PARTICLE PRODUCTION AND EQUILIBRATION IN HEAVY ION COLLISIONS*

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The application of statistical model in heavy ion collisions is discussed in the energy range from SIS/GSI through AGS/BNL up to SPS/CERN and RHIC/BNL. It is shown that in this broad energy range hadronic yields and their ratios resemble a thermal equilibrium population along a unified freeze-out curve determined by the condition of fixed energy per particle $\simeq 1$ GeV. The role of exact conservation of quantum numbers in the kinetic description of rarely produced particles is also analyzed.

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1. Introduction

One of the main objective of the experiments with ultra-relativistic heavy ion collisions is to study the properties of strongly interacting matter under the extreme condition of high energy density. Of particular interest in this context is the question of equilibration of QCD medium created during the collision [1–4]. This question has been considered in different context; by analyzing conditions required for a perturbative QCD medium to reach equilibrium [5–8] or by studying the level of particle equilibration in the final state [3, 9–20]. From the theoretical point of view to discuss equilibration one needs to formulate the kinetic equation for particle production and evolution. In the partonic medium this requires, in general, the formulation of

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the transport equation involving color degrees of freedom and a non-abelian structure of QCD dynamics [6–8]. In the hadronic medium, on the other hand, one needs to account for the conservation of abelian charges related with internal symmetries of the system [21–24].

In this article we present the phenomenological motivation for the statistical thermal nature of particle yields measured in heavy ion collisions and discuss the importance of the conservation laws in this analysis.

1.1. Critical conditions in heavy ion collisions

In ultra-relativistic heavy ion collisions, the knowledge of the critical energy density $\varepsilon_{\rm c}$ required for deconfinement as well as the Equation of State (EoS) of strongly interacting matter is of particular importance. The value of $\varepsilon_{\rm c}$ characterizes the necessary initial conditions in heavy ion collisions to create the QGP, whereas EoS is required as an input to describe the space–time evolution of the collision fireball.

Both information can be obtained today from the first principal calculations by formulating QCD on the lattice and performing numerical Monte Carlo simulations. The energy density obtained in Lattice Gauge Theory (LGT) exhibits a typical behavior in a system with a phase transition [25]: an abrupt change within a very narrow temperature range. In the region below $T_{\rm c}$ the basic constituents of QCD, quarks and gluons, are confined within their hadrons and here the EoS is well parameterized by the hadron resonance gas. Above $T_{\rm c}$ the system appears in the QGP phase where quarks and gluons can penetrate distances that substantially exceed the typical size of hadrons. The results of improved perturbative expansion of thermodynamical potential in the continuum QCD [26] are showing that at some distance above $T_{\rm c}$ the EoS of QGP can be well described by a gas of massive quasi-particles whose mass is temperature dependent. In the near vicinity of $T_{\rm c}$ the relevant degrees of freedom were argued to be described by the Polyakov loops [27].

Lattice Gauge Theory predicts that in two and two+one flavor QCD the critical temperature $T_{\rm c} \sim 175 \pm\ 15$ MeV and the corresponding critical energy density $\varepsilon_{\rm c} = 0.6 \pm 0.3$ GeV/fm^3 are required for deconfinement. This value of $\varepsilon_{\rm c}$ is relatively low and quantitatively corresponds to the energy density inside the nucleon.

The initial energy density reached in heavy ion collisions can be estimated within the Bjorken model [28] or by applying different saturation models [29–32]. A detailed study shows that already at SPS the energy density inside the collision fireball exceeds the critical value required for deconfinement. At RHIC and LHC it is larger by more than one order of magnitude. Thus, the necessary conditions to create the partonic medium in a deconfined phase are satisfied from the top SPS up to LHC energies. Large energy density is, however, still not sufficient to create a QGP. The distribution of initially produced gluons is very far from being thermal and the system needs time to equilibrate. Recently, it was shown [5] in the framework of perturbative QCD and kinetic theory that the equilibration of partons should happens quite fast at LHC and most likely at RHIC.

The thermal nature of the partonic medium could be preserved during hadronization. Consequently, the particle yields measured in the final state of heavy ion collisions could resemble the thermal equilibrium population.

2. Thermal analysis of secondaries in heavy ion collisions

The basic quantity required to verify thermal composition of particles measured in heavy ion collisions is the partition function $Z(T,V)$. In the Grand Canonical (GC) ensemble,

$$Z^{\mathrm{GC}}(T,V,\mu_Q) \equiv \mathrm{Tr}\left[\mathrm{e}^{-\beta(H-\sum_i \mu_{Q_i} Q_i)}\right], \tag{1}$$

where H is the Hamiltonian of the system, Q_i are the conserved charges and μ_{Q_i} are the chemical potentials that guarantee that the charges Q_i are conserved on the average in the whole system. Finally, $\beta = 1/T$ is the inverse temperature.

The Hamiltonian is usually described by the hadron resonance gas, which contains the contributions from all mesons with masses below ~1.8 GeV and baryons with masses below ~2 GeV. In this mass range the hadronic spectrum is well established and the decay properties of resonances are known. This mass cut in the contribution to partition function limits, however, the maximal temperature to $T_{\max} < 190$ MeV, up to which the model predictions could be trustworthy. For higher temperatures the contributions of heavier resonances are not negligible.

The main motivation of using the Hamiltonian of hadron resonance gas in the partition function is that it contains all relevant degrees of freedom of confined, strongly interacting medium and implicitly includes interactions that results in resonance formation. Secondly, this model is consistent with the equation of state obtained from LGT below the critical temperature.

Of particular importance is to account for resonances and their decay into lighter particles. The overall production of light hadrons is substantially dominated by resonances. Finally, at lower energies the momentum-dependent width of resonances have to be included, as there, an appreciate amount of particles are produced from the tail of the resonance distribution [13, 33].

In strongly interacting medium, one includes the conservation of electric charge, baryon number and strangeness. Thus, the partition function depends in general on five parameters. However, only three are independent, since the isospin asymmetry in the initial state fixes the charge chemical potential and strangeness neutrality conditions eliminate the strange chemical potential. On the level of particle multiplicity *ratios* derived from the above partition function, we are thus left with only temperature T and baryon chemical potential μ_B as independent parameters.

The statistical model, described above, was applied to test equilibration of secondaries in A–B collisions at the SPS. In the following we only concentrate on the most recent comparison of the model with Pb–Pb data at $\sqrt{17}$ GeV SPS energy [10, 13–15]. The model was compared with almost all experimental data obtained by NA44, NA49 and WA97 Collaboration (see Fig. 1, upper panel) and with selected fully integrated NA49 and midrapidity NA49 and WA97 yields. Hadron multiplicities ranging from pion to omega and their ratios were used to verify if there is a set of thermal parameters (T, μ_B) that simultaneously reproduces all measured yields. A detailed analysis (see Fig. 1, upper panel) has shown [13] that choosing the temperature $T = 168 \pm 4$ MeV and a baryon chemical potential $\mu_B = 266 \pm 8$ MeV, the statistical model with only two parameters can indeed describe seventeen different particle multiplicity ratios within an accuracy of one to two standard deviations. In comparison to the data obtained at midrapidity, particle ratios over 4π integrated lead to slightly lower thermal parameter, however, being consistent within statistical and systematic error with the above quoted values. At midrapidity the value of $T = 164 \pm 4$ MeV and $\mu_B = 231 \pm 8$ MeV were obtained [14] whereas 4π data [15] imply $T = 160 \pm 4$ MeV and $\mu_B = 238 \pm 8$ MeV. A satisfactory agreement of the model with fully integrated data could, however, be achieved only when introducing the new parameter γ_s that accounts for strangeness undersaturation. This parameter suppresses a thermal phase–space of particles composed of n_s strange or (anti)strange quarks by a factor $(\gamma_s)^{n_s}$.

The equilibrium statistical model was recently also applied to Au–Au collisions at $\sqrt{s} = 130$ GeV at RHIC [11]. The results of the STAR, PHENIX, PHOBOS and BRAHMS Collaborations for different particle multiplicity ratios have been used to test chemical equilibration at RHIC. Fig. 1 (lower panel), shows the comparison of the thermal model with experimental data. The data are reproduced by the model within the experimental errors. In Au–Au collisions at $\sqrt{s} = 130$ GeV the chemical freeze-out appears at $T = 174 \pm 7$ MeV and $\mu_B = 46 \pm 6$ MeV. This temperature is only slightly higher than that previously found at SPS ($T \sim 160$–170 MeV). This relatively moderate increase of temperature could be expected since, in the limit of vanishing baryon density, the temperature should not exceed the

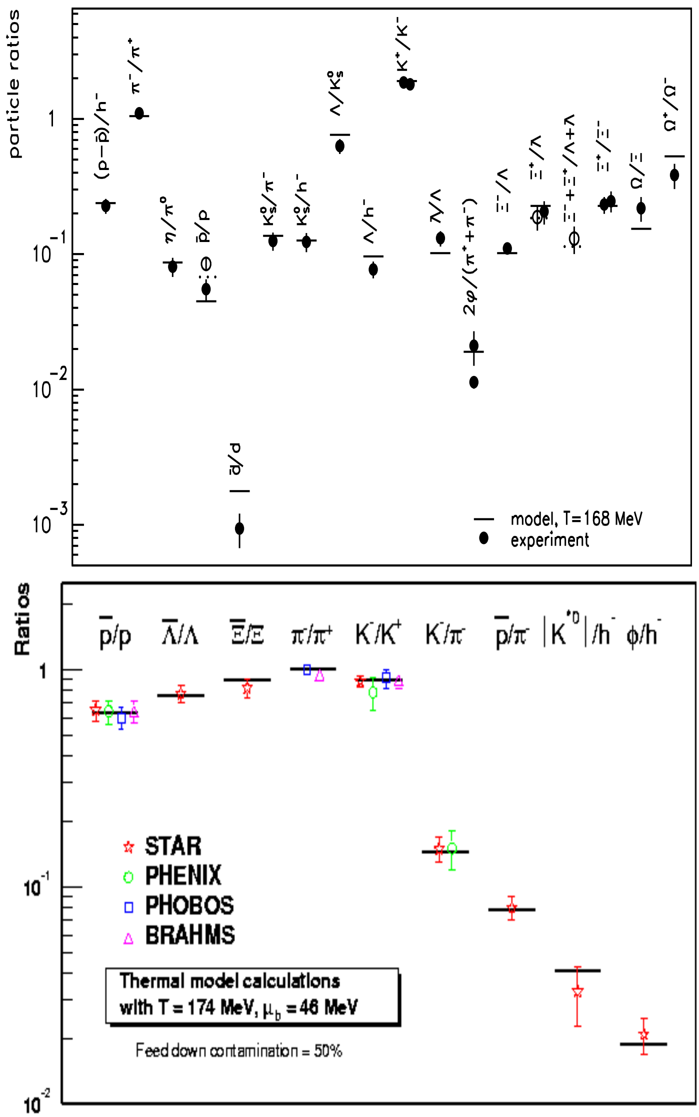

Fig. 1. Comparison of the experimental data on different particle multiplicity ratios obtained at SPS and RHIC with thermal model predictions [11, 13].

critical value required for deconfinement. The substantial decrease of the baryon chemical potential from $\mu_B \simeq 230$–$270\,\mathrm{MeV}$ at SPS to $\mu_B \simeq 50\,\mathrm{MeV}$ at RHIC shows that we are dealing with a low net-baryon density medium.

From Fig. 1 one can thus conclude that, with respect to the statistical operator formulated for equilibrium hadron resonance gas, the experimental data at SPS and RHIC are showing a high level of chemical equilibration.

The question arises whether this statistical operator provides a unique description of the data. Alternatively, the possible influence of in-medium effects on the chemical equilibrium description of particle yields at the SPS was studied [34]. Also a non-equilibrium scenario of explosive hadronization of a QGP fireball was proposed [35] giving satisfactory agreement with SPS data, however, with larger deviations for multistrange particles. We concentrate on the equilibrium description (admitting only possible undersaturation of strangeness) since this approach, as will be demonstrated, provides the systematic agreement with almost all heavy ion data from SIS to RHIC.

The chemical freeze-out temperature, found from a thermal analysis [11, 13, 15] of experimental data in Pb–Pb collisions at the SPS and Au–Au at RHIC is remarkably close to the critical temperature $T_{\rm c} \simeq 173 \pm 8\,{\rm MeV}$ obtained from lattice Monte Carlo simulations of QCD at vanishing baryon density [25]. Thus, the observed hadrons seem to be originating from a deconfined medium and the chemical composition of the system was most likely to be established during hadronization [3, 4].

Chemical equilibration of secondaries after hadronization is rather excluded by the time scale of the kinetics [36–38]. Thus, the equilibrium population of hadrons could appear since it was pre-established in the QGP phase. The equilibration of secondaries is, however, not a unique characteristic of only high energy heavy ion collisions. It was also found to appear at much lower energies. To test equilibration in low energy nucleus–nucleus collisions, one needs, however, to change the statistical operator from a grand canonical to a Canonical (C) ensemble with respect to strangeness conservation.

The conservation of quantum numbers related with abelian internal symmetry in statistical models can be described in the GC ensemble only if the number of produced particles per event carrying corresponding quantum number is much larger than 1. In the limit of rare particle production [21, 22, 39, 40], the corresponding conservation law must be implemented locally on an event-by-event basis, *i.e.* a canonical ensemble of conservation laws must be used. The C ensemble is relevant in the statistical description of particle production in low-energy heavy ion collisions [33], and in high-energy hadron–hadron or e^+e^- reactions [44] as well as in peripheral heavy ion collisions [22].

The exact conservation of quantum numbers, that is the canonical approach, is known to severely reduce the thermal phase space available for particle production [39, 40]. Consequently, the chemical equilibrium limit of rarely produced particles is changed and it is different from the one obtained in the asymptotic GC limit.

In the following we formulate the kinetic equation for charged particles that are produced in thermal environment of strongly interacting matter.

We show the importance of the conservation laws in the time evolution and chemical equilibration of particle multiplicities and their probability distributions. We argue that the constraints imposed by the charge conservation are of crucial importance, particularly, for rarely produced particle species.

To study equilibration in hadronic medium one introduces a kinetic model that takes into account the production and annihilation of particle–anti-particle pairs $c\bar{c}$ carrying abelian quantum numbers like, *e.g.*, strangeness, baryon number or charm. We consider only the production of particles c as being due to a binary process $ab \to c\bar{c}$. Particles a and b are assumed to be charge neutral and contained in a thermal fireball of temperature T and volume V. In addition the particle momentum distribution is assumed to be described by the Boltzmann statistics.

3. Kinetic rate equation

Consider $P_{N_c}(t)$ as the probability to find N_c particles c, where $0 \leq N_c \leq \infty$. This probability will obviously change in time owing to production $ab \to c\bar{c}$ and absorption $c\bar{c} \to ab$ processes. The probability P_{N_c} tends to increase in time, following the transition from $N_c - 1$ and $N_c + 1$ states to the N_c state. It also tends to decrease since the state N_c makes transitions to $N_c + 1$ and $N_c - 1$. The transition probability per unit time from $N_c + 1 \to N_c$ is given by the product of the probability L/V that the single reaction $c\bar{c} \to ab$ takes place multiplied by the number of possible reactions which is, $(N_c+1)(N_{\bar{c}}+1)$. In the case when the U(1) charge carried by particles c and $\bar{c}$ is exactly and locally conserved, that is if $N_c - N_{\bar{c}} = 0$; then this number is just $(N_c+1)^2$. Similarly the transition probability from $N_c \to N_c+1$ is described by $G\langle N_a\rangle\langle N_b\rangle/V$, where one assumes that particles a and b are not correlated and their multiplicity is governed by the thermal averages. One also assumes that the multiplicity of a and b is not affected by the $ab \to c\bar{c}$ process.

The master equation for the time evolution of the probability $P_{N_c}(\tau)$ can be written in the following form [21]

$$\begin{aligned} \frac{dP_{N_c}}{d\tau} &= \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P_{N_c-1} + \frac{L}{V}(N_c+1)^2 P_{N_c+1} \\ &\quad - \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P_{N_c} - \frac{L}{V}N_c^2 P_{N_c}\,. \end{aligned} \tag{2}$$

The first two terms in Eq. (2) describe the increase of $P_{N_c}(\tau)$ due to the transition from $N_c - 1$ and $N_c + 1$ to the N_c state. The last two terms, on the other hand, represent the decrease of the probability function due to the transition from N_c to $N_c + 1$ and $N_c - 1$ states, respectively.

Multiplying Eq. (2) by N_c and summing over N_c, one obtains the general kinetic equation for the time evolution of the average number $\langle N_c\rangle \equiv \sum_{N_c=0}^{\infty} N_c P_{N_c}(\tau)$ of particles

$$\frac{d\langle N_c\rangle}{d\tau} = \frac{G}{V}\langle N_a\rangle\langle N_b\rangle - \frac{L}{V}\langle N_c^2\rangle\,. \tag{3}$$

The above equation cannot be obviously solved as it connects particle multiplicity $\langle N_c\rangle$ with its second moment $\langle N_c^2\rangle$. The appearance of the $\langle N^2\rangle$ term is a direct consequence of the exact charge neutrality constraints imposed through $N_c - N_{\bar{c}} = 0$ condition. However, when the conservation laws can be valid on the average[1], that is $\langle N_c\rangle - \langle N_{\bar{c}}\rangle = 0$, then the master equation (Eq. (2)) can be greatly simplified. Indeed, in this case the transition probability from N_c to the $(N_c - 1)$ state is no longer proportional to $(L/V)N_c^2$ but rather to $(L/V)N_c\langle N_{\bar{c}}\rangle$, since the exact conservation condition $N_c = N_{\bar{c}}$ is assumed to be no longer valid and the number of $\bar{c}$ particles can only be determined by its average value. Consequently, the master equation for the time evolution of the probability $P_{N_c}(\tau)$ takes the following form

$$\begin{aligned}\frac{dP_{N_c}}{d\tau} &= \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P_{N_c-1} + \frac{L}{V}(N_c+1)\langle N_{\bar{c}}\rangle P_{N_c+1} \\ &\quad - \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P_{N_c} - \frac{L}{V}N_c\langle N_{\bar{c}}\rangle P_{N_c}\,.\end{aligned} \tag{4}$$

Multiplying the above equation by N_c, summing over N_c and using the condition that $\langle N_c\rangle = \langle N_{\bar{c}}\rangle$, one gets

$$\frac{d\langle N_c\rangle}{d\tau} = \frac{G}{V}\langle N_a\rangle\langle N_b\rangle - \frac{L}{V}\langle N_c\rangle\,. \tag{5}$$

Comparing the above equation with Eq. (3), one sees that the absorption terms are now linear instead of quadratic in particle number. Thus, changing the conditions from the exact to the average conservation of the abelian charges results in the modification of the absorbtion term in the kinetic transport equation.

Eqs. (4) and (5) can be solved exactly. Indeed, introducing the generating function $g(x,\tau)$ for probabilities P_{N_c},

$$g(x,\tau) = \sum_{N_c=0}^{\infty} x^{N_c} P_{N_c}(\tau)\,, \tag{6}$$

[1] This is the case where the number of particles N_c, carrying non-vanishing quantum number, is much larger than one.

the iterative Eq. (4) can be converted into a differential equation for the generating function

$$\frac{\partial g(x,\tau)}{\partial \tau} = \frac{L}{V}\sqrt{\varepsilon}(1-x)\left[g' - \sqrt{\varepsilon}g\right] \tag{7}$$

with the general solution [21]

$$g(x,\tau) = g_0(1 - x\mathrm{e}^{-\tilde{\tau}})\,\mathrm{e}^{\sqrt{\varepsilon}(1-x)(\mathrm{e}^{-\tilde{\tau}}-1)}\,, \tag{8}$$

where $g' \equiv \partial g/\partial x$, $\tilde{\tau} = (L\sqrt{\varepsilon}/V)\tau$.

One can readily find the equilibrium solution to the above equation. Taking the limit $\tau = \infty$ in Eq. (8) leads to

$$g_{\mathrm{eq}}(x) = \mathrm{e}^{-\sqrt{\varepsilon}(1-x)} \tag{9}$$

with the corresponding equilibrium multiplicity distribution

$$P_{N_c,\mathrm{eq}} = \frac{(\sqrt{\varepsilon})^{N_c}}{N_c!}\mathrm{e}^{-\sqrt{\varepsilon}}\,. \tag{10}$$

This is a Poisson distribution with equilibrium average particle multiplicity $\sqrt{\varepsilon} \equiv \langle N_k\rangle_{\mathrm{eq}}$ which for the Boltzmann momentum distribution reads

$$\langle N_k\rangle_{\mathrm{eq}} = \frac{d_k}{2\pi^2}VTm_k^2K_2\left(\frac{m_k}{T}\right)\,, \tag{11}$$

where d_k's denote spin–isospin degeneracy factors, m_k the particle mass, and K_2 is the modified Bessel function. This is a well known result for the average number of particles as obtained in the grand canonical ensemble with respect to conservation laws. The chemical potential, which is usually present in the particle density, vanishes in our case, due to the requirement of charge neutrality.

The kinetic master equation (Eq. (2)) is the generalization of the well known asymptotic equilibrium solution (11). This can be seen when solving Eq. (2) for its equilibrium value. Converting Eq. (2) for P_{N_c}'s into a partial differential equation for the generating function $g(x,\tau)$ one gets [21]

$$\frac{\partial g(x,\tau)}{\partial \tau} = \frac{L}{V}(1-x)\left(xg'' + g' - \varepsilon g\right)\,. \tag{12}$$

In equilibrium the $g_{\mathrm{eq}}(x)$ function obeys the equation

$$xg_{\mathrm{eq}}'' + g_{\mathrm{eq}}' - \varepsilon g_{\mathrm{eq}} = 0 \tag{13}$$

with the following solution

$$g_{\rm eq}(x) = \frac{1}{I_0(2\sqrt{\varepsilon})} I_0(2\sqrt{\varepsilon x}) \tag{14}$$

that implies the equilibrium probability function P_{N_c} as

$$P_{N_c,{\rm eq}} = \frac{\varepsilon^{N_c}}{I_0(2\sqrt{\varepsilon})(N_c!)^2}\,. \tag{15}$$

We note that the above particle multiplicity distribution is not Poissonian [21,42]. This is a direct consequence of particle correlations appearing through the exact charge conservation[2]. The equilibrium average number of particles in this general case reads

$$\langle N_c \rangle_{\rm eq} = g'(1) = \sqrt{\varepsilon}\frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})}\,. \tag{16}$$

The above result coincide with the particle multiplicity obtained in the canonical ensemble with respect to exact charge conservation [39]. Thus, the rate equation (2) is valid for any arbitrary value of $\langle N_c \rangle$ *and* obviously reproduces the standard grand canonical limit of large $\langle N_c \rangle$.

When constructing the evolution equation (2) for probabilities, we have assumed that there is no net charge in the system. In the application of the statistical approach to particle production in heavy ion collisions, the above assumption is only justified when the initial state is U(1) charge neutral and when considering particle yields in the full phase space. However, because of experimental limitations, one often deals with data in restricted kinematical windows. Here the overall U(1) charge is no longer zero and the generalization of the above master equation is needed.

The presence of the net charge implies the modification of the absorption terms in Eq. (2). The transition probability per unit time from the N_c to the $N_c - 1$ state was proportional to $(L/V)N_c N_{\bar{c}}$. For the overall net charge S the exact charge conservation required that $N_c - N_{\bar{c}} = S$. The transition probability from N_c to $N_c - 1$ due to pair annihilation is thus $(L/V)N_c(N_c - S)$. Following the same procedure as in Eq. (2) one can formulate the master equation for the probability $P^S_{N_c}(t)$ to find N_c particles c in a thermal medium with a net charge S

$$\begin{aligned}\frac{dP^S_{N_c}}{d\tau} &= \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P^S_{N_c-1} + \frac{L}{V}(N_c+1)(N_c+1-S)P^S_{N_c+1} \\ &\quad - \frac{G}{V}\langle N_a\rangle\langle N_b\rangle P^S_{N_c} - \frac{L}{V}N_c(N_c-S)P^S_{N_c}\,,\end{aligned} \tag{17}$$

[2] A Poisson distribution is obtained from Eq. (14) if $\sqrt{\varepsilon} \gg 1$, that is for large particle multiplicity.

which obviously reduces to Eq. (2) for $S = 0$.

To get the equilibrium solution for the probability and multiplicity, we again convert the above equation to the differential form for the generating function $g^S(x,\tau) = \sum_{N_c=0}^{\infty} x^{N_c} P_{N_c}^S(\tau)$

$$\frac{\partial g^S(x,\tau)}{\partial \tau} = \frac{L}{V}(1-x)\left(x g_S'' + g_S'(1-S) - \varepsilon g_S\right) . \tag{18}$$

In equilibrium, $\partial g^S(x,\tau)\partial\tau = 0$ and the solution for $g_{\rm eq}^S$ can be found to be

$$g_{\rm eq}(x) = \frac{x^{S/2}}{I_S(2\sqrt{\varepsilon})} I_S(2\sqrt{\varepsilon x}) . \tag{19}$$

The master equation for the probability to find $N_{\bar{c}}$ anti-particles $\bar{c}$, its corresponding differential form and the equilibrium solution for the generating function can be obtained by replacing $S \to -S$ in Eqs. (17)–(19).

The result of the equilibrium average number of particles $\langle N_c\rangle_{\rm eq}$ and anti-particles $\langle N_{\bar{c}}\rangle_{\rm eq}$ is obtained from the generating function using the relation $\langle N_c\rangle_{\rm eq} = g'(1)$. The final expressions read

$$\langle N_c\rangle_{\rm eq} = \sqrt{\varepsilon}\,\frac{I_{S-1}(2\sqrt{\varepsilon})}{I_S(2\sqrt{\varepsilon})} , \qquad \langle N_{\bar{c}}\rangle_{\rm eq} = \sqrt{\varepsilon}\,\frac{I_{S+1}(2\sqrt{\varepsilon})}{I_S(2\sqrt{\varepsilon})} . \tag{20}$$

The charge conservation is explicitly seen by taking the difference of these equations which yields the net value of the charge S. The results of Eq. (20) could also be derived from the equilibrium partition function by using the projection method [39, 40].

The above analysis was restricted to the particular case of one kind of particle and anti-particle being produced by the binary process[3]. However, this example is of physics interest as it can be applied to study such problems like equilibration of heavy quarks in the QCD plasma or charm [41], baryon and strangeness production in hot hadronic matter. The last process is of relevance in the application of the statistical model to the description of particle productions in low energy central [21, 24, 39, 43] and in high energy peripheral heavy ion collisions [22] as well as in p–p collisions [44]. Thus the results are quite general and applicable for abundantly, as well as rarely, produced particles. We have seen that the kinetic differences between these two limiting situations correspond to the grand canonical approximation and exact canonical approach. The master kinetic equation derived here can be also used to study the time evolution and equilibration of particle multiplicities as well as their fluctuations [21, 24].

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[3] The results presented here can be extended to more general case of different particle species carrying quantum numbers related with internal symmetry [43].

4. Rarely produced particles and statistical hadronization

The results discussed in the last section indicate that the major difference between the C and the GC treatment of the conservation laws appears through a different volume dependence of particle densities as well as a strong suppression of a thermal particle phase–space in the former [21, 22, 39, 40, 45, 46]. In low-energy heavy ion collisions the number of pro-

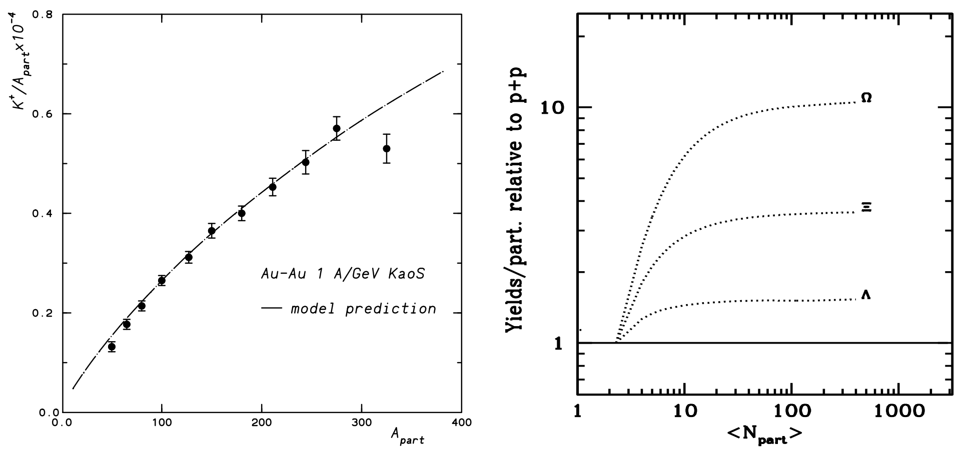

Fig. 2. In the left-hand side figure, the ratio of kaon to pion measured in Au–Au collisions at 1 A GeV [47]; the broken line represents the statistical model results [33]. The right-hand side figure shows statistical model predictions [16] for yield/participants in A–A collisions at $\sqrt{s} \simeq 130$ GeV normalized to the corresponding value in p–p collisions.

duced strange particles depends on the collision energy and *centrality* of these collisions. For example at SIS/GSI, the average number of strange particles produced in an event is of the order of 10^{-3}. Thus, we are in the regime of canonical ensemble. Fig. 2 (left panel) shows the experimental data on K^+ yield per participant $A_{\rm part}$ as a function of $A_{\rm part}$ measured in Au–Au collisions at $E_{\rm lab} \sim 1\,A\,$GeV [47]. The data are compared with the results of the canonical statistical model shown by the dashed line. The thermal parameters, the temperature and the baryon chemical potential were chosen in such a way as to reproduce measured particle multiplicity ratios of strangeness neutral particles [33]. The volume parameter in the statistical operator is assumed to scale with the number of participants. The results in Fig. 2 (left panel) clearly indicate that both the magnitude of the yield and the strong, almost quadratic, dependence of the kaon yield on the number of participants is well reproduced by the canonical model.

The importance of the canonical treatment of strangeness conservation has been shown also at higher collision energies, *e.g.* at the SPS or even RHIC, when considering the centrality dependence of *multistrange baryons* [16,22]. In very peripheral collisions the yield of strange particles is so small that the canonical description should be applied there as well. Fig. 2 (right panel) shows the multiplicity/participant of Ω, Ξ, and Λ relative to its value in p–p or p–A collisions [16] for the RHIC conditions. Fig. 2 indicates that the statistical model in the C ensemble reproduces the basic features of the WA97 data [48,49]: the enhancement pattern and enhancement saturation for large $A_{\rm part}$. The basic predictions of the canonical statistical model is that strangeness enhancement from p–p to A–A collisions should increase with decreasing energy [16]. This result is in contrast with UrQMD finding [50] and with previous heuristic predictions [36] that strangeness enhancement is only found if a quark–gluon plasma is formed during the collision.

5. Unified freeze-out curve in heavy ion collisions

A detailed analysis of the experimental data in heavy ion collisions from SIS to RHIC through AGS and SPS has shown that the canonical statistical model reproduces most of the measured hadron yields.

Fig. 3 is a compilation of chemical freeze-out parameters found to reproduce the measured particle yields in central A–A collisions at SIS, AGS, SPS and RHIC energies. The SIS/GSI results have the lowest freeze-out temperature and the highest baryon chemical potential. As the beam energy increases a clear shift towards higher T and lower μ_B occurs. There is a common feature to all these points, namely that the average energy per hadron is approximately 1 GeV. *Chemical freeze-out* in A–A collisions is thus reached *when the energy per particle drops below* 1 GeV at all collision energies [10]. The above phenomenological freeze-out condition provides the relation between temperature and chemical potential at all collision energies. This relation together with one particle ratio, *e.g.* the ratio of pion/participant establishes the energy dependence of the two thermal parameters T and μ_B. Consequently, predictions of particle excitation functions can be given in terms of this model. Figs. 4, 5 are showing two examples of the statistical model results for different particle multiplicity ratios along the unified freeze-out curve in comparison with experimental data.

The statistical model predicts that the particle/anti-particle ratio are independent from centrality for all collision energies[4]. Dynamically this is a rather surprising result as particles and their anti-particle are generally

[4] This is the case only when if thermal parameters are independent on centrality.

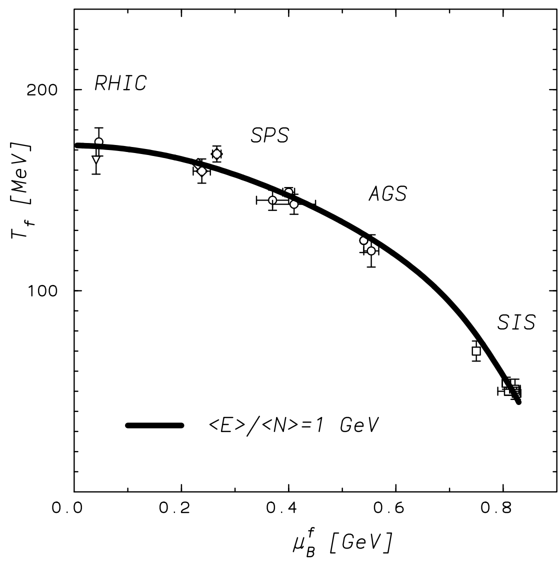

Fig. 3. Compilation of chemical freeze-out parameters at SIS [33], AGS [15], at the SPS at 40 A GeV [16–18] and 160 A GeV [13–15] and RHIC [11, 58, 59]. The solid lines represent the phenomenological condition of chemical freeze-out at fixed energy/particle $\simeq 1.0$ GeV [10].

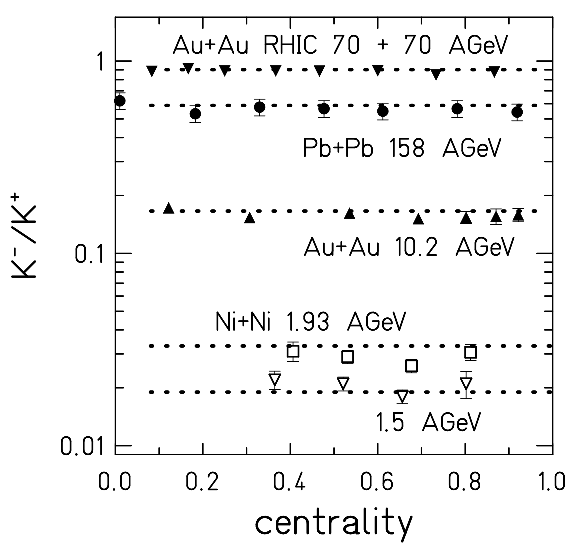

Fig. 4. The K^-/K^+ ratio appears to be constant as a function of centrality from SIS up to RHIC. Data are from the STAR, NA49, E866, and KaoS Collaborations. The broken lines are statistical model results.

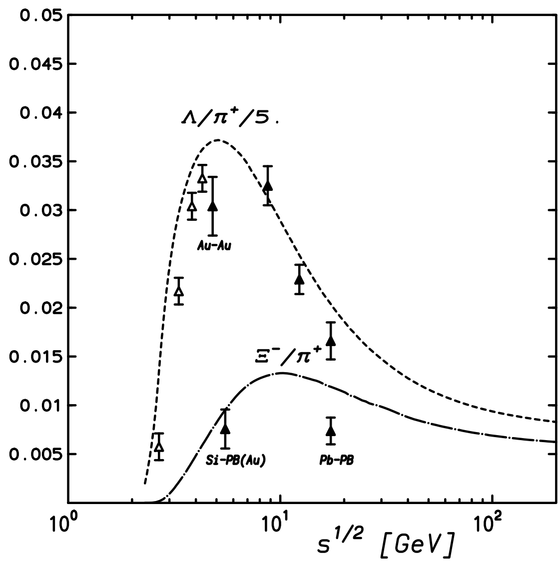

Fig. 5. Particle ratios in A–A *versus* energy. Data at the SPS are fully integrated NA49 results. The corresponding ratio at the top AGS was obtained from E810 results on Ξ^- measured in Si–Pb collisions in the rapidity interval $1.4 < y < 2.9$ [60], normalized to the full phase–space values of π^+ and K^- yield obtained in Si–Au collisions by E802 [61]. The lines represent statistical model results [45] along the unified freeze-out curve from Fig. 3.

produced and absorbed in surrounding nuclear medium in different ways. Fig. 4 represents the energy and centrality dependence of the K^-/K^+ ratio from SIS to RHIC. The statistical model predictions are seen in Fig. 4 to agree remarkably well with the data. The most striking result is the constancy at SIS energies. Here, both K^+ and K^- yields vary strongly with centrality as shown in Fig. 2 and explained by the volume dependence in the canonical description. It turns out that the volume dependence just drops out when making the ratio K^-/K^+.

The measured K^+/π^+ ratio [54] is a very rapidly rising function of collision energy between SIS up to top AGS energy. At higher energies it reaches a broad maximum between 20–40 A GeV and gradually decreases up to RHIC energy [51]. In the microscopic transport models [52, 53] the increase of the kaon yield with collision energy is qualitatively expected as being due to a change in the production mechanism from associated production of K^+ with strange baryons to direct K^+K^- pair production. However, the hadronic cascade transport models do not, until now, provide quantitative explanation of the experimental data in the whole energy range. The statistical model in the C formulation, on the other hand, provides an excellent description of K/π *midrapidity* data in the whole energy range from SIS up to RHIC [33]. The abrupt increase from SIS to AGS reflects the rise of T and the reduction due to the canonical description at SIS. At incident energies above AGS, T is hardly increasing but the baryon-chemical potential starts dropping rapidly.

In general, at lower energies the statistical model result should be rather compared with 4π-integrated yields, since strangeness does not have to be conserved in a limited portion of a phase–space. A drop in the K^+/π^+ ratio for 4π yields has been reported from preliminary results of the NA49 Collaboration at 15 *A* GeV [54]. This decrease is, however, not reproduced by the statistical model without further modifications, *e.g.* by introducing an additional strangeness undersaturation parameter $\gamma_s \sim 0.75$ [15] or by formulating a statistical model *of the early stage* [55].

The appearance of the maximum in the relative strange/non-strange particle multiplicity ratios already seen in K^+/π^+ is even more pronounced for strange baryon/meson ratios. Fig. 5 shows the energy dependence of Λ/π^+ and Ξ^-/π^+. There is a very clear pronounced maximum especially in the Λ/π^+. This maximum is related with a rather strong decrease of chemical potential coupled with an only moderate increase in associated temperature with increasing energy. The relative enhancement of Λ is stronger than that of Ξ^-. There is also a shift of the maximum to higher energies for particles with increasing strangeness quantum number. The enhanced strangeness content of hadrons suppresses the dependence of the corresponding ratio on μ_B. The actual experimental data both for Λ/π^+ and Ξ^-/π^+ ratios shown in Fig. 5 are following the predictions of the statistical model [45]. However, as in the case of kaons, midrapidity results are better reproduced by the model than 4π data.

The appearance of the maximum can be also recognized in the excitation function of the Wroblewski factor [56]. In p–p collisions and in the energy range from SPS up to RHIC a value of $\lambda_s \sim 0.2$ was extracted from the data [18, 44]. The canonical model is able to describe these findings. In high-energy heavy-ion collisions the statistical model predicts $\lambda_s \simeq 0.45$, *i.e.* an increase by a factor of two caused by the ratio between canonical and grand-canonical description. It is interesting to note that recent results of lattice gauge theory [57] (with $\mu_B = 0$) gives $\lambda_s \sim 0.45$ at T_c. This could be an additional quantitative argument that chemical composition of secondaries is to be established on the phase boundary.

6. Concluding remarks

The experimental data on particle yields and their spectra measured in A–A collisions from SIS up to RHIC energy are well described within a statistical approach. Thus at these energies particles seem to be produced according to the principle of maximal entropy, showing the statistical order of the multiplicities. Particle spectra, on the other hand, can be satisfactorily described by introducing in addition to the thermal also a transverse collective motion [62]. A large degree of thermalization and collectivity in experimental data is particularly evident at RHIC and the SPS [2, 63, 64]. Here chemical freeze-out conditions are remarkably consistent with those expected for deconfinement, and particle spectra are well described by transverse collective flow. At RHIC and the SPS the appearance of the QGP in the initial state could be the driving force towards equilibration. At low collision energies the necessary conditions for deconfinement are most likely not satisfied, thus here thermalization can take place through production and rescattering of hadronic constituents.

Statistical order of secondaries is also seen in high energy elementary collisions. Following statistical mechanics the particle thermal phase space is described here within canonical ensemble with respect to conservation laws. Strangeness enhancement from p–p to A–A collisions is qualitatively a direct consequence of a transition from the canonical in p–p to the grand-canonical limit in A–A.

The phenomenological observation of chemical particle phase–space in high energy A–A and p–p collisions could be thus the hadronization feature of a non-perturbative and excited QCD vacuum. Until now there is no rigorous theoretical understanding of this interesting phenomenological observation.

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