INELASTIC RESCATTERING IN B DECAYS TO $\pi\pi$, πK, AND $K\bar{K}$, AND EXTRACTION OF γ

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We discuss all contributions from inelastic SU(3)-symmetric rescattering in B decays into a final pair of pseudoscalar mesons $PP = \pi\pi$, $K\bar{K}$, πK. FSI-induced modifications of amplitudes obtained from the quark-line approach are described in terms of a few parameters which take care of all possible SU(3)-symmetric forms relevant for final-state interactions. Although in general it appears impossible to uniquely determine FSI effects from the combined set of all $\pi\pi$, $K\bar{K}$, and πK data, drawing some conclusions is feasible. In particular, it is shown that in leading order the amplitudes of strangeness-changing B decays depend on only one additional complex FSI-related parameter apart from those present in the definitions of penguin and tree amplitudes. It is also shown that joint considerations of U-spin-related $\Delta S = 0$ and $|\Delta S| = 1$ decay amplitudes are modified when non-negligible SU(3)-symmetric FSI are present. In particular, if rescattering in $B^+ \to K^+\bar{K}^0$ is substantial, determination of the CP-violating weak angle γ from $B^+ \to \pi^+K^0$, $B^0_d \to \pi^-K^+$, $B^0_s \to \pi^+K^-$, and their CP counterparts might be susceptible to important FSI-induced corrections.

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1. Introduction

Most of the literature analysing CP-violating effects in B decays (with $B \to PP = \pi\pi$, $K\bar{K}$, πK in particular) deals with quark-diagram Short-Distance (SD) amplitudes and assumes that Final State Interactions (FSI) are negligible. On the other hand, it has been argued that this neglect is not justified and that any reliable analysis must take FSI into account [1–3]. Indeed, recent analyses seem to show that even in $B \to D^*X$ decays FSI must play an important role (see *e.g.* [4]). Accordingly, various authors have tried to estimate FSI in $B \to PP$ decays by analysing the contribution from elastic or quasi-elastic rescattering [5]. The main problem, however, is posed by the sequence $B \xrightarrow{\text{weak}} i \xrightarrow{\text{FSI}} PP$ involving *inelastic* rescattering processes $i \xrightarrow{\text{FSI}} PP$, where i denote all kinds of multiparticle states. Arguments have been given that these inelastic processes constitute the main source of soft FSI phases [1, 6]. Since estimates of the size of these effects are model-dependent, one may envisage various scenarios, with the contributions from different intermediate states cancelling in an approximate way or renormalising SD prescriptions without changing their form, having random phases [6], or adding coherently [7], just to mention a few possibilities. With our insufficient knowledge of PP interactions at $\sqrt{s} = m_B \approx 5.2$ GeV, there is virtually no hope that a reliable calculation of inelastic FSI can be performed.

Consequently, various authors have argued that perhaps one should try to determine FSI effects directly from the data. For example, decays $B^0_d \to K^+K^-$ are thought to provide a measure on the size of FSI effects [8]. With many different decay channels and three varieties of B mesons (B^+, B^0_d, B^0_s) one may hope that the FSI effects can be untangled, especially if simple SU(3)-symmetric FSI is accepted. As FSI are oblivious of the original decay mechanism, various decays (for example, independently of whether the decay is strangeness-conserving or changing) are affected by the same SU(3)-symmetric FSI. If these FSI can be described with the help of a few parameters only, one may hope that the number of measurable decay types might be sufficient to permit determination of these parameters. Learning the size of FSI directly from the data would be certainly important as there are various papers which fit the present data on $B \to \pi\pi, \pi K, K\bar{K}$ decays both without and with FSI (*e.g.* [9, 10]).

The SD approaches attempt to include all strong interaction effects by assigning different phase parameters to different quark-line diagrams (*e.g.* tree T, penguin P, *etc.*). However, it was argued that this prescription violates such tenets of strong interactions as isospin symmetry [11, 12]. The origin of the problem pointed out in Ref. [12] is the lack of any (isospin) correlation between the spectator quark and the products of b quark decay. By its very nature such correlation cannot be provided by SD dynamics. A Long-Distance (LD) mechanism which ensures that quarks "know" about each other must be involved here. The inelastic rescattering effects considered in the present paper will provide both such a correlation and a generalisation of the formulas of Ref. [12]. We shall show how the standard formulas of the SD approach to B decay amplitudes are modified when FSI are not negligible. In particular, assuming the dominance of SD dynamics by a few (2 or 3) quark-line amplitudes (as it is usually done) we will discuss ways in which deviations from these formulas can be used to indicate the size of Inelastic FSI (IFSI). It will be also shown that rescattering may affect considerations based on analyses of U-spin related decays, including the method of extracting the value of the CP-violating weak angle γ from $B \to \pi K$ decays.

2. General

If one accepts that final state interactions cannot modify the probability of the original SD weak decay, it follows that vector $\boldsymbol{W}$ representing the set of all FSI-corrected amplitudes is related to vector $\boldsymbol{w}$ of the original amplitudes driven by the SD dynamics through [7]:

$$\boldsymbol{W} = \boldsymbol{S}^{1/2}\boldsymbol{w} \approx \left(1 + \tfrac{1}{2}(\boldsymbol{S}-1) + \ldots\right)\boldsymbol{w}. \tag{1}$$

After the SD-driven $B \to PP$ decay whose description is included in $\boldsymbol{w}$, the PP pair produced may undergo further scattering into many non-PP states. This out-of-PP-channel process provides absorption in the PP channel, *i.e.* it reduces the original decay amplitudes. This is described by (mainly imaginary) Pomeron exchange contribution in $\boldsymbol{T}$ ($\boldsymbol{S} - 1 = i\boldsymbol{T} \to -\mathrm{Im}\, T$).

Pomeron contributions in direct channels belonging to different SU(3) multiplets are related using $u-d-s$ symmetry of the quark diagram approach. This approach relates absolute magnitudes and phases of FSI amplitudes in various direct channels corresponding to different SU(3) multiplets. (SU(3) itself, on the other hand, relates amplitudes only within — but not between — these channels.) For Pomeron, the FSI effects in all possible SU(3) channels (**1**, **8**, **27**) are identical. Thus, Pomeron exchange between departing pseudoscalar mesons amounts to rescaling down the overall size of all quark-line decay amplitudes without modifying any other SD predictions.

The $b \to u\bar{u}q$ and $b \to c\bar{c}q$ SD decay processes lead directly also to non-PP states composed of two higher-mass states (resonances) M_1 and M_2. The latter may rescatter into PP yielding an "indirect" contribution to $B \to PP$. Thus, the set of FSI-corrected decay amplitudes $\boldsymbol{W} = [W_j]$ is composed of the direct and indirect parts as follows (amplitudes w_i are already absorption-rescaled):

$$W_j = w_j + \sum_{k,\alpha} F_{j,k\alpha} w_{k\alpha}\,, \tag{2}$$

where the indirect contributions are described by the sum on the r.h.s. In Eq. (2) the subscripts denote decay channels rather schematically: j,k are SU(3)-related indices, while α labels inelastic channels. SD decays to multi-meson states occur after $q\bar{q}$ pairs leading to resonances M_1M_2 are formed, when the quarks and antiquarks from these pairs radiate off gluons and further quark pairs. In other words SD decays to multi-meson states proceed via decays of resonances M_1 and M_2. We do not consider these decays explicitly, but assume that they are included in our description of rescattering processes via

$$\sum |M_k\rangle\langle M_k| = \sum |M_k \text{ decay products}\rangle\langle M_k \text{ decay products}| \,. \quad (3)$$

In this paper we are interested in finding the pattern of inelastic FSI contributions following the original SD decay $b \to u\bar{u}q$. Rescattering from the $b \to c\bar{c}q$-generated intermediate states leads to charming penguins [13], whose amplitudes may be added to those of SD penguins in the final formulas.

Formally, the choice of decay channels j (*i.e.* a basis in the flavour space) is irrelevant, and one may use either a Cartesian basis (where all mesons in PP states have definite $q\bar{q}$ content), or SU(3) basis (in which j correspond to — belonging to different SU(3) multiplets — linear combinations of $(q\bar{q})(q\bar{q})$). However, as resonances appear only in the octet channel, FSI in the octet and the 27-plet channels are different. Consequently, it is natural to use the SU(3) basis, only at the end transforming everything to the basis of interest.

Consider now the simple case when SU(3) is replaced by SU(2) and $j, k = \mathbf{1}, \mathbf{3}, \mathbf{5}, \ldots$ label SU(2) multiplets. Furthermore, in order to simplify the argument, let us assume that for all $\alpha = 1, \ldots N$ one has $w_{k\alpha} = w_k$ and $F_{j,k\alpha} = F_{j,k}$. Clearly, we must have $F_{j,k} = f_j \delta_{jk}$ with f_j complex in general. One obtains then

$$W_j = (1 + N f_j) w_j \,. \quad (4)$$

If $f_j = f$ for all j, one has $\boldsymbol{W} = (1+Nf)\boldsymbol{w}$, *i.e.* all FSI-induced modifications are contained in one, experimentally not discernible, overall complex factor $1 + Nf$, identical for all isopin channels. If strong interactions in different isospin channels are different (*i.e.* $f_j \neq f_i$ for $j \neq i$), the differences between f_j's will lead to a modification of the SD pattern: the magnitudes and phases of FSI effects will depend on isospin.

One expects the SU(3) case to be similar: for an appropriate choice of F's in Eq. (2), no FSI should be discernible in the final W_j amplitudes. Modifications of the predictions of the SD quark-line approach may appear only when FSI in different SU(3) channels differ from this particular choice. The relevant conditions on the SU(3) analogues of f_i are derived in Section 4.

3. SD amplitudes for decays into inelastic SU(3) eigenstates

In this paper we accept SU(3) in both direct and indirect terms as we do not attempt to fit any data as yet. When doing the latter, SU(3) breaking should probably be first introduced in the direct term, as one may argue that no corrections to corrections (*i.e.* no SU(3)-breaking in FSI effects) should be considered in the first attempt.

Our conventions and definitions for the (final, symmetrised) PP states are given in the Appendix, where PP states with mesons of definite charges, PP states of definite isospin, and PP states belonging to definite SU(3) multiplets (*i.e.* direct-channel SU(3) eigenstates) are listed.

In quasi-elastic FSI the intermediate state is also a PP state, and thus the intermediate mesons have to be symmetrised. In the inelastic case the original SD weak decay produces two $q\bar{q}$ pairs, which transform into a pair of resonances M_1M_2. These M_1 and M_2 mesons are different in general (we neglect the case when the two mesons are identical as the bulk of inelastic rescattering must come from $M_1 \neq M_2$). We may define M_1 to be the state of lower mass. In the Appendix we call the first (second) meson M_1 (M_2) a P (V) meson. Here P and V are only labels denoting different SU(3) multiplets of mesons, such as pseudoscalar, vector, axial, tensor *etc.* (including heavier and heavier) mesons. With $P \neq V$, there is no need to symmetrise. In particular, the PV states do not have to be symmetric in SU(3) indices. Thus, while in the case of quasi-elastic FSI the mesons V and P are both pseudoscalars and only states $(P_aP_b + P_bP_a)/\sqrt{2}$ (with P representing a pseudoscalar and a, b being SU(3) indices) are admissible, in general we must distinguish cases when $M_1M_2 = P_aV_b$ and $M_1M_2 = P_bV_a$. Using the PV labels to denote all such situations, the Appendix lists all the relevant PV states in the SU(3) basis. In the preparation of this list one has to consider both SU(3)-symmetric and SU(3)-antisymmetric combinations of octet mesons P and V in particular. In order to prevent any misunderstanding, we note that the replacement $P \rightleftharpoons V$ has nothing to do with this SU(3) (anti)symmetrisation: indices P, V do not belong to the SU(3) group as is explicit in the Appendix. Note that while the **27**-plet can be obtained only in the $\mathbf{8}\times\mathbf{8}$ PV channel, the octet may be obtained not only as a symmetric or antisymmetric combination of two octets, but also from a singlet P and octet V (or *vice versa*). Similar possibilities exist for the singlet PV channel. Since in each of these PV channels $((\mathbf{8}\times\mathbf{8}) \to \mathbf{27}, \mathbf{8}_s, \mathbf{8}_a, \mathbf{1}$; $(\mathbf{8}\times\mathbf{1}) \to \mathbf{8}$, *etc.*) rescattering of generally unknown form may take place, one is forced to use a free parameter to describe FSI in each such given channel. This proliferation of free parameters constitutes the main obstacle on the way of their determination from data.

Possible types of SD diagrams are shown in Fig. 1. For T (tree), C (colour-suppressed), P (penguin), S (singlet penguin) amplitudes only these diagrams are shown in which short-distance b decay consists in the emis-

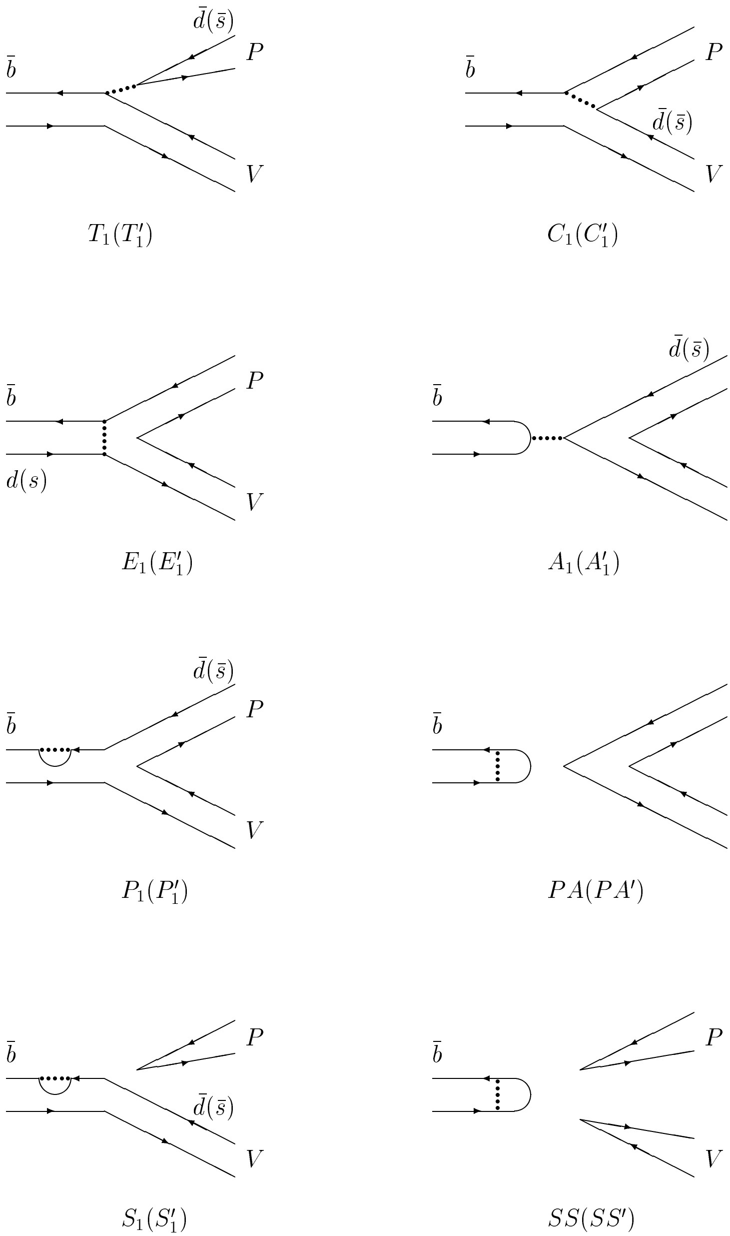

Fig. 1. Quark-line diagrams for B decays.

sion of meson $M_1 = P$ off the decaying quark line (*i.e.* when the spectator quark is not taken into account). These amplitudes are denoted by T_1, P_1, C_1, ...for strangeness-conserving processes (T'_1, P'_1, C'_1,...for strangeness-changing processes). When short-distance b decay produces meson $M_2 = V$, the corresponding amplitudes (not shown in Fig. 1) are denoted by T_2, P_2 *etc.* (T_1 does not have to be equal to T_2). Although we keep the distinction between E_1 and E_2 as well as A_1 and A_2, in these cases quarks produced in $\bar{b}d(\bar{b}s)$ should enter P and V mesons with equal probabilities. For the penguin annihilation amplitudes (PA and SS) there does not seem to be any reason why $PA_1 \neq PA_2$ or $SS_1 \neq SS_2$, hence PA and SS do not carry a subscript.

With the above preparations, the amplitudes for strangeness-conserving $\Delta S = 0$ (strangeness-violating $\Delta S = 1$) decays into quasi-two-body "M_1M_2" SU(3) channels may be calculated in terms of unprimed (primed) SD quark-line amplitudes T_i, P_i, ...(T'_i, P'_i, ...). We label channels by their SU(3) and isospin characteristics, *e.g.* $(\mathbf{8}_a, 1)$ denotes an isospin-1 octet channel formed as an antisymmetric combination of P_8 and V_8.

With the channels being specified on the l.h.s. and denoting $T_1 + T_2 = 2T$, $P_1 + P_2 = 2P$, $C_1 + C_2 = 2C$, $A_1 + A_2 = 2A$, $E_1 + E_2 = 2E$, and similarly for the primed amplitudes, one obtains the following expressions

a) for B^+ decays

$$
\begin{array}{rl}
(\mathbf{27}, 2) & -(T + C)\,, \\
(\mathbf{27}, 3/2) & \frac{2}{\sqrt{6}}(T' + C')\,, \\
(\mathbf{27}, 1) & -\frac{1}{\sqrt{5}}(T + C)\,, \\
(\mathbf{27}, 1/2) & 2\sqrt{\frac{2}{15}}(T' + C')\,, \\
(\mathbf{8}_s, 1) & -\frac{2}{\sqrt{30}}(T + C + 5P + 5A)\,, \\
(\mathbf{8}_s, 1/2) & \frac{2}{\sqrt{30}}(T' + C' + 5P' + 5A')\,, \\
(\mathbf{8}_a, 1) & -\frac{2}{\sqrt{6}}(T - C + 3P + 3A)\,, \\
(\mathbf{8}_a, 1/2) & \frac{2}{\sqrt{6}}(T' - C' + 3P' + 3A')\,, \\
(\mathbf{8}_{81}, 1) & -\frac{1}{\sqrt{3}}(T_1 + C_2 + 2P + 2A + S_2)\,,
\end{array}
$$

$$
\begin{array}{rl}
(\mathbf{8}_{81},1/2) & \dfrac{1}{\sqrt{3}}(T'_1+C'_2+2P'+2A'+S'_2)\,, \\
(\mathbf{8}_{18},1) & -\dfrac{1}{\sqrt{3}}(T_2+C_1+2P+2A+S_1)\,, \\
(\mathbf{8}_{18},1/2) & \dfrac{1}{\sqrt{3}}(T'_2+C'_1+2P'+2A'+S'_1)\,;
\end{array}
\tag{5}
$$

b) for B_d^0 decays

$$
\begin{array}{rl}
(\mathbf{27},2) & -\dfrac{2}{\sqrt{6}}(T+C)\,, \\
(\mathbf{27},3/2) & \dfrac{2}{\sqrt{6}}(T'+C')\,, \\
(\mathbf{27},1) & 0\,, \\
(\mathbf{27},1/2) & \dfrac{2}{\sqrt{30}}(T'+C')\,, \\
(\mathbf{27},0) & -\dfrac{1}{\sqrt{30}}(T+C)\,, \\
(\mathbf{8}_s,1) & \sqrt{\dfrac{5}{3}}(E-P)\,, \\
(\mathbf{8}_s,1/2) & \dfrac{2}{\sqrt{30}}(3T'-2C'+5P')\,, \\
(\mathbf{8}_s,0) & -\dfrac{2}{3\sqrt{20}}(6T-4C+5P+5E)\,, \\
(\mathbf{8}_a,1) & -\dfrac{1}{\sqrt{3}}(2T+3P-3E)\,, \\
(\mathbf{8}_a,1/2) & \dfrac{2}{\sqrt{6}}(T'+3P')\,, \\
(\mathbf{8}_a,0) & -(E+P)\,, \\
(\mathbf{8}_{81},1) & \dfrac{1}{\sqrt{6}}(C_1-C_2-2P+2E-S_2)\,, \\
(\mathbf{8}_{81},1/2) & \dfrac{1}{\sqrt{3}}(C'_2+2P'+S'_2)\,, \\
(\mathbf{8}_{81},0) & -\dfrac{1}{3\sqrt{2}}(2C+2P+2E+S_2)\,, \\
(\mathbf{8}_{18},1) & \dfrac{1}{\sqrt{6}}(-C_1+C_2-2P+2E-S_1)\,, \\
(\mathbf{8}_{18},1/2) & \dfrac{1}{\sqrt{3}}(C'_1+2P'+S'_1)\,,
\end{array}
$$

$$
\begin{array}{ll}
(\mathbf{8}_{18},0) & -\dfrac{1}{3\sqrt{2}}(2C+2P+2E+S_1)\,,\\
(\mathbf{1}_{88},0) & \dfrac{1}{3\sqrt{2}}(3T-C+8P+8E+12PA)\,,\\
(\mathbf{1}_{11},0) & \dfrac{1}{3}(2C+2P+2E+3PA+2S+SS)\,;
\end{array}
\tag{6}
$$

c) for B_s^0 decays

$$
\begin{array}{rl}
(\mathbf{27},2) & 0\,,\\
(\mathbf{27},3/2) & -\dfrac{2}{\sqrt{6}}(T+C)\,,\\
(\mathbf{27},1) & -\dfrac{2}{\sqrt{10}}(T'+C')\,,\\
(\mathbf{27},1/2) & -\dfrac{2}{\sqrt{30}}(T+C)\,,\\
(\mathbf{27},0) & \sqrt{\dfrac{3}{10}}(T'+C')\,,\\
(\mathbf{8}_s,1) & \dfrac{1}{\sqrt{15}}(3T'+5E'-2C')\,,\\
(\mathbf{8}_s,1/2) & -\dfrac{2}{\sqrt{30}}(3T-2C+5P)\,,\\
(\mathbf{8}_s,0) & \dfrac{1}{3\sqrt{5}}(3T'-2C'+10P'-5E')\,,\\
(\mathbf{8}_a,1) & -\dfrac{1}{\sqrt{3}}(T'-3E')\,,\\
(\mathbf{8}_a,1/2) & -\dfrac{2}{\sqrt{6}}(T+3P)\,,\\
(\mathbf{8}_a,0) & (T'+2P'-E')\,,\\
(\mathbf{8}_{81},1) & \dfrac{1}{\sqrt{6}}(C_1'+2E')\,,\\
(\mathbf{8}_{81},1/2) & -\dfrac{1}{\sqrt{3}}(C_2+2P+S_2)\,,\\
(\mathbf{8}_{81},0) & -\dfrac{1}{3\sqrt{2}}(C_1'-2C_2'-4P'+2E'-2S_2')\,,\\
(\mathbf{8}_{18},1) & \dfrac{1}{\sqrt{6}}(C_2'+2E')\,,
\end{array}
$$

$$\begin{array}{ll}
(\mathbf{8}_{18},1/2) & -\frac{1}{\sqrt{3}}(C_1+2P+S_1)\,, \\
(\mathbf{8}_{18},0) & -\frac{1}{3\sqrt{2}}(C'_2-2C'_1-4P'+2E'-2S'_1)\,, \\
(\mathbf{1}_{88},0) & \frac{1}{3\sqrt{2}}(3T'-C'+8P'+8E'+12PA')\,, \\
(\mathbf{1}_{11},0) & \frac{1}{3}(2C'+2P'+2E'+3PA'+2S'+SS')\,.
\end{array} \tag{7}$$

4. Modifications of SD amplitudes due to inelastic rescattering

Usually, the SD quark-diagram analyses of $B \to PP$ decays start with an assumption that only two or three diagram types are dominant, while the remaining ones are negligible. Thus, in strangeness-conserving ($b \to ud\bar{u}$) decays one expects the hierarchy $|T| > |P|, |C| > \ldots$ [14], while in the strangeness-violating decays one expects $|P'| > |T'| > \ldots$. Denoting the amplitudes for decays into a given M_1M_2 state with superscript $^{(\alpha)}$, we substitute in Eqs. (5)–(7) $T \to T^{(\alpha)}$, $P \to P^{(\alpha)}$, *etc.* Since at the level of short-distance decay it is not yet decided whether the particular quark-level state will hadronize as the PP state or one of the M_1M_2 states, one expects that quark-level amplitudes for the $B \to M_1M_2$ and $B \to PP$ transitions exhibit the same hierarchy pattern. Thus, transition amplitudes $T^{(\alpha)}$, $C^{(\alpha)}$, $P^{(\alpha)}$ should satisfy $T^{(\alpha)} = \eta^{(\alpha)}T > C^{(\alpha)} = \eta^{(\alpha)}C, P^{(\alpha)} = \eta^{(\alpha)}P > \ldots$ with T, C, P now describing transitions into pseudoscalar pairs, and analogously for primed amplitudes (η_a takes care of an overlap between quark-level and hadron-level states).

We will consider IFSI corrections resulting from the inelastic rescattering of the M_1M_2 states generated by these dominant amplitudes ($T^{(\alpha)}, P^{(\alpha)}$, $C^{(\alpha)}$) and ($P'^{(\alpha)}$,$T'^{(\alpha)}$) into PP. We will not keep any other terms, even though there are known problems with the description of $B \to \eta, \eta'$ decays, which indicate that in these decays the contributions from singlet penguin amplitudes may be significant. One expects, however, that contributions in which intermediate states are generated by Zweig-rule-violating SD amplitudes should be negligible for general (non-PP) inelastic states.

We describe inelastic final state interactions by introducing several complex free parameters as follows:

$$\begin{array}{rcll}
(M_1(\mathbf{8})M_2(\mathbf{8}))_{\mathbf{27}} & \to & (PP)_{\mathbf{27}} & f_{27}^{(\alpha)}\,, \\
(M_1(\mathbf{8})M_2(\mathbf{8}))_{\mathbf{8}_s} & \to & (PP)_{\mathbf{8}} & f_{s}^{(\alpha)}\,, \\
(M_1(\mathbf{8})M_2(\mathbf{8}))_{\mathbf{8}_a} & \to & (PP)_{\mathbf{8}} & f_{a}^{(\alpha)}\,, \\
M_1(\mathbf{1})M_2(\mathbf{8}) & \to & (PP)_{\mathbf{8}} & f_{1,8}^{(\alpha)}\,,
\end{array}$$

$$\begin{aligned}
M_1(\mathbf{8})M_2(\mathbf{1}) &\rightarrow (PP)_{\mathbf{8}} \qquad f^{(\alpha)}_{8,1}\,,\\
M_1(\mathbf{8})M_2(\mathbf{8}) &\rightarrow (PP)_{\mathbf{1}} \qquad f^{(\alpha)}_{8,8}\,,\\
M_1(\mathbf{1})M_2(\mathbf{1}) &\rightarrow (PP)_{\mathbf{1}} \qquad f^{(\alpha)}_{1,1}\,.
\end{aligned} \tag{8}$$

Upper indices label inelastic intermediate states in the direct channel (some $f^{(\alpha)}$ may be zero).

Let us now consider as an example the B^+ decay into the **27**-plet PP state. One calculates that (with the direct term already including absorption-induced rescaling)

$$W(B^+ \to PP(\mathbf{27},1)) = -\frac{1}{\sqrt{10}}(T+C) - \frac{1}{\sqrt{10}}\sum_{\alpha} f^{(\alpha)}_{27}(T^{(\alpha)}+C^{(\alpha)})\,. \tag{9}$$

Using $T^{(\alpha)} = \eta^{(\alpha)}T$ *etc.*, the above equation may be reduced to

$$W(B^+ \to PP(\mathbf{27},1)) = -\frac{1}{\sqrt{10}}(T+C)(1+f_{27})\,, \tag{10}$$

where $f_{27} \equiv \sum_\alpha f^{(\alpha)}_{27}\eta^{(\alpha)}$. We observe that the original amplitude has been multiplied by an inessential complex factor $1+f_{27}$, which may be absorbed into the definition of T and C.

Following the above example, one introduces complex parameters f_s, f_a, $f_{1,8}$, $f_{8,1}$, $f_{8,8}$ and $f_{1,1}$. As these parameters are free, in order to keep the formulas simple we define some of the parameters with additional purely numerical factors included. Furthermore we use $f_{1,8} = f_{8,1}$ as required by nonet symmetry.

Proceeding as in the example leading to Eq. (10), we may derive (after transforming to the basis in which final mesons are in states of definite charge):

$$\begin{aligned}
W(B^+ \to \pi^+\pi^0) &= -\frac{1}{\sqrt{2}}(T+C)(1+f_{27})\,,\\
W(B^+ \to K^+\bar{K}^0) &= -P(1+f_{27})\\
&\quad -\frac{1}{5}\{T\varDelta_1 + P\varDelta_2 + C\varDelta_3\}\,,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\varDelta_1 &= (f_s - f_{27}) + f_a + f_{1,8}\,,\\
\varDelta_2 &= 5(f_s - f_{27}) + 3f_a + 2f_{1,8}\,,\\
\varDelta_3 &= (f_s - f_{27}) - f_a + f_{1,8}\,.
\end{aligned} \tag{12}$$

The above equations reduce to standard SD prescriptions (with an overall factor of $1+f_{27}$) when $\Delta_1=\Delta_2=\Delta_3=0$, *i.e.* when $f_s-f_{27}=f_a=f_{1,8}=0$. This is the explicit form of the condition for no observable FSI effect, mentioned in Section 2.

Having presented the general idea, we now list all the relevant formulas. The decays in which at least one pseudoscalar produced is η or η' involve additional uncertainties at the direct level. Consequently, using these decays to help untangle the FSI is risky. Thus, we restrict ourselves to B decays into $\pi\pi$, $\pi K(\bar{K})$, and $K\bar{K}$.

In the $\Delta S=0$ sector, keeping only the T, P, C terms, we have

$$
\begin{aligned}
W(B^+\to\pi^+\pi^0) &= -\frac{1}{\sqrt{2}}(T+C)(1+f_{27})\,,\\
W(B^+\to K^+\bar{K}^0) &= -P(1+f_{27})\,,\\
&\quad -\frac{1}{5}\{T\Delta_1+P\Delta_2+C\Delta_3\}\,,\\
W(B_s^0\to\pi^+K^-) &= -(T+P)(1+f_{27})\,,\\
&\quad -\frac{1}{5}\{T(\Delta_2-2\Delta_1)+P\Delta_2+C(3\Delta_1-\Delta_2)\}\,,\\
W(B_s^0\to\pi^0\bar{K}^0) &= -\frac{1}{\sqrt{2}}(C-P)(1+f_{27})\\
&\quad +\frac{1}{5\sqrt{2}}\{T(\Delta_2-2\Delta_1)+P\Delta_2+C(3\Delta_1-\Delta_2)\}\,,\\
W(B_d^0\to\pi^+\pi^-) &= -(T+P)(1+f_{27})\\
&\quad -\frac{1}{15}\{T(-5\Delta_1+2\Delta_2+\Delta_3+\Delta_4)\\
&\quad +P(\Delta_2+\Delta_5)+C(6\Delta_1-2\Delta_2-3\Delta_4+\Delta_5)\}\,,\\
W(B_d^0\to K^+K^-) &= -\frac{1}{15}\{T(-\Delta_1+\Delta_2-\Delta_3-\Delta_4)\\
&\quad +P(2\Delta_2-\Delta_5)+C(3\Delta_1-\Delta_2+3\Delta_4-\Delta_5)\}\,,\\
W(B_d^0\to\pi^0\pi^0) &= -\frac{1}{\sqrt{2}}(C-P)(1+f_{27})\\
&\quad +\frac{1}{15\sqrt{2}}\{T(-5\Delta_1+2\Delta_2+\Delta_3+\Delta_4)\\
&\quad +P(\Delta_2+\Delta_5)+C(6\Delta_1-2\Delta_2-3\Delta_4+\Delta_5)\}\,,\\
W(B_d^0\to K^0\bar{K}^0) &= -P(1+f_{27})\\
&\quad -\frac{1}{15}\{T(4\Delta_1-\Delta_2-2\Delta_3+\Delta_4)\\
&\quad +P(\Delta_2+\Delta_5)+C(3\Delta_1-\Delta_2+3\Delta_4-\Delta_5)\}\,, \qquad (13)
\end{aligned}
$$

where the influence of FSI in the singlet channel is parametrised through

$$\begin{aligned} \Delta_4 &= \frac{15}{4}(f_{8,8} - f_{27})\,, \\ \Delta_5 &= 10(f_{8,8} - f_{27}) + 5f_{1,1}\,. \end{aligned} \tag{14}$$

Similarly, in the $\Delta S = 1$ sector (keeping only the dominant P', T' in the FSI contribution) we have:

$$\begin{aligned}
W(B^+ \to \pi^+ K^0) &= -P'(1 + f_{27}) \\
&\quad -\frac{1}{5}\{P'\Delta_2 + T'\Delta_1\}\,, \\
W(B^+ \to \pi^0 K^+) &= \frac{1}{\sqrt{2}}(T' + C' + P')(1 + f_{27}) \\
&\quad +\frac{1}{5\sqrt{2}}\{P'\Delta_2 + T'\Delta_1\}\,, \\
W(B_d^0 \to \pi^- K^+) &= (T' + P')(1 + f_{27}) \\
&\quad +\frac{1}{5}\{P'\Delta_2 + T'(\Delta_2 - 2\Delta_1)\}\,, \\
W(B_d^0 \to \pi^0 K^0) &= \frac{1}{\sqrt{2}}(C' - P')(1 + f_{27}) \\
&\quad -\frac{1}{5\sqrt{2}}\{P'\Delta_2 + T'(\Delta_2 - 2\Delta_1)\}\,, \\
W(B_s^0 \to \pi^+\pi^-) &= \frac{1}{15}\{P'(2\Delta_2 - \Delta_5) + T'(-\Delta_1 + \Delta_2 - \Delta_3 - \Delta_4)\}\,, \\
W(B_s^0 \to \pi^0\pi^0) &= -\frac{1}{15\sqrt{2}}\{P'(2\Delta_2 - \Delta_5) + T'(-\Delta_1 + \Delta_2 - \Delta_3 - \Delta_4)\}\,, \\
W(B_s^0 \to K^+K^-) &= (T' + P')(1 + f_{27}) \\
&\quad +\frac{1}{15}\{P'(\Delta_2 + \Delta_5) + T'(-5\Delta_1 + 2\Delta_2 + \Delta_3 + \Delta_4)\}\,, \\
W(B_s^0 \to K^0\bar{K}^0) &= -P'(1 + f_{27}) \\
&\quad -\frac{1}{15}\{P'(\Delta_2 + \Delta_5) + T'(4\Delta_1 - \Delta_2 - 2\Delta_3 + \Delta_4)\}\,.
\end{aligned} \tag{15}$$

Equations (13), (15) quantify explicitly what is already well known, *i.e.* that the presence of significant FSI can be seen most directly in $B_d^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-, \pi^0\pi^0$.

For any FSI the above formulas satisfy the following three triangle relations [15]:

$$W(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}}W(B_s^0 \to \pi^+K^-) + W(B_s^0 \to \pi^0\bar{K}^0)\,,$$

$$W(B_s^0 \to \pi^+ K^-) = W(B_d^0 \to \pi^+\pi^-) + W(B_d^0 \to K^+K^-)\,,$$
$$W(B_d^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}} W(B_d^0 \to K^+K^-) + W(B_s^0 \to \pi^0\overline{K}^0)\,. \quad (16)$$

Alternatively, one of the three relations above may be replaced by the isospin relation

$$W(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}} W(B_d^0 \to \pi^+\pi^-) + W(B_d^0 \to \pi^0\pi^0) \quad (17)$$

(not independent of the previous three).

In the $\Delta S = 1$ sector we have the following relations

$$\begin{aligned} W(B_d^0 \to \pi^- K^+) + \sqrt{2} W(B_d^0 \to \pi^0 K^0) &= (T' + C')(1 + f_{27})\,, \\ W(B^+ \to \pi^+ K^0) + \sqrt{2} W(B^+ \to \pi^0 K^+) &= (T' + C')(1 + f_{27})\,, \\ W(B_s^0 \to \pi^+\pi^-) + \sqrt{2} W(B_s^0 \to \pi^0\pi^0) &= 0\,, \\ W(B_s^0 \to K^+K^-) + W(B_s^0 \to \pi^+\pi^-) &= W(B_d^0 \to \pi^- K^+) \end{aligned} \quad (18)$$

as discussed in [15], with the first two relations leading to

$$\begin{aligned} W(B_d^0 \to \pi^- K^+) &+ \sqrt{2} W(B_d^0 \to \pi^0 K^0) \\ &= W(B^+ \to \pi^+ K^0) + \sqrt{2} W(B^+ \to \pi^0 K^+)\,. \end{aligned} \quad (19)$$

All these relations are FSI-independent.

Consequently, although the same five unknown complex parameters Δ_i ($i = 1, \ldots 5$) enter into both $\Delta S = 0$ and $\Delta S = 1$ sectors, the number of all independent and in principle measurable data (*i.e.* decay widths) is not sufficient to determine all these parameters, unless some additional input (like knowledge of sizes and relative phases of $T, P, \ldots$ and $T', P', \ldots$ and/or Δ's, assumption of higher-symmetry relations between Δ's, or justified neglect of some terms) is accepted.

5. Compatibility of quark-level parametrisation with isospin

In Ref. [12] it was argued that quark-diagram parametrisation in which T' and P' are given strong phases $\delta_{T'}$ and $\delta_{P'}$ is not compatible with isospin invariance, unless $\delta_{T'} - \delta_{P'} = \delta_{I=3/2} - \delta_{I=1/2} = 0$ (see also Ref. [11]).

From the previous section we have

$$W(B^+ \to \pi^+ K^0) = -P' \left(1 + f_{27} + \frac{1}{5}\Delta_2\right) - \frac{1}{5} T' \Delta_1\,,$$
$$W(B_d^0 \to \pi^- K^+) = (T' + P') \left(1 + f_{27} + \frac{1}{5}\Delta_2\right) - \frac{2}{5} T' \Delta_1\,,$$

$$\begin{aligned} W(B^+ \to \pi^0 K^+) &= \frac{1}{\sqrt{2}}(T'+P')\left(1+f_{27}+\frac{1}{5}\varDelta_2\right)+\frac{1}{5\sqrt{2}}T'(\varDelta_1-\varDelta_2)\,, \\ W(B_d^0 \to \pi^0 K^0) &= -\frac{1}{\sqrt{2}}P'\left(1+f_{27}+\frac{1}{5}\varDelta_2\right)-\frac{1}{5\sqrt{2}}T'(\varDelta_2-2\varDelta_1)\,. \end{aligned} \quad (20)$$

This should be compared with the approach of [12] which, after adjustment to our notation, inclusion of weak phase γ into the definition of T', C', and the neglect of C' terms, yields (the first two equations below are Eqs. (6a), (6b) of [12], $\delta = \delta_{3/2} - \delta_{1/2}$):

$$\begin{aligned} A(B^+ \to \pi^+ K^0) &= -P' - \frac{1}{3}(1-\mathrm{e}^{i\delta})T'\,, \\ A(B_d^0 \to \pi^- K^+) &= (T'+P') + \frac{2}{3}(1-\mathrm{e}^{i\delta})T'\,, \\ A(B^+ \to \pi^0 K^+) &= \frac{1}{\sqrt{2}}(T'+P') - \frac{1}{\sqrt{2}}\frac{2}{3}(1-\mathrm{e}^{i\delta})T'\,, \\ A(B_d^0 \to \pi^0 K^0) &= -\frac{1}{\sqrt{2}}P' - \frac{1}{\sqrt{2}}\frac{2}{3}(1-\mathrm{e}^{i\delta})T'\,. \end{aligned} \quad (21)$$

We see that the two sets of equations (20) and (21) are identical if we make the following replacements

$$\begin{aligned} P'\left(1+f_{27}+\frac{1}{5}\varDelta_2\right) &\to P'\,, \\ T'\left(1+f_{27}+\frac{1}{5}\varDelta_2\right) &\to T'\,, \end{aligned} \quad (22)$$

and appropriately choose $\varDelta_1$ and $\varDelta_2$, separately in each of the rightmost (and proportional to T') terms in Eqs. (20). The need for separate choices results from the oversimplified prescription for FSI used in [12]: a naive multiplication of quark-diagram amplitudes for $I = 1/2$ and $I = 3/2$ by two different phases only. The latter prescription does not allow for differences in various $I = 1/2$ phases (*e.g.* from **27** and from **8**, see also Ref. [10]), or possible different changes in the absolute size of the amplitudes.

Still, the general conclusion of [12] is correct: quark-diagram parametrisation in which P' and T' are given different strong phases is compatible with isospin symmetry in the $B \to \pi K$ decay channel only if terms proportional to $T'\varDelta_i$ (corresponding to $(1-\mathrm{e}^{i\delta})T'$) are neglected. As one expects that $|T'| < |P'|$, neglecting $T'\varDelta_i$ terms might seem a reasonable approximation for strangeness-violating $B \to \pi K$ decays. However, when $\varDelta S = 1$ decays $B \to \pi\pi, K\bar{K}$ are also considered, a glance at Eqs. (15) shows that different modifications of P' are needed there.

For $\Delta S = 0$ decays, the dominant FSI-induced correction terms should be proportional to T. Eqs. (13) show then that FSI-induced terms proportional to T enter different amplitudes with different coefficients and no universal renormalisation of quark-level amplitudes T, P, C can work. In general, therefore, parametrisation of FSI effects by endowing quark-diagram amplitudes T, P, C with additional universal phases cannot take the whole complexity of FSI into account.

6. Restriction to leading FSI corrections

If final-state interactions may be treated as a correction to the direct SD amplitudes, it seems natural to keep leading terms only in such a correction. Assuming then that Δ_i are all of similar sizes, we may neglect in Eqs. (13), (15) all FSI-induced terms but the leading ones, *i.e.* those proportional to T and P'. Thus, the $\Delta S = 0$ decay amplitudes depend on four Δ_i (Δ_5 drops out), while the $\Delta S = 1$ amplitudes on two Δ_i: Δ_2 and Δ_5.

In the $\Delta S = 0$ sector, with amplitudes still depending on four Δ_i, no relations between amplitudes in addition to those of Eq. (16) are generated. The number of undetermined parameters is too large to permit their clear-cut determination from data. Thus, additional input is necessary.

In the $\Delta S = 1$ sector it is instructive to rewrite the amplitudes in terms of redefined quark-diagram amplitudes:

$$\begin{aligned} \tilde{T'} &\equiv T'(1+f_{27}), \\ \tilde{C'} &\equiv C'(1+f_{27}), \\ \tilde{P'} &\equiv P'(1+f_{27}+\frac{1}{5}\Delta_2) \end{aligned} \tag{23}$$

and

$$\tilde{\Delta} \equiv \frac{1}{15}(2\Delta_2 - \Delta_5)/(1+f_{27}+\frac{1}{5}\Delta_2)\,. \tag{24}$$

One then obtains

$$\begin{aligned} W(B^+ \to \pi^+ K^0) &= -\tilde{P'}, \\ W(B^+ \to \pi^0 K^+) &= \frac{1}{\sqrt{2}}(\tilde{T'}+\tilde{C'}+\tilde{P'}), \\ W(B_d^0 \to \pi^- K^+) &= \tilde{T'}+\tilde{P'}, \\ W(B_d^0 \to \pi^0 K^0) &= \frac{1}{\sqrt{2}}(\tilde{C'}-\tilde{P'}), \\ W(B_s^0 \to \pi^+\pi^-) &= \tilde{P'}\tilde{\Delta}, \\ W(B_s^0 \to \pi^0\pi^0) &= -\frac{1}{\sqrt{2}}\tilde{P'}\tilde{\Delta}, \end{aligned}$$

$$\begin{aligned} W(B_s^0 \to K^+K^-) &= \tilde{T}' + \tilde{P}' - \tilde{P}'\tilde{\Delta}\,, \\ W(B_s^0 \to K^0\bar{K}^0) &= -\tilde{P}' + \tilde{P}'\tilde{\Delta}\,. \end{aligned} \tag{25}$$

Note that the first four equations above have the structure used in the SD quark-diagram approach: the FSI effects can be identified only with additional help from B_s^0 decays. With eight decays and four parameters ($\tilde{T}'$, $\tilde{P}'$, $\tilde{C}'$, $\tilde{\Delta}$) there are four relations between the amplitudes. In addition to the three relations of Eqs. (18), (19), we have one new relation involving $B_s^0 \to K^0\bar{K}^0$:

$$W(B^+ \to \pi^+K^0) + W(B_s^0 \to \pi^+\pi^-) = W(B_s^0 \to K^0\bar{K}^0)\,. \tag{26}$$

This relation yields information on the phase of the FSI-related parameter $\tilde{\Delta}$. Note that the ratio $|\sqrt{2}W(B_s^0 \to \pi^+\pi^-)/W(B^+ \to \pi^+K^0)|$ measures the (relative) size of observable FSI effects.

7. Relating $\Delta S = 0$ and $\Delta S = 1$ amplitudes

In the SD quark-diagram approach the $\Delta S = 0$ and $\Delta S = 1$ decay amplitudes are related. Consequently, simultaneous analyses of these amplitudes have been considered as a means to provide important tests of the approximations made in the SD approach, and as a way to extract weak angle γ. An important question is how such analyses are affected by FSI effects.

It appears that rescattering may upset expectations related to $s \leftrightarrow d$ flavour U-spin reflection arguments [16]. Consider for example the amplitudes for the four decays

$$\begin{aligned} B^+ &\to K^+\bar{K}^0\,, \\ B_s^0 &\to \pi^+K^-\,, \\ B^+ &\to \pi^+K^0\,, \\ B_d^0 &\to \pi^-K^+\,. \end{aligned} \tag{27}$$

Introducing

$$\begin{aligned} \tilde{P} &= P\left(1 + f_{27} + \frac{1}{5}\Delta_2\right)\,, \\ \tilde{T} &= T\left(1 + f_{27} + \frac{1}{5}\Delta_2\right)\,, \\ \tilde{\Delta}_1 &= \frac{\Delta_1}{\left(1 + f_{27} + \frac{1}{5}\Delta_2\right)}\,, \\ \tilde{\Delta}_2 &= \frac{\Delta_2}{\left(1 + f_{27} + \frac{1}{5}\Delta_2\right)} \end{aligned} \tag{28}$$

in addition to $\tilde{P}' = P'(1 + f_{27} + \Delta_2/5)$ (Eq. (23)), the amplitudes for the first two ($\Delta S = 0$) decays in Eq. (27) may be re-expressed as

$$\begin{aligned} W(B^+ \to K^+\bar{K}^0) &= -\tilde{P} - \frac{1}{5}\tilde{T}\tilde{\Delta}_1 \,, \\ W(B_s^0 \to \pi^+K^-) &= -\tilde{P} - \tilde{T} + \frac{2}{5}\tilde{T}\tilde{\Delta}_1 \,, \end{aligned} \tag{29}$$

when the FSI-induced terms proportional to C are neglected. Note that we have kept terms of order $P\Delta_2$ even though they represent nonleading FSI effects. Similarly, we could have kept nonleading terms of order $T'\Delta_2$ in the definition of T' in Eq. (23), *i.e.*

$$\tilde{T}' = T'\left(1 + f_{27} + \frac{1}{5}\Delta_2\right) \tag{30}$$

so that

$$\tilde{T} = \frac{1}{\lambda}\tilde{T}' \,, \tag{31}$$

where $\lambda \approx 0.22$ is the parameter in the Wolfenstein's parametrisation of the CKM matrix.

From Eqs. (15) the two $\Delta S = 1$ decays of Eqs. (27) are then described by

$$\begin{aligned} W(B^+ \to \pi^+K^0) &= -\tilde{P}' - \frac{1}{5}\tilde{T}'\tilde{\Delta}_1 \,, \\ W(B_d^0 \to \pi^-K^+) &= \tilde{P}' + \tilde{T}' - \frac{2}{5}\tilde{T}'\tilde{\Delta}_1 \,. \end{aligned} \tag{32}$$

Corrections from electroweak penguin diagrams to the right-hand sides of Eqs. (29) (Eqs. (32)) are proportional to $P^c_{\rm EW}/3$ and $-2P^c_{\rm EW}/3$ ($P'^c_{\rm EW}/3$ and $-2P'^c_{\rm EW}/3$), respectively. (Actually, using the substitutions $T \to T + P^c_{\rm EW}$, $P \to P - P^c_{\rm EW}/3$, $C \to C + P_{\rm EW}$, and the analogous ones for the $\Delta S = 1$ transitions, we could have started our calculations from SD amplitudes corrected for electroweak penguins.) Since one expects that $|P^c_{\rm EW}| < |E|, |A|, |P_{\rm EW}| < |C|, |P| < |T|$, and $|P'^c_{\rm EW}| < 0.05|P'| < |T'| \approx (0.1 \text{ to } 0.2)|P'|$ [17] (see also [18]), any such contributions have to be neglected in our approximation. Only the $P'_{\rm EW}$ terms (of order T') should be included in the non-FSI-induced terms in Eqs. (27). However, in FSI-induced terms the corrections from the $P'_{\rm EW}$ should be neglected if those from T' are. Thus, when terms of order $\tilde{T}\tilde{\Delta}_1 = \tilde{T}'\tilde{\Delta}_1/\lambda$ (in $B^+ \to K^+\bar{K}^0$ and $B_s^0 \to \pi^+K^-$) are kept, but those of order $\tilde{T}'\tilde{\Delta}_1$ (in $B^+ \to \pi^+K^0$ and $B_d^0 \to \pi^-K^+$) are neglected, our final form of Eqs. (29), (32) is

$$\begin{aligned}
W(B^+\to K^+\bar{K}^0) &= -\tilde{P}-\frac{1}{5}\tilde{T}\tilde{\Delta}_1 \qquad = -\tilde{P}-\frac{1}{5}\frac{1}{\lambda}\tilde{T}'\tilde{\Delta}_1\,,\\
W(B^0_s\to\pi^+K^-) &= -\tilde{P}-\tilde{T}+\frac{2}{5}\tilde{T}\tilde{\Delta}_1 = -\tilde{P}-\frac{1}{\lambda}\tilde{T}'+\frac{2}{5}\frac{1}{\lambda}\tilde{T}'\tilde{\Delta}_1\,,\\
W(B^+\to\pi^+K^0) &= -\tilde{P}'\,,\\
W(B^0_d\to\pi^-K^+) &= \quad\tilde{P}'+\tilde{T}'
\end{aligned}\tag{33}$$

with $\tilde{P}$, $\tilde{T}$ defined in Eq. (28), $\tilde{P}'$ defined in Eq. (23) and $\tilde{T}'$ defined in Eq. (30). In Eqs. (33) a part of rescattering effects is included into the definition of effective "penguin" and "tree" amplitudes $\tilde{P}$, $\tilde{P}'$ and $\tilde{T}$, $\tilde{T}'$ through a common multiplicative factor of $(1+f_{27}+\Delta_2/5)$. It is only the term $-\frac{1}{5}\tilde{T}\tilde{\Delta}_1$ in the expression for $W(B^+\to K^+\bar{K}^0)$ (and a similar one in $W(B^0_s\to\pi^+K^-)$) which represents "visible" FSI effects (*i.e.* those not removable through a redefinition of P, T amplitudes). This term may influence the equality

$$W(B^+\to K^+\bar{K}^0) = -\lambda W(B^+\to\pi^+K^0)\tag{34}$$

obtained (for SU(3) symmetric P and P') either when charming penguins are dominant, or in SD approaches when $\beta\approx 0$ (see later). Comparison of $B^+\to K^+\bar{K}^0$ and $B^+\to\pi^+K^0$ was considered as a test for the presence of the contribution from the annihilation diagram or FSI effects [19, 20]. Indeed, the relative size of the FSI-generated correction term to $\tilde{P}$ in $W(B^+\to K^+\bar{K}^0)$ is proportional to $\frac{1}{5}|T/P|$ and, with $|P/T|\approx 0.3\pm 0.1$, it might be sizable. Note that by including two terms of different weak phases, the first of Eqs. (33) explicitly indicates the appearance of a rescattering-induced CP-violating asymmetry $\Gamma(B^+\to K^+\bar{K}^0)-\Gamma(B^-\to K^-K^0)$. Great importance of $W(B^+\to K^+\bar{K}^0)$ for gathering information on rescattering effects was also noted in [8]. The present approach places such considerations in a framework which quantifies the connections between all FSI effects in B decays into $\pi\pi$, $K\bar{K}$, and πK.

Qualitatively, violation of equality (34) by FSI effects may be understood as follows. The amplitude $W(B^+\to K^+\bar{K}^0)$ receives contributions from inelastic intermediate states with flavour content "PV" = "$\pi^+\omega_8$", "PV" = "$\pi^+\omega_1$", "PV" = "$\eta_8\rho^+$", *etc.* These amplitudes involve tree amplitudes proportional to the SD tree amplitude T (in addition to the amplitudes proportional to P, *etc.*). The approximations involved when deriving the first of Eqs. (29) leave the $T\Delta$ term as the only sizable FSI-induced term (as $|T|>|P|\approx|C|>\ldots$). On the other hand, although the FSI-induced corrections to $W(B^+\to\pi^+K^0)$ also contain (compare Eqs. (32)) analogous terms proportional to the SD tree amplitude T' (originating from inelastic intermediate states "PV" = "$\pi^0 K^{*+}$", "PV" = "$K^+\omega_8$" *etc.*), the approximations involved neglect these terms on account of $|P'| > |T'| > |C'| \ldots$.

Combined analysis of decays $B^+ \to \pi^+ K^0$, $B^0_d \to \pi^- K^+$, and $B^0_s \to \pi^+ K^-$ (together with their CP counterparts) was proposed in Ref. [21] as a means to provide information on the value of the CP-violating angle γ. From the form of the expressions for relevant amplitudes in the presence of FSI (the last three equations in Eqs. (33)), we see that rescattering might affect the determination of γ (see also [22]): the FSI-induced term in the $B^0_s \to \pi^- K^+$ amplitude is of the order of $\frac{2}{5}|T/P|$ of the penguin amplitude P, *i.e.* twice the size of a similar term in $B^+ \to K^+ \bar{K}^0$. Thus, if rescattering effects in $B^+ \to K^+ \bar{K}^0$ are substantial, one should seriously worry about the FSI corrections to the method of Ref. [21].

In Ref. [21], using unitarity of the CKM matrix, *i.e.* $V^*_{tb} V_{ti} = -V^*_{cb} V_{ci} - V^*_{ub} V_{ui}$, only the $-V^*_{cb} V_{ci}$ part of the penguins is included into the redefined penguins p and p':

$$
\begin{aligned}
P &= p\left(1 - \frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma}\right) = p\frac{\sin\gamma}{\sin(\beta+\gamma)} \mathrm{e}^{-i\beta} , \\
P' &= p'\left(1 + \lambda^2 \frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma}\right)
\end{aligned}
\tag{35}
$$

with

$$
p = -\lambda p' , \tag{36}
$$

while the $-V^*_{ub} V_{ui}$ parts are absorbed into the redefined tree amplitudes t, t'

$$
\begin{aligned}
T &= t + p\frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma} , \\
T' &= t' - \lambda^2 p' \frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma}
\end{aligned}
\tag{37}
$$

with

$$
t = \frac{1}{\lambda} t' . \tag{38}
$$

The approximation of Ref. [21] consists in neglecting the λ^2 terms in the expression relating P' and p', *i.e.* it corresponds to $\beta \to 0$ [23].

With FSI taken into account, by replacing the $\tilde{T}'\tilde{\Delta}_1$ terms with $\tilde{t}'\tilde{\delta}_1 \equiv \tilde{T}'\tilde{\Delta}_1$, where $\tilde{t}'$ is related to $\tilde{T}'$ through an analogon of Eqs. (37), and with $\tilde{p}' = p'(1 + f_{27} + \Delta_2/5)$, we have

$$
W(B^+ \to K^+ \bar{K}^0) = \lambda \tilde{p}' \left(1 - \frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma}\right) - \frac{1}{5}\frac{1}{\lambda} \tilde{t}' \tilde{\delta}_1 ,
$$

$$\begin{aligned}
W(B_s^0 \to \pi^+ K^-) &= \lambda\tilde{p}' - \frac{1}{\lambda}\tilde{t}' + \frac{2}{5}\frac{1}{\lambda}\tilde{t}'\tilde{\delta}_1 \, , \\
W(B^+ \to \pi^+ K^0) &= -\tilde{p}'\left(1 + \lambda^2 \frac{\sin\beta}{\sin(\beta+\gamma)} \mathrm{e}^{i\gamma}\right) , \\
W(B_d^0 \to \pi^- K^+) &= \tilde{p}' + \tilde{t}' \, . \qquad (39)
\end{aligned}$$

If the charming penguins of Ref. [13] are substantial, they may be included into the definition of β-independent parts of redefined penguins above, effectively suppressing the β-dependent parts (and leading to Eq. (34)).

In the following formulas we accept that β is small, so that terms proportional to $\sin\beta$ may be neglected; in reality, a nonzero value of β would have to be used in any attempt to extract the angle γ from data on the basis of Eqs. (39) [23].

The equality $A_0 = -A_s$, expected to hold (for any β) in SU(3) [21] between the CP-violating rate pseudo-asymmetries

$$A_0 \equiv \frac{\Gamma(B_d^0 \to K^+\pi^-) - \Gamma(\bar{B}_d^0 \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \bar{K}^0\pi^-)} \, , \qquad (40)$$

and

$$A_s \equiv \frac{\Gamma(B_s^0 \to K^-\pi^+) - \Gamma(\bar{B}_s^0 \to K^+\pi^-)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \bar{K}^0\pi^-)} \qquad (41)$$

may be affected by FSI even when the latter is SU(3) symmetric. Indeed, using Eqs. (39) one derives (for $\beta \approx 0$)

$$A_0 = -2r\sin\delta\sin\gamma \qquad (42)$$

and

$$A_s = 2r\kappa\sin(\delta+\varepsilon)\sin\gamma \, , \qquad (43)$$

where δ (γ) denotes relative strong (weak) phase of t' ($\tilde{t}'$) with respect to p' ($\tilde{p}'$), $r = |\tilde{t}'/\tilde{p}'| = |t'/p'|$, and (with $\kappa = |\kappa|$)

$$1 - \frac{2}{5}\tilde{\delta}_1 \equiv \kappa\mathrm{e}^{i\varepsilon} \, . \qquad (44)$$

Since from Eqs. (39) the charge-averaged ratios

$$R \equiv \frac{\Gamma(B_d^0 \to K^+\pi^-) + \Gamma(\bar{B}_d^0 \to K^-\pi^+)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \bar{K}^0\pi^-)} \qquad (45)$$

and

$$R_s \equiv \frac{\Gamma(B_s^0 \to K^-\pi^+) + \Gamma(\bar{B}_s^0 \to K^+\pi^-)}{\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \bar{K}^0\pi^-)} \qquad (46)$$

are given by

$$R = 1 + r^2 + 2r\cos\delta\cos\gamma\,, \tag{47}$$

$$R_s = \lambda^2 + \frac{r^2}{\lambda^2} - 2r\kappa\cos\gamma\cos(\delta+\varepsilon)\,, \tag{48}$$

there are now four equations (42), (43), (47), (48) for five unknowns (r, γ, δ, κ, ε). If $\varepsilon \ll \delta$, the four equations may be solved after neglecting ε. For ε of order δ, additional constraints would be needed. The ratios $(\Gamma(B^+ \to K^+\bar{K}^0) \pm \Gamma(B^- \to K^-K^0))/(\Gamma(B^+ \to K^0\pi^+) + \Gamma(B^- \to \bar{K}^0\pi^-))$ may be expressed in terms of $r,\ldots,\varepsilon$ (and β when its nonzero value is taken into account), and seem to provide such constraints. Thus, if $A_0 \neq -A_s$, their usefulness would have to be studied. Such an analysis requires a detailed consideration of SU(3) breaking which is outside the scope of this paper.

Similar effects of apparent SU(3) breaking are observed for other pairs of U-spin-related decays. According to Eqs. (13), (15), when E and PA (E' and PA') SD amplitudes are neglected, the processes $B^0_d \to K^+K^-$ and $B^0_s \to \pi^+\pi^-$, related to one another by this reflection, are described by rescattering-induced amplitudes:

$$\begin{aligned} W(B^0_d \to K^+K^-) &= -\frac{1}{15}\{T(-\Delta_1+\Delta_2-\Delta_3-\Delta_4)+P(2\Delta_2-\Delta_5)+\ldots\},\\ W(B^0_s \to \pi^+\pi^-) &= \frac{1}{15}\{T'(-\Delta_1+\Delta_2-\Delta_3-\Delta_4)+P'(2\Delta_2-\Delta_5)\}. \end{aligned} \tag{49}$$

If P/T were equal to P'/T', we would indeed expect for $|T/T'| = |V_{ud}/V_{us}|$ that

$$\frac{\Gamma(B^0_d \to K^+K^-)}{\Gamma(B^0_s \to \pi^+\pi^-)} = \left|\frac{V_{ud}}{V_{us}}\right|^2, \tag{50}$$

as obtained in SD approaches. However, as one expects that $|T'/T| \approx |P/P'|$ with dominant T- and P'-terms, relation (50) may be violated. Thus, Eq. (50) may help distinguish between rescattering effects and genuine short-distance E and PA contributions.

A look at Eqs. (13), (15) shows that the method of Ref. [24], based on the U-spin-related decays $B^0_d \to \pi^+\pi^-$ and $B^0_s \to K^+K^-$, is also affected by rescattering. Indeed, keeping only the dominant FSI-induced terms (*i.e.* those proportional to T and P') introduces two unrelated linear combinations of Δ's into the game. Thus, FSI-induced modifications of this method are less easily controlled than those of Ref. [21].

When specific models for rescattering relations (and thus, definite relations between Δ's) are considered, further relations between FSI-induced corrections to various decays should appear. The analysis of such models and their predictions is outside the scope of this paper.

8. Conclusions

In this paper we analysed the influence of SU(3)-symmetric inelastic rescattering onto the predictions of short-distance quark-diagram approach to B decays into two pseudoscalars PP when the tree and penguin amplitudes are assumed dominant. Final-state interactions were described with the help of a few parameters corresponding to all possible SU(3)-symmetric forms of inelastic rescattering into PP. We found that the combined set of experimental data on all $B \to \pi\pi$, $K\bar{K}$, πK decays is not sufficient to determine all relevant FSI-related parameters. Still, some important information on inelastic FSI effects may be extracted from the data. Apart from providing explicit expressions for the amplitudes of the FSI-driven decays $B_d^0 \to K^+K^-$, $B_s^0 \to \pi^+\pi^-$, and $B_s^0 \to \pi^0\pi^0$, it was shown that the $\Delta S = 1$ decays may provide quantitative information on the magnitude and phase of the single FSI-indicating effective parameter appearing in this sector. FSI-induced modification of the connection between $B^+ \to K^+\bar{K}^0$ and $B^+ \to \pi^+K^0$ amplitudes was also given explicitly. Furthermore, it was shown that rescattering affects the analyses of U-spin-related decays. In particular, by modifying the SD prescription for the amplitudes of $B^+ \to \pi^+K^0$, $B_d^0 \to \pi^-K^+$, and $B_s^0 \to \pi^+K^-$ decays, FSI may affect the method of determining the CP-violating angle γ, which uses the corresponding decay rates as input. Deviation from equality $A_0 = -A_s$ may indicate SU(3) breaking induced by SU(3)-symmetric FSI effects.

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Appendix

A.1 Mesons

$$
\begin{array}{lll}
\pi^+ = -u\bar{d} & K^+ = u\bar{s} & B^+ = u\bar{b} \\
\pi^0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}} & K^0 = d\bar{s} & B_d^0 = d\bar{b} \\
\pi^- = d\bar{u} & K^- = s\bar{u} & B_s^0 = s\bar{b} \\
\eta_8 = \frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}} & \bar{K}^0 = -s\bar{d} & \\
\eta_1 = \frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}\,. & &
\end{array}
$$

Analogous conventions hold for vector- and other mesons. In the following we denote $K = (K^+, K^0)$, $\bar{K} = (\bar{K}^0, K^-)$.

A.2 Two-meson PP states

Two-meson PP states of definite isospin I are denoted as $(ab)_I$. Since the charge of state $(ab)_I$ must correspond to the charge of the decaying B-meson, the value of charge is suppressed whenever this does not lead to ambiguity. States $(ab)_I$ with mesons a and b in definite charge states are defined according to the following example for charge $Q=+1$:

$$(\pi\pi)_2 = +\{\pi^+\pi^0\}\,, \tag{51}$$

where $\{a^{q_1}b^{q_2}\}$ denotes a properly symmetrised state, *i.e.* $\{a^{q_1}b^{q_2}\} = (a^{q_1}b^{q_2}+b^{q_2}a^{q_1})/\sqrt{2}$. If $b^{q_2}=a^{q_1}$, $\{a^q a^q\}=a^q a^q$. (All relations of type (51) have a positive sign on the right-hand side). States in which mesons a and b are not in definite charge states are represented as linear combinations of states with definite charges of mesons a and b. All relevant states of given charge, strangeness and definite isospin are listed below.

a) Strangeness $S=0$, charges $Q=+1,0$

$$\begin{aligned}
(\pi\pi)_2 &= \begin{cases} \{\pi^+\pi^0\} & \text{if } Q=+1 \\ \frac{\{\pi^+\pi^-\}+\sqrt{2}\{\pi^0\pi^0\}}{\sqrt{3}} & \text{if } Q=0 \end{cases}, \\
(K\bar{K})_1 &= \begin{cases} \{K^+\bar{K}^0\} & \text{if } Q=+1 \\ \frac{\{K^+K^-\}+\{K^0\bar{K}^0\}}{\sqrt{2}} & \text{if } Q=0 \end{cases}, \\
&\left.\begin{matrix}(\pi\eta_8)_1 \\ (\pi\eta_1)_1\end{matrix}\right\} \quad \text{both charges}\,, \\
(\pi\pi)_0 &= \frac{\sqrt{2}\{\pi^+\pi^-\}-\{\pi^0\pi^0\}}{\sqrt{3}}\,, \\
(K\bar{K})_0 &= \frac{\{K^+K^-\}-\{K^0\bar{K}^0\}}{\sqrt{2}}\,, \\
&(\eta_8\eta_8)_0\,, \\
&(\eta_1\eta_1)_0\,, \\
&(\eta_8\eta_1)_0\,.
\end{aligned} \tag{52}$$

b) Strangeness $S=+1$, charges $Q=+1,0$

$$\begin{aligned}
(\pi K)_{3/2} &= \begin{cases} \sqrt{\frac{2}{3}}\{\pi^0K^+\}+\frac{1}{\sqrt{3}}\{\pi^+K^0\} & \text{if } Q=+1 \\ \frac{1}{\sqrt{3}}\{\pi^-K^+\}+\sqrt{\frac{2}{3}}\{\pi^0K^0\} & \text{if } Q=0 \end{cases}, \\
(\pi K)_{1/2} &= \begin{cases} \frac{1}{\sqrt{3}}\{\pi^0K^+\}-\sqrt{\frac{2}{3}}\{\pi^+K^0\} & \text{if } Q=+1 \\ \sqrt{\frac{2}{3}}\{\pi^-K^+\}-\frac{1}{\sqrt{3}}\{\pi^0K^0\} & \text{if } Q=0 \end{cases},
\end{aligned}$$

$$\left.\begin{array}{l}(\eta_8 K)_{1/2}\\(\eta_1 K)_{1/2}\end{array}\right\}\quad \text{both charges}\,. \tag{53}$$

c) Strangeness $S=-1$, charge $Q=0$

$$\begin{aligned}(\pi\bar{K})_{3/2} &= \frac{1}{\sqrt{3}}\{\pi^+K^-\}+\sqrt{\frac{2}{3}}\{\pi^0\bar{K}^0\}\,,\\ (\pi\bar{K})_{1/2} &= \sqrt{\frac{2}{3}}\{\pi^+K^-\}-\frac{1}{\sqrt{3}}\{\pi^0\bar{K}^0\}\,,\\ &(\eta_8\bar{K})_{1/2}\,,\\ &(\eta_1\bar{K})_{1/2}\,.\end{aligned} \tag{54}$$

A.3 States in definite SU(3) representations

Notation used: (**SU(3) multiplet**, isospin)

a) Strangeness $S=0$

Isospin 2, charges $Q=+1,0$

$$(\mathbf{27},2) = (\pi\pi)_2\,. \tag{55}$$

Isospin 1, charges $Q=+1,0$

$$\begin{aligned}\begin{bmatrix}(\mathbf{27},1)\\(\mathbf{8},1)\end{bmatrix} &= \frac{1}{\sqrt{5}}\begin{bmatrix}\sqrt{3} & -\sqrt{2}\\ \sqrt{2} & \sqrt{3}\end{bmatrix}0\begin{bmatrix}(\pi\eta_8)_1\\(K\bar{K})_1\end{bmatrix},\\ (\mathbf{8}',1) &= (\pi\eta_1)_1\,.\end{aligned} \tag{56}$$

Isospin 0, charge $Q=0$

$$\begin{aligned}\begin{bmatrix}(\mathbf{27},0)\\(\mathbf{8},0)\\(\mathbf{1},0)\end{bmatrix} &= \begin{bmatrix}\frac{1}{2\sqrt{10}} & \sqrt{\frac{3}{10}} & -\frac{3\sqrt{3}}{2\sqrt{10}}\\ \sqrt{\frac{3}{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}}\end{bmatrix}\begin{bmatrix}(\pi\pi)_0\\(K\bar{K})_0\\(\eta_8\eta_8)_0\end{bmatrix},\\ (\mathbf{1}',0) &= (\eta_1\eta_1)_0\,.\end{aligned} \tag{57}$$

b) Strangeness $S=+1$, charges $Q=+1,0$

$$\begin{aligned}(\mathbf{27},3/2) &= (\pi K)_{3/2}\,,\\ \begin{bmatrix}(\mathbf{27},1/2)\\(\mathbf{8},1/2)\end{bmatrix} &= \frac{1}{\sqrt{10}}\begin{bmatrix}1 & 3\\ 3 & -1\end{bmatrix}\begin{bmatrix}(\pi K)_{1/2}\\(\eta_8 K)_{1/2}\end{bmatrix},\\ (\mathbf{8}',1/2) &= (\eta_1 K)_{1/2}\,.\end{aligned} \tag{58}$$

c) Strangeness $S=-1$, charge $Q=0$

$$\begin{aligned}
(\mathbf{27},3/2) &= (\pi\bar{K})_{3/2}\,,\\
\begin{bmatrix} (\mathbf{27},1/2) \\ (\mathbf{8},1/2) \end{bmatrix} &= \frac{1}{\sqrt{10}}\begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} (\pi\bar{K})_{1/2} \\ (\eta_8\bar{K})_{1/2} \end{bmatrix},\\
(\mathbf{8}',1/2) &= (\eta_1\bar{K})_{1/2}\,.
\end{aligned} \tag{59}$$

A.4 Two-meson "PV" states in definite SU(3) representations

The labels P and V (π, ρ *etc.*) denote two different types of resonances of appropriate flavour.

A.4.1 Intermediate states in B^+ decays

a) Strangeness $S=0$

$$\begin{aligned}
(\mathbf{27},2) &= \frac{\pi^+\rho^0+\pi^0\rho^+}{\sqrt{2}}\,,\\
\begin{bmatrix} (\mathbf{27},1) \\ (\mathbf{8},1)_s \end{bmatrix} &= \frac{1}{\sqrt{5}}\begin{bmatrix} \sqrt{3} & -\sqrt{2} \\ \sqrt{2} & \sqrt{3} \end{bmatrix}\begin{bmatrix} \frac{\pi^+\omega_8+\eta_8\rho^+}{\sqrt{2}} \\ \frac{K^+\bar{K}^{*0}+\bar{K}^0K^{*+}}{\sqrt{2}} \end{bmatrix},\\
(\mathbf{8},1)_a &= \frac{\sqrt{2}(\pi^+\rho^0-\pi^0\rho^+)-(K^+\bar{K}^{*0}-\bar{K}^0K^{*+})}{\sqrt{6}},\\
(\mathbf{8},1)_{81} &= \pi^+\omega_1\,,\\
(\mathbf{8},1)_{18} &= \eta_1\rho^+\,.
\end{aligned} \tag{60}$$

b) Strangeness $S=+1$

$$\begin{aligned}
(\mathbf{27},3/2) &= \frac{K^0\rho^++\pi^+K^{*0}+\sqrt{2}(K^+\rho^0+\pi^0K^{*+})}{\sqrt{6}},\\
\begin{bmatrix} (\mathbf{27},1/2) \\ (\mathbf{8},1/2)_s \end{bmatrix} &= \frac{1}{\sqrt{10}}\begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} \frac{K^+\rho^0+\pi^0K^{*+}-\sqrt{2}(K^0\rho^++\pi^+K^{*0})}{\sqrt{6}} \\ \frac{K^+\omega_8+\eta_8K^{*+}}{\sqrt{2}} \end{bmatrix},\\
(\mathbf{8},1/2)_a &= -\frac{1}{2\sqrt{3}}(\pi^0K^{*+}-K^+\rho^0)\\
&\quad +\frac{1}{\sqrt{6}}(\pi^+K^{*0}-K^0\rho^+)-\frac{1}{2}(\eta_8K^{*+}-K^+\omega_8),\\
(\mathbf{8},1/2)_{81} &= K^+\omega_1\,,\\
(\mathbf{8},1/2)_{18} &= \eta_1K^{*+}\,.
\end{aligned} \tag{61}$$

A.4.2 Intermediate states in B_d^0, B_s^0 decays

a) Strangeness $S=0$

$$
\begin{aligned}
(\mathbf{27},2) &= \frac{\pi^+\rho^- + \pi^-\rho^+ + 2\pi^0\rho^0}{\sqrt{6}}\,,\\
\left[\begin{array}{c}(\mathbf{27},1)\\(\mathbf{8},1)_s\end{array}\right] &= \frac{1}{\sqrt{5}}\left[\begin{array}{cc}\sqrt{3} & -\sqrt{2}\\ \sqrt{2} & \sqrt{3}\end{array}\right]\left[\begin{array}{c}\frac{\pi^0\omega_8+\eta_8\rho^0}{\sqrt{2}}\\ \frac{K^0\bar{K}^{*0}+K^+K^{*-}+\bar{K}^0K^{*0}+K^-K^{*+}}{2}\end{array}\right],\\
\left[\begin{array}{c}(\mathbf{27},0)\\(\mathbf{8},0)_s\\(\mathbf{1},0)\end{array}\right] &= \left[\begin{array}{ccc}\frac{1}{2\sqrt{10}} & \sqrt{\frac{3}{10}} & -\frac{3\sqrt{3}}{2\sqrt{10}}\\ \sqrt{\frac{3}{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ -\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}}\end{array}\right]\left[\begin{array}{c}\frac{\pi^+\rho^-+\pi^-\rho^+-\pi^0\rho^0}{\sqrt{3}}\\ \frac{K^+K^{*-}+K^-K^{*+}-K^0\bar{K}^{*0}-\bar{K}^0K^{*0}}{2}\\ \eta_8\omega_8\end{array}\right],\\
(\mathbf{8},1)_a &= \frac{2(\pi^+\rho^- - \pi^-\rho^+) - (K^+K^{*-}+K^0\bar{K}^{*0}-K^-\bar{K}^{*+}-\bar{K}^0K^{*0})}{2\sqrt{3}},\\
(\mathbf{8},0)_a &= \frac{K^+K^{*-} - K^0\bar{K}^{*0} - K^-K^{*+} + \bar{K}^0K^{*0}}{2}\,,\\
(\mathbf{8},1)_{81} &= \pi^0\omega_1\,,\\
(\mathbf{8},0)_{81} &= -\eta_8\omega_1\,,\\
(\mathbf{8},1)_{18} &= \eta_1\rho_0\,,\\
(\mathbf{8},0)_{18} &= -\eta_1\omega_8\,,\\
(\mathbf{1},0)_{11} &= \eta_1\omega_1\,.
\end{aligned}
\tag{62}
$$

b) Strangeness $S=+1$

$$
\begin{aligned}
(\mathbf{27},3/2) &= \frac{K^+\rho^- + \pi^-K^{*+} + \sqrt{2}(K^0\rho^0 + \pi^0K^{*0})}{\sqrt{6}}\,,\\
\left[\begin{array}{c}(\mathbf{27},1/2)\\(\mathbf{8},1/2)_s\end{array}\right] &= \frac{1}{\sqrt{10}}\left[\begin{array}{cc}1 & 3\\ 3 & -1\end{array}\right]\left[\begin{array}{c}\frac{\sqrt{2}(\pi^-K^{*+}+K^+\rho^-)-(\pi^0K^{*0}+K^0\rho^0)}{\sqrt{6}}\\ \frac{\eta_8K^{*0}+K^0\omega_8}{\sqrt{2}}\end{array}\right],\\
(\mathbf{8},1/2)_a &= \frac{\sqrt{2}(K^+\rho^- - \pi^-K^{*+}) - (K^0\rho^0 - \pi^0K^{*0})}{\sqrt{12}} + \frac{K^0\omega_8 - \eta_8K^{*0}}{2},\\
(\mathbf{8},1/2)_{81} &= K^0\omega_1\,,\\
(\mathbf{8},1/2)_{18} &= \eta_1K^{*0}\,.
\end{aligned}
\tag{63}
$$

c) Strangeness $S=-1$

$$
\begin{aligned}
(\mathbf{27},3/2) &= \frac{\pi^+K^{*-}+K^-\rho^+ +\sqrt{2}(\pi^0K^{*0}+\bar{K}^0\rho^0)}{\sqrt{6}}, \\
\begin{bmatrix} (\mathbf{27},1/2) \\ (\mathbf{8},1/2)_s \end{bmatrix} &= \frac{1}{\sqrt{10}}\begin{bmatrix} 1 & 3 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} \frac{\sqrt{2}(\pi^+K^{*-}+K^-\rho^+)-(\pi^0\bar{K}^{*0}+\bar{K}^0\rho^0)}{\sqrt{6}} \\ \frac{\eta_8\bar{K}^{*0}+\bar{K}^0\omega_8}{\sqrt{2}} \end{bmatrix}, \\
(\mathbf{8},1/2)_a &= \frac{\sqrt{2}(\pi^+K^{*-}-K^-\rho^+)-(\pi^0\bar{K}^{*0}-\bar{K}^0\rho^0)}{\sqrt{12}}+\frac{\eta_8\bar{K}^{*0}-\bar{K}^0\omega_8}{2}, \\
(\mathbf{8},1/2)_{81} &= \bar{K}^0\omega_1\,, \\
(\mathbf{8},1/2)_{18} &= \eta_1\bar{K}^{*0}\,. \qquad (64)
\end{aligned}
$$

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