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ANISOTROPY OF THE COSMIC MICROWAVE BACKGROUND RADIATION AND TOPOLOGY OF THE UNIVERSE

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We show that the presently observed anisotropy of the cosmic microwave background radiation already restricts the size of the fundamental domain. It turns out that if the Universe is multiply connected then the size of the fundamental domain is comparable or larger than the diameter of the surface of last scattering.

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The generally accepted cosmological models are based on the assumption that on a sufficiently large scale the Universe is homogeneous and isotropic. This assumption strongly restricts the geometry of spacelike sections of the Universe allowing only three possibilities: flat ($k = 0$) Euclidean geometry, hyperbolic ($k = -1$) geometry, and spherical ($k = +1$) geometry. The geometry of the Universe however does not determine its topology. It has been established that a flat 3 dimensional manifold can have 18 different topological structures (Hantzsche & Wendt 1935) while homogeneous and isotropic 3 manifolds of constant positive or negative curvature allow infinitely many different topological structures (Thurston 1997; Weeks 1985; Wolf 1999). It is therefore of prime interest to find out or at least restrict the topology of the Universe.

Non trivial topological structure of the Universe is generated by appropriate identifications and gluing. To get a feeling of richness of the possible topological structures let us consider as an example the simplest case of a square drawn on a flat 2-dimensional plane. By identifying opposite edges of the square one obtains a 2-dimensional torus and by changing orientation of one of the sides in one pair of opposite edges and identifying opposite edges one obtains a Klein bottle. The square in this example plays a role of the fundamental domain (for more examples see Weeks 1985).

Early attempts to restrict the topology of the Universe were based on the simple observation that in a multiply connected space one should be able to observe images of the same object (Lachiéze-Rey & Luminet 1995). Galaxies, clusters of galaxies and quasars do evolve and the problem of proper identification of images has never been solved, besides even quasars are observed only up to $z \approx 7$. It means that from catalogs of galaxies, clusters of galaxies and quasars it was only possible to conclude that the scale of the elementary cell is most probably larger than the distance to the farthest quasar.

More promising possibilities of restricting the topology of the Universe opened up with the COBE detection of the Cosmic Microwave Background radiation (CMB) (Bennett *et al.* 1996). CMB allows to probe much larger volume of the Universe since the surface of last scattering is at $z_{\rm r} \approx 1300$. In 1996 Cornish, Spergel and Starkman proposed an interesting and effective method of determining the topology of the Universe. Their idea is based on the following consideration. If the scale of the fundamental domain L is smaller than the distance to the surface of last scattering $R_{\rm sls}$ then an observer will see specific pattern in the distribution of CMB anisotropies on the celestial sphere. Depending on the ratio $R_{\rm sls}/L$ an observer will see several sections of the last scattering surface intersecting. Spheres do intersect along circles and therefore information about the topological structure of the Universe will be encoded in the positions and radii of the circles with identical pattern of temperature fluctuations. The positions and radii of the circles can be used to determine the shape, size and orientation of the fundamental domain (Cornish, Spergel & Starkman 1998). The COBE results have been used by several groups to put some restrictions on topology of the Universe, see for example Stevens, Scott & Silk 1993, de Oliveira-Costa & Smoot 1995, and Levin, Scannapieco & Silk 1998.

Here we would like to present a simple method of restricting the scale of the fundamental domain using the presently available CMB data. In a multiply connected Universe, depending on the ratio $R_{\rm sls}/L$, an observer will see many sections of the surface of last scattering intersecting on circles, but he will also see sections of the surface of last scattering at slightly different redshifts.

As was already mentioned form the analysis of catalogs of quasars and clusters of galaxies it follows that the scale of the elementary cell cannot be smaller than the distance to the farthest quasar, therefore we have $L_{\min} \approx \frac{2}{3}R_{\rm H}$, where $R_{\rm H}$ is the Hubble radius.

The recently released BOOMERANG and MAXIMA data cover correspondingly $100^\circ \times 30^\circ$ and $30^\circ \times 10^\circ$ sections of the celestial sphere (Netterfield *et al.* 2001; Lee *et al.* 2001; Stompor *et al.* 2001). Let us first consider the case when $L \approx R_{\rm sls}$. The maximal possible angular diameter of an intersection circle in the BOOMERANG data is about 30°. In the simple case of flat geometry, favored by the recent observational data, and cubical fundamental domain, we have

$$L = 2R_{\rm sls} \cos\frac{\theta}{2}\,. \tag{1}$$

Substituting $\theta = 30^\circ$ we see that $L \geq 1.9R_{\rm sls}$. Of course, to get some information about the topology of the Universe, L should be smaller than $2R_{\rm sls}$.

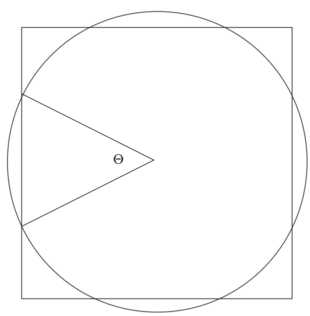

Fig. 1. The surface of last scattering intersecting the fundamental domain represented by a square.

Considering the case when the size of the elementary cell is comparable to the size of the last scattering surface let us assume that the circles are so small that they cannot be resolved by the BOOMERANG and MAXIMA instruments. The angular resolution of the MAXIMA instrument is about $10'$. In this case the size of the elementary cell is constrained by

$$L = 2R_{\rm sls} \cos\frac{\theta}{2} \approx 2R_{\rm sls}\left(1 - \frac{\theta^2}{8}\right)\,. \tag{2}$$

Substituting here $\theta \approx 3 \times 10^{-3}$ we see that

$$L \geq 2R_{\rm sls}(1 - 10^{-6})\,. \tag{3}$$

There is of course the other possibility that the angular size of the circles is small because $L \ll R_{\rm sls}$ but this case is excluded by the analysis of distributions of galaxies, clusters of galaxies and quasars.

Non trivial multiply connected topology of the Universe generates still another effect. As was already mentioned an observer in such a universe will see many self intersections of the surface of last scattering. Different sections will appear at slightly different redshifts creating small temperature anisotropy.

Due to topological imaging a small section of the surface of last scattering will be shifted by $\Delta R = 2R_{\rm sls}(1 - \cos\frac{\theta}{2})$, where θ is its angular scale, what leads to a small displacement in redshift. To estimate the amplitude of temperature anisotropy generated in this way we average this effect over the section. The volume enclosed by the section of surface of last scattering and the edge of the fundamental domain is $V_{\rm s} = \pi R_{\rm sls} h^2$, where $h = R_{\rm sls}(1 - \cos\frac{\theta}{2}) \approx R_{\rm sls}\frac{\theta^2}{8}$. The surface area of this section is $S_{\rm s} = \pi R^2_{\rm sls}\frac{\theta^2}{4}$. The average $\langle h \rangle$, therefore, is $\langle h \rangle = \frac{V_{\rm s}}{S_{\rm s}} = R_{\rm sls}\frac{\theta^2}{16}$. Let us now use redshift to describe distance, we have $1 + z = \frac{R_0}{R(t)}$. In a flat $(k = 0)$ cosmological model filled with matter and the cosmological constant ($\Omega_{\rm m} + \Omega_\Lambda = 1$, where $\Omega_{\rm m}$ and Ω_Λ are correspondingly the density parameters of matter and the cosmological constant) distance is related to redshift by

$$d(z) = \frac{c}{H_0}\int_0^z [(1+z)^2(1+\Omega_{\rm m} z) - z(z+2)\Omega_\Lambda]^{-1/2} dz\,, \tag{4}$$

where H_0 is the present value of the Hubble constant. Therefore, we have

$$\frac{\Delta R}{R_{\rm sls}} = \frac{2h}{R_{\rm sls}} = \frac{\Delta z}{z_{\rm r}^{3/2}\sqrt{\Omega_{\rm m}}}\frac{c}{R_{\rm sls}H_0}\,, \tag{5}$$

where $z_{\rm r}$ is the redshift of the recombination epoch. To estimate temperature anisotropy generated by this topological effect we use in this formula $< h >$ instead of h, we obtain

$$\frac{\theta^2}{8} = \frac{\Delta z}{z_{\rm r}^{3/2}\sqrt{\Omega_{\rm m}}}\frac{c}{R_{\rm sls}H_0}. \tag{6}$$

Assuming that the temperature of CMB at the surface of last scattering is uniform we obtain

$$\frac{\Delta T}{T_0} = \frac{\Delta z}{1+z}\,. \tag{6}$$

Combining (6) and (7) we have

$$\frac{\Delta T}{T_0} = \left(\frac{\theta^2}{8}\sqrt{z_{\rm r}\Omega_{\rm m}}\frac{R_{\rm sls}H_0}{c}\right)\frac{\theta^2}{\theta_A^2}\,, \tag{7}$$

where we have introduced an additional factor $\frac{\theta^2}{\theta_A^2}$ to take into account the fact that the angular size of the antenna beam θ_A is much larger than the angular scale of anisotropy. Using conservative estimate from the COBE data we restrict the maximal variation of $\frac{\Delta T}{T_0} \approx 10^{-4}$, $\theta_A = 10°$, $z_{\rm r} = 1300$ and for the density parameters we take $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$, we obtain

$$\theta \approx 1^\circ\,. \tag{8}$$

This restricts the possible scale of the elementary cell to

$$L \geq 1.9999 R_{\rm sls}\,. \tag{9}$$

These estimates indicate that searching for topological signature of the Universe will require subtle and specific methods of analysis. Our estimates can also be applied to more complicated geometries of the Universe.

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