quad (1)$$

where m is a quark mass and ψ_β^j is the field operator of the quark. The Greek indices describe the color and the Latin indices describe the isospin quantum numbers. The matrix τ_2 is the antisymmetric Pauli matrix and t is one of the three antisymmetric Gell-Mann matrices. For our calculations we chose it arbitrarily as the λ_2 . The gap parameter $\Delta(\vec{r})$ depends on the position. It vanishes inside the QGP phase and it takes a nonzero value in the 2SC phase. The equation of motion describing our system takes the form:

$$\begin{aligned} i\dot{\psi}_\beta^j &= \left(-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu \right) \psi_\beta^j + \Delta(\vec{r}) (\tau_2)_k^j t_\beta^\sigma C \gamma_5 \psi_\sigma^{k\dagger T}, \\ i\dot{\psi}_\alpha^{j\dagger} &= -\vec{\nabla} \psi_\alpha^{j\dagger} \cdot i\vec{\alpha} - \psi_\alpha^{j\dagger} (m\gamma_0 - \mu) + \Delta(\vec{r})^* \psi_\beta^{k\dagger} (\tau_2)_k^j t_\alpha^\beta C \gamma_5. \end{aligned} \quad (2)$$

The equations (2) describe the interaction of the 6 “families” of quarks coupled through the non-zero value of the gap parameter. Using standard conventions (*e.g.* [5]), one can find that the up “red” quark couples to the down “green” quark, the up “green” to the down “red”, whereas the “blue” quarks remain free. For the first “family” the equations (2) read:

$$\begin{aligned} i\dot{\psi}_R^u &= \left(-i\vec{\alpha} \cdot \vec{\nabla} + m\gamma_0 - \mu\right) \psi_R^u - \Delta(\vec{r})C\gamma_5\psi_G^{d\dagger T}, \\ i\dot{\psi}_G^{d\dagger} &= -\vec{\nabla}\psi_G^{d\dagger} \cdot \vec{\alpha} - \psi_G^{d\dagger}(m\gamma_0 - \mu) - \Delta(\vec{r})^*\psi_R^{uT}C\gamma_5, \end{aligned} \quad (3)$$

where u and d are isospin and R and G color degrees of freedom. The equations for the second “family” (up “green” and down “red” quarks) can be evaluated exactly the same way and the “blue” quarks are not interesting in the first approximation. One can change the operator equations (3) to c -number equations by taking the expectation values $f \equiv \langle\Psi_0|\psi_R^u|\Psi_1\rangle$ and $g^\dagger \equiv \langle\Psi_0|\psi_G^{d\dagger}|\Psi_1\rangle$, where $|\Psi_0\rangle, |\Psi_1\rangle$ are ground and excited state, respectively. In the QGP phase ($|\Delta| = 0$) the fields f and $g^\dagger$ describe a “particle” and a “hole”. Let us now find the quasiparticle wavefunctions in the case of $\Delta = \text{const.}$ It is convenient to decompose the Fermi fields in the plane wave basis. Remembering that the relativistic effects are suppressed in the presence of the Fermi sea one can neglect the contribution from the antiquarks:

$$\begin{aligned} f(t, \vec{r}) &= \sum_s u_s(\vec{q})\alpha_s(\vec{q})\exp(-iEt + i\vec{q} \cdot \vec{r}), \\ g^\dagger(t, \vec{r}) &= \sum_s d_s^\dagger(-\vec{q})\beta_s^*(\vec{q})\exp(-iEt + i\vec{q} \cdot \vec{r}), \end{aligned} \quad (4)$$

where s describes the spin degrees of freedom, $\alpha_s(\vec{q})$ and $\beta_s(\vec{q})$ are some c -numbers. The energy E describes the energy of the quark with respect to the Fermi surface in the absence of the gap parameter Δ . The Dirac bispinors u_s and d_s satisfy the equations:

$$\begin{aligned} (\vec{\alpha} \cdot \vec{q} + m\gamma_0 - \mu)u_s(\vec{q}) &= \varepsilon_q u_s(\vec{q}), \\ d_s^\dagger(-\vec{q})(\vec{\alpha} \cdot \vec{q} - m\gamma_0 + \mu) &= d_s^\dagger(-\vec{q})\bar{\varepsilon}_q, \end{aligned} \quad (5)$$

and because momentum $\vec{q}$ can be in general the complex number one has to carefully distinguish between the left and the right eigenvectors. Inserting (4) into the equations (3), using bispinors algebraic relations and assuming constant value of the gap parameter one arrives at the Bogolubov–de Gennes equations for α and β parameters:

$$\begin{aligned} E\alpha_\uparrow(\vec{q}) &= \varepsilon_q\alpha_\uparrow(\vec{q}) + \Delta\beta_\downarrow^*(\vec{q}), \\ E\beta_\downarrow^*(\vec{q}) &= -\varepsilon_q\beta_\downarrow^*(\vec{q}) + \Delta^*\alpha_\uparrow(\vec{q}), \end{aligned} \quad (6)$$

and the similar set of equations with reverse spin projection on the momentum $\vec{q}$:

$$\begin{aligned} E\alpha_{\downarrow}(\vec{q}) &= \varepsilon_q\alpha_{\downarrow}(\vec{q}) - \Delta\beta_{\uparrow}^*(\vec{q}), \\ E\beta_{\uparrow}^*(\vec{q}) &= -\varepsilon_q\beta_{\uparrow}^*(\vec{q}) - \Delta^*\alpha_{\downarrow}(\vec{q}), \end{aligned} \quad (7)$$

where $\varepsilon_q = \sqrt{\vec{q}^2 + m^2} - \mu = -\bar{\varepsilon}_q$. We have two possible solutions of the uniform equations (6) (the case of (7) is similar): for $\varepsilon_q = \xi$

$$\begin{aligned} \alpha_{\uparrow} &= \sqrt{\frac{1}{2} \left(1 + \frac{\xi}{E}\right)} \exp\left(i\frac{\delta}{2}\right), \\ \beta_{\downarrow}^* &= v\sqrt{\frac{1}{2} \left(1 - \frac{\xi}{E}\right)} \exp\left(-i\frac{\delta}{2}\right), \end{aligned} \quad (8)$$

and for $\varepsilon_q = -\xi$

$$\begin{aligned} \alpha_{\uparrow} &= \sqrt{\frac{1}{2} \left(1 - \frac{\xi}{E}\right)} \exp\left(i\frac{\delta}{2}\right), \\ \beta_{\downarrow}^* &= \sqrt{\frac{1}{2} \left(1 + \frac{\xi}{E}\right)} \exp\left(-i\frac{\delta}{2}\right), \end{aligned} \quad (9)$$

where $\xi = \sqrt{E^2 - |\Delta|^2}$ and δ is a phase of the gap. Let us point out that only for $E > |\Delta|$ the solution describes the propagating excitations. Finally, the wavefunction describing the quasiparticle excitation of energy E in the 2SC phase takes the form:

$$\begin{aligned} \psi(t, \vec{r}) &\equiv \begin{pmatrix} f \\ g^{\dagger} \end{pmatrix} \\ &= \begin{pmatrix} \exp\left(i\frac{\delta}{2}\right) \sqrt{\frac{E+\xi}{2E}} \left(A u_{\uparrow}(\vec{q}_1) - B u_{\downarrow}(\vec{q}_1)\right) \\ \exp\left(-i\frac{\delta}{2}\right) \sqrt{\frac{E-\xi}{2E}} \left(A d_{\downarrow}^{\dagger T}(-\vec{q}_1) + B d_{\uparrow}^{\dagger T}(-\vec{q}_1)\right) \end{pmatrix} \exp(-iEt + i\vec{q}_1 \cdot \vec{r}) \\ &+ \begin{pmatrix} \exp\left(i\frac{\delta}{2}\right) \sqrt{\frac{E-\xi}{2E}} \left(C u_{\uparrow}(\vec{q}_2) + D u_{\downarrow}(\vec{q}_2)\right) \\ \exp\left(-i\frac{\delta}{2}\right) \sqrt{\frac{E+\xi}{2E}} \left(C d_{\downarrow}^{\dagger T}(-\vec{q}_2) - D d_{\uparrow}^{\dagger T}(-\vec{q}_2)\right) \end{pmatrix} \exp(-iEt + i\vec{q}_2 \cdot \vec{r}), \end{aligned} \quad (10)$$

where $q_{1,2} = \pm\sqrt{(\mu \pm \xi)^2 - m^2}$ and A, B, C and D are arbitrary constants.

3. Andreev reflection

Let us consider the simple physical problem of the quark scattering on the superconducting plane surface placed at $z = 0$ in space. If we approximate the boundary as the step function ($\Delta = 0$ for $z < 0$ and $\Delta = \text{const.}$ for $z > 0$) then we are looking for stationary solution of equations (3), independently for negative and positive z , supplemented with the condition that matches the wavefunction at $z = 0$.

Let us consider in detail the situation when the “red” quark, with energy E and spin projection $\uparrow$ falls at the plane boundary from the left. Then the wavefunctions (for $\Delta = 0$) take the form

$$\psi_{<}(t, z) = \begin{pmatrix} u_{\uparrow}(k) \exp(ikz) + F u_{\uparrow}(-k) \exp(-ikz) \\ H d_{\downarrow}^{\dagger T}(-p) \exp(ipz) \end{pmatrix} \exp(-iEt). \quad (11)$$

The coefficients F and H describe the amplitude of the reflection of the u -particle and d -hole, respectively. The first equation from (6) determines the value of the momentum of the u quark as $k^2 = (\mu + E)^2 - m^2$. In the similar fashion the second equation from (6) gives the value of the momentum of the reflected d hole as $p^2 = (\mu - E)^2 - m^2$. For $z > 0$ the quasi-particles excitation are described by the wavefunction $\psi_{>}(t, z)$ given by the expression (10) with $B = D = 0$ (no spin flip in the scattering process). Let us note that for $0 < E < |\Delta|$, the momenta of the quasi-particles has to be complex number

$$q_{1,2} = \pm \sqrt{\mu^2 - m^2 - |\xi|^2 \pm 2i|\xi|\mu}. \quad (12)$$

Particularly simple form of momenta one can find in the massless limit. In that case $q_{1,2} = \pm \mu + i|\xi|$, where the signs are chosen as to describe the vanishing of the wavefunction in 2SC phase for $z \rightarrow \infty$. Now it is seen that for $E < |\Delta|$ in the 2SC phase the quasiparticle wavefunction is exponentially decaying with the suppression factor $|\xi| = \sqrt{|\Delta|^2 - E^2}$.

The continuity conditions matching the wavefunctions are of the form

$$\psi_{<}(t, z = 0) = \psi_{>}(t, z = 0). \quad (13)$$

Using this condition one can find the amplitudes of the scattering process:

$$\begin{aligned}
A &= \sqrt{\frac{2E}{E+\xi}} \exp\left(-i\frac{\delta}{2}\right) + O\left(\frac{1}{\mu}\right), \\
C &= \sqrt{\frac{E-\xi}{2E}} \frac{mE}{\mu^2} \exp\left(-i\frac{\delta}{2}\right) + O\left(\frac{1}{\mu^3}\right), \\
F &= \frac{m(E-\xi)}{\mu^2} + O\left(\frac{1}{\mu^3}\right), \\
H &= \sqrt{\frac{E-\xi}{E+\xi}} \exp(i\delta) + O\left(\frac{1}{\mu}\right).
\end{aligned} \tag{14}$$

In the limit where $|\Delta|, E, m \ll \mu$ only A and H contribute which means that only hole of the d -green quark can be reflected into the QGP phase and the only quasiparticle with momentum q_1 can propagate in the 2SC phase. It is interesting to point out that in the massless limit $F = C = 0$ exactly. Additional insight we gain from the calculation of the probability current. From the equations (3) it follows that the probability current $\vec{j} = \psi_R^{u\dagger} \vec{\alpha} \psi_R^u + \psi_G^{d\dagger} \vec{\alpha} \psi_G^d$ is conserved. Using (10), (14) one finds:

$$j_z = 2\mu \begin{cases} 0 & \text{for } E < |\Delta|, \\ \frac{2\xi}{E+\xi} + O\left(\frac{1}{\mu}\right) & \text{for } E > |\Delta|. \end{cases} \tag{15}$$

The result (15) has simple interpretation. If the quark u has energy below the gap it cannot excite quasiparticle inside the 2SC phase. In that case the matter waves does not propagate through the superconducting medium. However, if the u quark possesses energy above the gap it excites the quasiparticles with a given transition coefficient. This result may be of importance for the physics of the Neutron Stars.

4. Conclusions

In this paper we describe the phenomena of Andreev reflection at the junction between the cold quark-gluon plasma and superconducting 2SC phase. We found that the hole of different flavor and color than the incoming particle is reflected toward the QGP phase. Inside the 2SC phase the quasi-particles are excited, which for energy E of the incoming particle below the gap $|\Delta|$, penetrate the 2SC phase for depth of the order of $1/\sqrt{|\Delta|^2 - E^2}$ whereas for the case of $E > |\Delta|$ they propagate freely inside the superconducting phase. This result is similar to the situation one

encounters in the condensed matter systems. This can be expected because in the high density QCD the purely relativistic phenomena are suppressed by the powers of quark chemical potential.

From the equation (15) one can see that the transport phenomena (like heat transport or density waves) is strongly affected by the presence of the interfaces inside the Neutron Stars. The suppression is exponential of the form $\exp(-|\Delta|/T)$ where T is a temperature of the Neutron Star, usually much smaller than the expected superconducting gap. This phenomena certainly require more extensive analysis¹.

The structure of the Neutron Stars can be complicated, thus more work is needed for better understanding of the Andreev reflection phenomena in superconducting QCD. In particular one has to consider other possible superconducting phases like CFL or other possible interfaces. Existence of such interfaces certainly influences the dynamics of matter flow inside the Neutron Stars.

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¹ Let us note that probability current through the interface is not suppressed completely, because in the 2SC phase we have unpaired blue quarks.