

UNINTEGRATED GLUON DISTRIBUTIONS FROM THE TRANSVERSE COORDINATE REPRESENTATION OF THE CCFM EQUATION IN THE SINGLE LOOP APPROXIMATION

J. KWIECIŃSKI

Henryk Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland

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We utilise the fact that the Catani–Ciafaloni–Fiorani–Marchesini (CCFM) equation in the single loop approximation can be diagonalised by the Fourier–Bessel transform. The analytic solution of the CCFM equation for the moments $f_\omega(b, Q)$ of the scale dependent gluon distribution is obtained, where b is the transverse coordinate conjugate to the transverse momentum of the gluon. The unintegrated gluon distributions obtained from this solution are analysed. It is shown how the approximate treatment of the exact solution makes it possible to express the unintegrated gluon distributions in terms of the integrated ones. The corresponding approximate expressions for the unintegrated gluon distribution are compared with exact solution of the CCFM equation in the single loop approximation.

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1. Introduction

The basic, universal quantities of the QCD improved parton model are the scale dependent parton distributions, like the gluon distribution $g(x, Q^2)$, where x denotes the momentum fraction. The (integrated) parton distributions can be related to the less inclusive distributions $f(x, Q_t, Q)$ unintegrated over transverse momentum Q_t of the parton. Those *unintegrated* distributions are often needed in less inclusive measurements which are sensitive to the transverse momentum of the parton [1–6].

The unintegrated, scale dependent distributions are described in QCD by the CCFM equation [7–17] based upon quantum coherence which implies angular ordering [18]. Its very nice feature is the fact that it embodies both the DGLAP and BFKL evolutions at low x . In the region of large and

moderately small values of x , where the small x effects can be neglected the CCFM equation becomes equivalent to the (LO) DGLAP evolution. This approximation corresponds to the so called “single loop” approximation [9, 10].

The CCFM equation interlocks in a rather complicated way the two relevant scales *i.e.* the transverse momentum Q_t of the parton and the hard scale Q . The main purpose of this paper is to explore the fact that in the “single loop” approximation the CCFM equation can be solved exactly in the transverse coordinate representation conjugate to the transverse momentum of the parton. The unintegrated distributions can then be obtained from the Fourier–Bessel transform of this solution. Although the “single loop” approximation neglects small x effects and so it is not valid at very small x it can be a reasonable approximation at large and moderately small values of x ($x \geq 0.01$ or so), which is certainly the region of phenomenological interest [2, 6]. Analytic insight into the *exact* solution of the CCFM equation in the single loop approximation will also make it possible to critically examine and justify approximate formulas linking the unintegrated distributions to the integrated ones in the region where the (LO) DGLAP dynamics should be adequate [1, 2, 6].

The content of our paper is as follows: In the next section we recall the CCFM equation for the unintegrated gluon distribution. In Sec. 3 we discuss the “single loop” approximation of this equation in the transverse coordinate representation. We show that it can be solved exactly for the moment function $f_\omega(b, Q)$, where b denotes the transverse coordinate conjugate to the transverse momentum Q_t of the gluon. We do also show how the approximate forms of this solution expressing the unintegrated distributions in terms of the integrated ones [1, 2] originate from the exact solution. In Sec. 4 we present numerical results for the unintegrated gluon distributions based on the solution of the CCFM equation in the transverse coordinate representation. We also confront exact solution with its approximate forms. Finally in Sec. 5 we summarise our main results and give our conclusions.

2. The CCFM equation

Parton cascade with angular ordering generates the Catani–Ciafaloni–Fiorani–Marchesini (CCFM) equation [7] for the unintegrated, scale dependent gluon distribution $f(x, Q_t, Q)$ in the proton, where x , Q_t and Q denote the longitudinal momentum fraction carried by the gluon, transverse momentum of the gluon and the hard scale, respectively. The latter is specified by the maximal value of the emission angle. The CCFM equation has the following form:

$$\begin{aligned}
f(x, Q_t, Q) &= \bar{f}^0(x, Q_t, Q) \\
&+ \int \frac{d^2 \mathbf{q}}{\pi q^2} \int_x^1 \frac{dz}{z} \Theta(Q - qz) \Theta(q - q_0) \frac{\alpha_s}{2\pi} \Delta_S(Q, q, z) \\
&\times \left[2N_c \Delta_{NS}(Q_t, q, z) + \frac{2N_c z}{(1-z)} + z \bar{P}_{gg}(z) \right] f\left(\frac{x}{z}, |\mathbf{Q}_t + (1-z)\mathbf{q}|, q\right), \quad (1)
\end{aligned}$$

where $\Delta_S(Q, q, z)$ and $\Delta_{NS}(Q_t, q, z)$ are the Sudakov and non-Sudakov form factors. They are given by the following expressions:

$$\Delta_S(Q, q, z) = \exp \left[- \int_{(qz)^2}^{Q^2} \frac{dp^2}{p^2} \frac{\alpha_s}{2\pi} \int_0^{1-q_0/p} dz z P_{gg}(z) \right], \quad (2)$$

$$\Delta_{NS}(Q_t, q, z) = \exp \left[- \int_z^1 \frac{dz'}{z'} \int_{(qz')^2}^{Q_t^2} \frac{dp^2}{p^2} \frac{2N_c \alpha_s}{2\pi} \right]. \quad (3)$$

For simplicity we neglect possible quark contributions. The function $\bar{P}_{gg}(z)$ is:

$$\bar{P}_{gg}(z) = 2N_c [-2 + z(1-z)] \quad (4)$$

and corresponds to the non-singular part of the $g \rightarrow gg$ splitting function $P_{gg}(z)$

$$P_{gg}(z) = 2N_c \left[\frac{1}{z} + \frac{1}{1-z} \right] + \bar{P}_{gg}(z). \quad (5)$$

The argument of α_s will be specified later.

In principle the CCFM equation has been obtained using only the singular parts of the splitting function proportional to $1/z$ and $1/(1-z)$. We add the non-singular part $\bar{P}_{gg}(z)$ to the kernel of this equation in order to obtain the complete DGLAP evolution in the “single loop” approximation. The two-dimensional vector $\mathbf{q}$ in equation (1) is related to the transverse momentum $\mathbf{q}_t$ of the emitted gluon

$$\mathbf{q}_t = (1-z)\mathbf{q}. \quad (6)$$

The constraint $Q > qz$ reflects the angular ordering and the inhomogeneous term $\bar{f}^0(x, Q_t, Q)$ is related to the input non-perturbative gluon distribution. It also contains effects of both the Sudakov and non-Sudakov form factors.

It should be observed that if the cut-off qz' in the definition of the non-Sudakov form factor is replaced by the fixed cut-off q_0 then the non-Sudakov form factor reduces to the form factor reflecting the reggeisation of the gluon, *i.e.*:

$$qz' \rightarrow q_0 \rightarrow \Delta_{\text{NS}} = \exp \left[(2\alpha_{\text{G}}(Q_{\text{t}}^2) - 2) \ln \left(\frac{1}{z} \right) \right],$$

$$\alpha_{\text{G}}(Q_{\text{t}}^2) = 1 - \int_{q_0^2}^{Q_{\text{t}}^2} \frac{dp^2}{p^2} \frac{N_c \alpha_{\text{s}}}{2\pi}.$$

The unintegrated scale dependent gluon distribution $f(x, Q_{\text{t}}, Q)$ is related in the following “standard” way to the conventional (scale dependent) integrated gluon distribution $xg(x, Q^2)$:

$$xg(x, Q^2) = \int^{Q^2} dQ_{\text{t}}^2 f(x, Q_{\text{t}}, Q). \quad (7)$$

In the “single loop” approximation of the CCFM equation (1) the angular ordering constraint $\Theta(Q - qz)$ is replaced by $\Theta(Q - q)$ and the non-Sudakov form factor Δ_{NS} is set equal to unity [9, 10]. Equation (1) then reads:

$$\begin{aligned} f(x, Q_{\text{t}}, Q) &= \bar{f}^0(x, Q_{\text{t}}, Q) \\ &+ \int \frac{d^2 \mathbf{q}}{\pi q^2} \int_x^1 \frac{dz}{z} \Theta(Q - q) \Theta(q - q_0) \frac{\alpha_{\text{s}}}{2\pi} \Delta_{\text{S}}(Q, q, z=1) \\ &\times \left[2N_c + \frac{2N_c z}{(1-z)} + z\bar{P}_{gg}(z) \right] f\left(\frac{x}{z}, |\mathbf{Q}_{\text{t}} + (1-z)\mathbf{q}|, q\right). \end{aligned} \quad (8)$$

It is useful to “unfold” the Sudakov form factor in equation (8) in order to treat the real emission and virtual corrections terms on equal footing. Unfolded CCFM equation in the single loop approximation takes the following form:

$$\begin{aligned} f(x, Q_{\text{t}}, Q) &= f^0(x, Q_{\text{t}}) \\ &+ \int \frac{d^2 \mathbf{q}}{\pi q^2} \Theta(q - q_0) \frac{\alpha_{\text{s}}(q^2)}{2\pi} \int_0^1 \frac{dz}{z} z P_{gg}(z) \\ &\times \left[\Theta(Q - q) \Theta(z - x) f\left(\frac{x}{z}, |\mathbf{Q}_{\text{t}} + (1-z)\mathbf{q}|, q\right) - z\Theta(Q - q) f(x, Q_{\text{t}}, q) \right]. \end{aligned} \quad (9)$$

The inhomogeneous term $f^0(x, Q_{\text{t}})$ is equal to the input non-perturbative gluon distribution in x and Q_{t} .

3. Transverse coordinate representation of the CCFM equation in the single loop approximation

It can be easily observed that the CCFM equation in the single loop approximation (9) can be diagonalised by the Fourier–Bessel transform:

$$f(x, Q_t, Q) = \int_0^\infty db b J_0(Q_t b) \bar{f}(x, b, Q), \quad (10)$$

with the function $\bar{f}(x, b, Q)$ given by:

$$\bar{f}(x, b, Q) = \int_0^\infty dQ_t Q_t J_0(Q_t b) f(x, Q_t, Q), \quad (11)$$

where $J_0(u)$ is the Bessel function. From equations (7) and (11) we get:

$$\bar{f}(x, b = 0, Q) = \frac{1}{2} x g(x, Q^2). \quad (12)$$

The corresponding equation for $\bar{f}(x, b, Q)$, which follows from equation (9) after taking the Fourier–Bessel transform of both sides of this equation reads:

$$\begin{aligned} \bar{f}(x, b, Q) = & \bar{f}^0(x, b) + \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 \frac{dz}{z} z P_{gg}(z) \\ & \times \left\{ \Theta(z - x) J_0[bq(1 - z)] \bar{f}\left(\frac{x}{z}, b, q\right) - z \bar{f}(x, b, q) \right\}, \quad (13) \end{aligned}$$

where we put q^2 as the argument of α_s . This choice of scale gives standard (LO) DGLAP equation for the integrated gluon distribution for

$$xg(x, Q^2) = 2\bar{f}(x, b = 0, Q).$$

In order to solve equation (13) it is useful to introduce the moment function $\bar{f}_\omega(b, Q)$

$$\bar{f}_\omega(b, Q) = \int_0^1 dx x^{\omega-1} \bar{f}(x, b, Q). \quad (14)$$

Equation (13) implies the following equation for the moment function $\bar{f}_\omega(b, Q)$:

$$\bar{f}_\omega(b, Q) = \bar{f}_\omega^0(b) + \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) \left\{ z^{\omega-1} J_0[bq(1-z)] \bar{f}_\omega(b, q) - \bar{f}_\omega(b, q) \right\}. \quad (15)$$

The solution of this equation reads:

$$\bar{f}_\omega(b, Q) = \bar{f}_\omega^0(b) \exp[S_\omega(b, Q)], \quad (16)$$

where

$$S_\omega(b, Q) = \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) \left\{ z^{\omega-1} J_0[(1-z)bq] - 1 \right\}. \quad (17)$$

At small values of b (*i.e.* $b \ll 1/q_0$) we can neglect b dependence in $\bar{f}_\omega^0(b)$ and set

$$\bar{f}_\omega^0(b) \simeq \bar{f}_\omega^0(b=0).$$

We can identify $\bar{f}_\omega^0(b=0)$ with the moment of the input (non-perturbative) integrated distribution, *i.e.*

$$\bar{f}_\omega^0(b=0) = \frac{1}{2} g_\omega^0, \quad (18)$$

where

$$g_\omega^0 = \int dQ_t^2 \int_0^1 dx x^{\omega-1} f^0(x, Q_t). \quad (19)$$

We note that at $b=0$ solution (16) reduces to the solution of the DGLAP equation for the moment function $g_\omega(Q^2)$ of the integrated gluon distribution $g(x, Q^2)$, *i.e.*

$$g_\omega(Q^2) = \int_0^1 dx x^\omega g(x, Q^2). \quad (20)$$

To be precise we get

$$f(b=0, Q) = \frac{1}{2}g_\omega(Q^2),$$

$$g_\omega(Q^2) = g_\omega^0 \exp \left\{ \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) [z^{\omega-1} - 1] \right\}. \quad (21)$$

It is useful to rearrange solution (16) as below:

$$\bar{f}_\omega(b, Q) = \tilde{f}_\omega(b, Q) T_g(b, Q), \quad (22)$$

where

$$\tilde{f}_\omega(b, Q) = \bar{f}_\omega^0(b) \exp \left\{ \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) J_0[(1-z)bq] [z^{\omega-1} - 1] \right\} \quad (23)$$

and the Sudakov-like form factor $T_g(b, q)$ is given by:

$$T_g(b, Q) = \exp \left\{ \int_{q_0^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) [J_0[(1-z)bq] - 1] \right\}. \quad (24)$$

In order to obtain more insight into the structure of the unintegrated distribution which follows from the CCFM equation in the single loop approximation it is useful to adopt the following approximation of the Bessel function:

$$J_0(u) \simeq \Theta(1-u). \quad (25)$$

Using this approximation in equation (11) we get:

$$f(x, Q_t, Q) \simeq 2 \frac{d\bar{f}(x, b=1/Q_t, Q)}{dQ_t^2}. \quad (26)$$

It may be useful to analyse solution (16) using approximation (25) which gives

$$\bar{f}_\omega(b, Q) \simeq \frac{g_\omega^0}{2} T_g \left(\frac{1}{b}, Q \right) \exp \left[S_\omega^r(b, Q) + \Delta S_\omega^r(b, Q) \right], \quad (27)$$

where

$$S_{\omega}^r(b, Q) \simeq \int_{q_0^2}^{\min(1/b^2, Q^2)} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^1 dz z P_{gg}(z) [z^{\omega-1} - 1] \quad (28)$$

and

$$\Delta S_{\omega}^r(b, Q) = \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_{1-1/(bq)}^1 dz z P_{gg}(z) [z^{\omega-1} - 1], \quad (29)$$

$$T_g(b, Q) \simeq \exp(S^0(b, Q)), \quad (30)$$

where $S^0(b, Q)$ is given by:

$$S^0(b, Q) = - \int_{1/b^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \int_0^{1-1/(bq)} dz z P_{gg}(z). \quad (31)$$

It may be seen that the form factor $T_g(1/b, Q)$ given by equations (30), (31) has the structure of the Sudakov form factor. It can also be seen that the factor $g_{\omega}^0 \exp[S_{\omega}^r(b, Q)]$ in equation (27) with $S_{\omega}^r(b, Q)$ defined by equation (28) can be identified with the moment function $g_{\omega}(\mu^2)$ of the integrated gluon distribution at the scale $\mu^2 = \min(1/b^2, Q^2)$. Neglecting the term $\Delta S_{\omega}^r(b, Q)$ we get:

$$\bar{f}_{\omega}(b, Q) \simeq T_g(b, Q) g_{\omega} \left(\min \left(\frac{1}{b^2}, Q^2 \right) \right) \quad (32)$$

that gives:

$$f_{\omega}(Q_t, Q) \simeq 2 \frac{\partial \bar{f}_{\omega}(b = 1/Q_t, Q)}{\partial Q_t^2} \simeq \frac{\partial [T_g(b = 1/Q_t, Q) g_{\omega}(Q_t^2)]}{\partial Q_t^2} \quad (33)$$

for $Q_t < Q$, and

$$f_{\omega}(Q_t, Q) = 0$$

for $Q_t > Q$.

Equation (33) gives:

$$f(x, Q_t, Q) \simeq \frac{\partial [T_g(b = 1/Q_t, Q) x g(x, Q_t^2)]}{\partial Q_t^2}. \quad (34)$$

It should be noted that equation (33) correctly reproduces the double logarithmic effects in the region $Q_t \ll Q$ [1]. The formalism presented above is similar to that used for the description of the p_T distributions in (for instance) Drell–Yan process (see *e.g.* [19]).

Taking approximately into account the remaining contribution in equation (27) gives:

$$f(x, Q_t, Q) \simeq \frac{T_g(b = 1/Q_t, Q)}{Q_t^2} \times \int_0^{1-Q_t/Q} dz P_{gg}(z) \frac{\alpha_s \left(\frac{Q_t^2}{(1-z)^2} \right)}{2\pi} \Theta(z-x) \frac{x}{z} g \left(\frac{x}{z}, \left(\frac{Q_t}{1-z} \right)^2 \right). \quad (35)$$

Derivation of equation (35) is sketched in the Appendix.

After replacement $Q_t^2/(1-z)^2 \rightarrow Q_t^2$ in the argument of α_s and in the gluon distribution $g(x/z, \mu^2)$ which introduces subleading effects, expression (35) coincides with the representation used in Ref. [2] (modulo subleading terms in the definition of the Sudakov form factor):

$$f(x, Q_t, Q) \simeq \frac{T_g(b = 1/Q_t, Q)}{Q_t^2} \times \int_0^{1-Q_t/Q} dz P_{gg}(z) \frac{\alpha_s (Q_t^2)}{2\pi} \Theta(z-x) \frac{x}{z} g \left(\frac{x}{z}, Q_t^2 \right). \quad (36)$$

4. Numerical results

In the previous section we have shown that the CCFM equation in the single loop approximation can be solved analytically in the b space, where b is the transverse coordinate conjugate to the transverse momentum Q_t of the gluon. We have also indicated approximations which make it possible to relate the unintegrated distributions to the integrated ones. In this section we present results of the numerical analysis of the exact solution of the CCFM equation utilising its diagonalisation in the transverse coordinate representation. We shall also confront this exact solution with the approximate expressions defined by equations (34) and (36).

To this aim we solved equation (13) for the distribution $\bar{f}(x, b, Q)$ and computed the unintegrated distribution $f(x, Q_t, Q)$ from the Fourier–Bessel transform:

$$f(x, Q_t, Q) = \int_0^\infty db b J_0(b Q_t) \bar{f}(x, b, Q). \quad (37)$$

We started from the input distribution $\bar{f}^0(x, b)$

$$\begin{aligned} \bar{f}^0(x, b) &= \frac{g_0(x)}{2} \exp\left(\frac{-b^2 q_0^2}{4}\right), \\ g_0(x) &= 3(1-x)^5, \end{aligned} \quad (38)$$

where we have set $q_0 = 1 \text{ GeV}$.

In figure 1 we plot the function $Q_t^2 f(x, Q_t, Q)$ as the function of Q_t for $Q^2 = 100 \text{ GeV}^2$ and for two values of x , $x = 0.01$ (upper curve) and $x = 0.1$ (lower curve). The transverse momentum Q_t is in GeV.

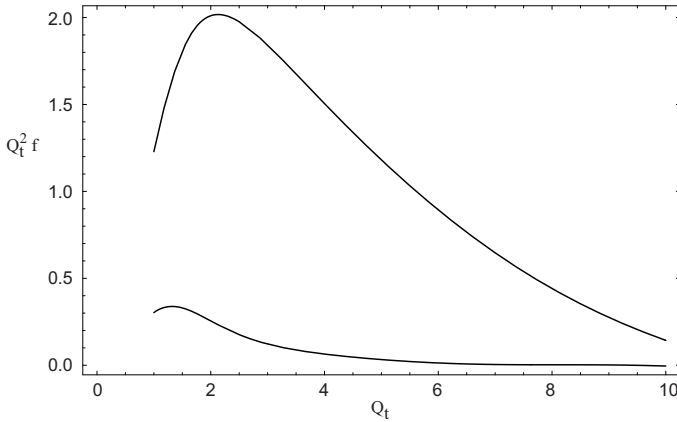


Fig. 1. Function $Q_t^2 f(x, Q_t, Q)$, where $f(x, Q_t, Q)$ is the unintegrated gluon distribution obtained from the exact solution of the CCFM equation in the single loop approximation, plotted as the function of the transverse momentum Q_t of the gluon for $Q^2 = 100 \text{ GeV}^2$. The upper and lower curves correspond to $x = 0.01$ and $x = 0.1$, respectively.

In figure 2 and 3 we compare those exact solutions with approximate expressions (34) and (36). We find that equation (36) gives somehow better approximation of the exact solution except for the “end points” $Q_t^2 \sim Q_0^2$ and $Q_t^2 \sim Q^2$. The simple formula (34) is a reasonable approximation of the exact solution for small values of Q_t . It may, however, give negative contribution at large x ($x \sim 0.1$) and large Q_t^2 .

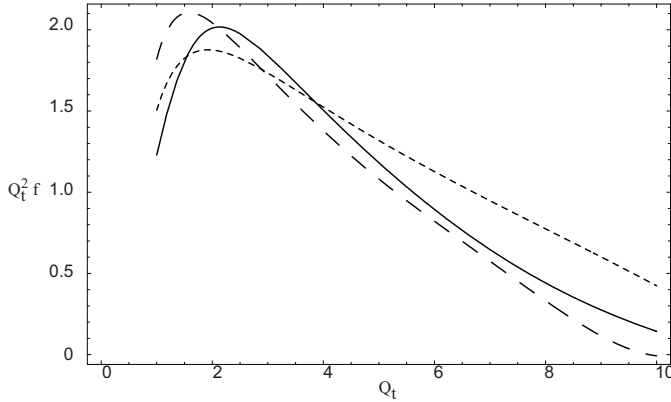


Fig. 2. Function $Q_t^2 f(x, Q_t, Q)$, where $f(x, Q_t, Q)$ is the unintegrated gluon distribution obtained from the exact solution of the CCFM equation in the single loop approximation, plotted as the function of the transverse momentum Q_t of the gluon for $Q^2 = 100 \text{ GeV}^2$ and $x = 0.01$. The solid curve corresponds to $f(x, Q_t, Q)$ obtained from exact solution of the CCFM equation in the single loop approximation, while the short dashed and long dashed curves correspond to approximate expressions for $f(x, Q_t, Q)$ given by equations (34) and (36), respectively.

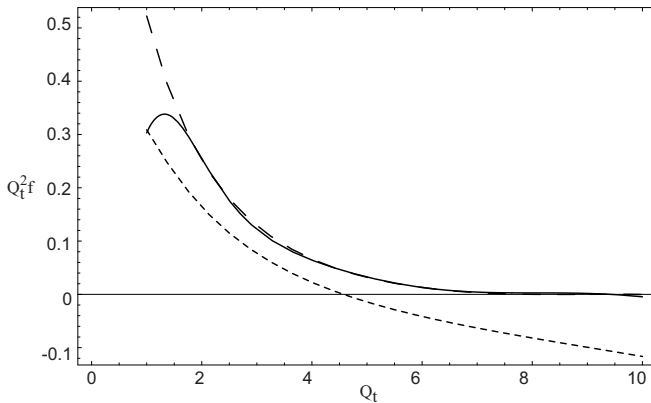


Fig. 3. Function $Q_t^2 f(x, Q_t, Q)$, where $f(x, Q_t, Q)$ is the unintegrated gluon distribution obtained from the exact solution of the CCFM equation in the single loop approximation, plotted as the function of the transverse momentum Q_t of the gluon for $Q^2 = 100 \text{ GeV}^2$ and $x = 0.1$. The solid curve corresponds to $f(x, Q_t, Q)$ obtained from the exact solution of the CCFM equation in the single loop approximation, while the short dashed and long dashed curves correspond to approximate expressions for $f(x, Q_t, Q)$ given by equations (34) and (36), respectively.

Summary and conclusions

In this paper we have utilised the transverse coordinate representation of the CCFM equation in order to get an analytical insight into its solution. The transverse coordinate representation has been widely used for the discussion of the soft gluon re-summation effects in the transverse momentum distribution of Drell–Yan pair *etc.*, [19]. In our paper we have utilised the fact that this representation diagonalises the CCFM equation in the single loop approximation and can be very helpful for obtaining unintegrated parton distribution satisfying the CCFM equation in this approximation. We have shown that the CCFM equation in the single loop approximation can be solved analytically for the moment function $f_\omega(b, Q)$, where b is the transverse coordinate conjugate to the transverse momentum of the gluon. We have also confronted the unintegrated gluon distributions with approximate expressions which were discussed in the literature. The single loop approximation neglects small x effects in the CCFM equation and, in particular, it neglects virtual corrections responsible for the non-Sudakov form factor. This form factor generates contributions which are no longer diagonal in the b space and so the merit of using this representation beyond the single-loop approximation is less apparent. However, in the leading $\ln(1/x)$ approximation at small x the CCFM equation reduces to the BFKL equation with no scale dependence and the kernel of the BFKL equation in the b space is the same as the BFKL kernel in the (transverse) momentum space. One can expect that the transverse coordinate representation of the CCFM equation may eventually appear to be helpful beyond the single loop and BFKL approximations.

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Appendix

In this Appendix we derive equation (35). To this aim we start from the following improved approximation of the derivative $\partial \bar{f}_\omega(b = 1/Q_t, Q)/\partial Q_t^2$:

$$2 \frac{\partial \bar{f}_\omega(b = 1/Q_t, Q)}{\partial Q_t^2} \simeq \frac{\partial [T_g(b = 1/Q_t, Q) g_\omega(Q_t^2)]}{\partial Q_t^2} + T_g(b = 1/Q_t, Q) g_\omega(Q_t^2) \frac{\partial \Delta S^r(b = 1/Q_t, Q)}{\partial Q_t^2}, \quad (39)$$

where the function $\Delta S^r(b = 1/Q_t, Q)$ is given by equation (29). Using equations (30) and (29) we get for $Q_t < Q$:

$$\begin{aligned} 2 \frac{\partial \bar{f}_\omega(b = 1/Q_t, Q)}{\partial Q_t^2} &\simeq \frac{T_g(b = 1/Q_t, Q)}{Q_t^2} \\ &\times \left[\int_0^{1-Q_t/Q} dz z P_{gg}(z) \frac{\alpha_s \left(\frac{Q_t^2}{(1-z)^2} \right)}{2\pi} g_\omega(Q_t^2) + \frac{dg_\omega(Q_t^2)}{d \ln(Q_t^2)} \right] + \frac{T_g(b = 1/Q_t, Q)}{Q_t^2} g_\omega(Q_t^2) \\ &\times \left\{ \int_0^{1-Q_t/Q} dz \frac{\alpha_s \left(\frac{Q_t^2}{(1-z)^2} \right)}{2\pi} z P_{gg}(z) [z^{\omega-1} - 1] - \int_0^1 dz \frac{\alpha_s(Q_t^2)}{2\pi} z P_{gg}(z) [z^{\omega-1} - 1] \right\}. \quad (40) \end{aligned}$$

Taking into account the DGLAP evolution equation:

$$\mu^2 \frac{dg_\omega(\mu^2)}{d\mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \int_0^1 dz z P_{gg}(z) [z^{\omega-1} - 1] g_\omega(\mu^2) \quad (41)$$

we get:

$$\begin{aligned} 2 \frac{\partial \bar{f}_\omega(1/Q_t, Q)}{\partial Q_t^2} \\ \simeq \frac{T_g(b = 1/Q_t, Q)}{Q_t^2} g_\omega(Q_t^2) \left\{ \int_0^{1-Q_t/Q} dz \frac{\alpha_s \left(\frac{Q_t^2}{(1-z)^2} \right)}{2\pi} z P_{gg}(z) z^{\omega-1} \right\}. \quad (42) \end{aligned}$$

Taking the inverse Mellin transform of both sides of equation (42) we get equation (35).

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