NEUTRINO MASSES: HIERARCHY WITHOUT HIERARCHY*

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A large hierarchy of the Dirac masses can result in a small hierarchy for the low energy masses of the active neutrinos. This can happen even if the Majorana masses of right-handed neutrinos are all equal. A realistic description of the observed neutrino masses and mixing can be obtained starting from a large hierarchy in the Dirac masses. A large mixing for solar neutrinos results from the neutrino sector. The small value of the MNS matrix element U_{e3} is a natural consequence of the scheme. The masses of the two lighter neutrinos are related to the solar neutrino mixing angle: $\mu_1/\mu_2 = \tan^2\theta_\odot$.

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1. Is there any mass hierarchy for active neutrinos?

Let us start with the remark that the active neutrinos are exactly these particles which experimentalists are studying. They couple to W and Z bosons. There are three of them known as ν_e, ν_μ and ν_τ, and they have very small masses which are reflected in mass scales governing neutrino oscillations. In the oscillations of the solar and atmospheric neutrinos only differences [1–3]

$$\Delta m^2_\odot = |\mu_2^2 - \mu_1^2| \sim 5.0 \times 10^{-5}\mathrm{eV}^2 \quad (1)$$

$$\Delta m^2_@ = |\mu_3^2 - \mu_2^2| \sim 2.5 \times 10^{-3}\mathrm{eV}^2 \quad (2)$$

* Dedicated to Stefan Pokorski on his 60th birthday.

can be measured. The ratio of these two mass scales

$$\rho_{\rm exp}^{-1} = \Delta m^2_{@}/\Delta m^2_{\odot} \sim 50 \tag{3}$$

seems to provide a clear answer to the question asked in the title of this section. Apparently yes. There is a hierarchy. However the correct answer may be more subtle. Let us compare what the Nature tells us through Eq. (3) with expectations based on a theory. The best theory of neutrino masses we know is the see-saw mechanism [4]. It explains why the masses of the active neutrinos are much smaller than the masses of all other fundamental fermions, *i.e.* charged leptons and quarks. The see-saw mechanism implies that the masses of the active neutrinos are composite low energy objects derived from more fundamental mass parameters. These more fundamental masses are the Dirac masses describing couplings between left-handed and right-handed neutrinos. and the Majorana masses of the right-handed neutrinos. The right-handed neutrinos are singlets of the standard model $\mathrm{SU}_3 \times \mathrm{SU}_2 \times \mathrm{U}_1$ local gauge symmetry, so their Majorana masses are not forbidden by gauge invariance. Majorana masses are not allowed for particles with non-zero electric charge. So, the masses of charged leptons and quarks are all of the Dirac type and they all exhibit a clear hierarchy

$$\begin{aligned} m_e &\ll m_\mu \ll m_\tau \\ m_u &\ll m_c \ll m_t \\ m_d &\ll m_s \ll m_b\,. \end{aligned} \tag{4}$$

If we assume that this hierarchical structure is a common feature of all fundamental fermions, the Dirac masses of neutrinos should be also hierarchical, *i.e.*

$$m_1 \ll m_2 \ll m_3\,. \tag{5}$$

We still have to say something about the Majorana masses of the right-handed neutrinos. The most natural thing is to assume that they are all equal. So let us assume that there are three right-handed neutrinos and their Majorana masses are equal to M:

$$M_{\rm R} = M\mathbf{1}\,. \tag{6}$$

Then the following sequence can be derived for the masses of three active neutrinos:

$$\mu_1 = \frac{m_1^2}{M}\,, \qquad \mu_2 = \frac{m_2^2}{M}\,, \qquad \mu_3 = \frac{m_3^2}{M}\,. \tag{7}$$

If $m_3/m_2 \sim m_t/m_c \sim 10^2$ is assumed, as suggested by many grand unified models, the ratio

$$\rho_{\rm th}^{-1} \approx \frac{\mu_3^2}{\mu_2^2} = \left(\frac{m_3}{m_2}\right)^4 \sim 10^8 \tag{8}$$

is obtained. When viewed from this perspective the hierarchy exhibited in Eq. (3) can be called a moderate one at best. It is much more appropriate in fact to consider this small hierarchy as a small perturbation of the situation without hierarchy.

2. Reducing hierarchy

Are we then forced to abandon the assumed hierarchy (5) of the Dirac masses or the nice and economic postulate (6) of equal Majorana masses? Let us repeat the standard derivation of the mass formula for the active neutrinos. Our guiding principle is to reduce the resulting hierarchy as much as possible. The Dirac masses of neutrinos are described by a 3×3 matrix

$$N = U_{\mathrm{R}} m^{(\nu)} U_{\mathrm{L}} \tag{9}$$

with

$$m^{(\nu)} = \mathrm{diag}\,(m_1, m_2, m_3)\ . \tag{10}$$

As an unitary matrix U_{L} cannot affect the resulting mass spectrum, we assume

$$U_{\mathrm{L}} = \mathbf{1} \tag{11}$$

for simplicity. We may be led to reconsidering this when discussing the lepton mixing matrix.

The mass spectrum of the active neutrinos is given by a dimension five operator $\mathcal{N}$. This operator is obtained as a low energy approximation of a term resulting from the underlying renormalizable theory in the next-to-leading order. The result is

$$\mathcal{N} = N^{\mathrm{T}} M_{\mathrm{R}}^{-1} N = \frac{1}{M} {m^{(\nu)}}^{\mathrm{T}} U_{\mathrm{R}}^{\mathrm{T}} U_{\mathrm{R}} m^{(\nu)}\ . \tag{12}$$

As the Majorana mass M in (12) is huge the resulting masses of the active neutrinos are small. The spectrum is extremely sensitive to the form of a symmetric unitary matrix

$$R = U_{\mathrm{R}}^{\mathrm{T}} U_{\mathrm{R}} \tag{13}$$

so, the matrix U_{R} plays a very important role in low energy physics and its structure is imprinted in the masses of the active neutrinos[1]. Unfortunately this mass spectrum is the only piece of information on U_{R} accessible at our low energies. So we have to guess some form of R and hope that the results obtained may to some extend justify our cavalier attempt. $R = \mathbf{1}$ is not

[1] It is interesting to note that the analogous matrices for up and down quarks play no role in low energy physics because they neither affect the spectra of Dirac masses nor the electroweak charged currents.

acceptable because this would lead us directly to the disastrous spectrum (7). Let us follow our guiding principle and try to reduce the hierarchy of the resulting spectrum as much as possible. Certainly

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = P_{13} \tag{14}$$

seems to be a good candidate to achieve this goal. From (12)–(14)

$$\mathcal{N} = \mu \begin{pmatrix} 0 & 0 & r \\ 0 & 1 & 0 \\ r & 0 & 0 \end{pmatrix} \tag{15}$$

is obtained with $r = m_1 m_3 / m_2^2$ and $\mu = m_2^2 / M$. There is a doublet (ν_1, ν_2) of mass μr and a singlet μ_3 of mass μ in the spectrum resulting from (15). When this spectrum is compared with those in (7) the reduction of hierarchy becomes evident. One may remark that this success is rather problematic. If we want to interpret the mass splitting between singlet and doublet as the origin of $\Delta m^2_{@}$ then $\Delta m^2_{\odot}$ is zero pushing our ρ_{th}^{-1} to infinity, which seems to be even worse than (8). We shall ignore this problem for a while. It can be solved by introducing a small off-diagonal element in (10) removing mass degeneracy for ν_1 and ν_2 and leading to non-zero $\Delta m^2_{\odot}$. These considerations dictate ordering of eigenvalues after diagonalization of $\mathcal{N}$ which partly fixes the form of a unitary matrix O' such that

$$O'^{\mathrm{T}} \mathcal{N} O' = \mathrm{diag}\,(\mu_1, \mu_2, \mu_3) \tag{16}$$

with $\mu_1 = \mu_2 = \mu r$, $\mu_3 = \mu$. The remaining freedom will be removed completely by the perturbation splitting the masses of ν_1 and ν_2. The result is

$$O' = P_{23} U_{12}\,(\pm\pi/4) \tag{17}$$

with

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{18}$$

$$U_{12}\,(\pm\pi/4) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \mp\frac{i}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \pm\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{19}$$

3. Lepton mixing matrix

The Maki–Nakagawa–Sakata lepton mixing matrix [5] can be expressed in terms of O' and $V_{\rm L}$, where $V_{\rm L}$ is a unitary matrix diagonalizing $L^\dagger L$ and L is the charged lepton mass matrix: $V_{\rm L} L^\dagger L V_{\rm L}^\dagger = {\rm diag}\left(m_e^2, m_\mu^2, m_\tau^2\right)$. Then

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U_{\rm MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{20}$$

and

$$U_{\rm MNS} = V_{\rm L} O' = V_{\rm L} P_{23} U_{12}\left(\pm\pi/4\right) . \tag{21}$$

The structure in Eq. (21) is striking. If $V_{\rm L}$ is a matrix with the element $(V)_{11} = 1$ and other non-zero elements in the 2-3 block the product $V_{\rm L} P_{23}$ has the very same structure[2]. Moreover, it is exactly this form of $V_{\rm L}$ that can account for the mixing of atmospheric neutrinos. Many authors considered lepton sector as the origin of maximal mixing for atmospheric neutrinos; see [6] and references therein. Particularly attractive models are based on lopsided mass matrices [6,7]. So we do not spend more time on that problem because up to some irrelevant redefinitions[3]

$$V_{\rm L} P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{22}$$

can be obtained following arguments of those papers. What we get from (21) and (22) is known as the bi-maximal mixing [8]. It was a lot of fun to get this structure four years ago. However, now the bi-maximal mixing is without any doubt excluded by the experimental data [2]. Is this a problem for the present scheme? Not really. The same perturbation which splits the masses of ν_1 and ν_2 can push the solar mixing angle $\theta_\odot$ away from $\pm\pi/4$ in U_{12}. A perturbation can be found producing $\Delta m^2_\odot$ and $\tan^2\theta_\odot$ in agreement with experiment [9].

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[2] In general the product $V_{\rm L} P_{23}$ is obtained from $V_{\rm L}$ by exchanging its second and third column. It may be considered as an efficient way to ruin predictions associated with some special forms of $V_{\rm L}$.

[3] Throughout this paper we ignore complex phases which are not important for oscillations. Of course, these phases are of crucial importance for $0\nu 2\beta$ transitions.

4. Can we test this picture?

The picture which we obtain is quite encouraging. Up to small corrections the lepton mixing matrix can be written as

$$U_{\mathrm{MNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{@} & \sin\theta_{@} \\ 0 & -\sin\theta_{@} & \cos\theta_{@} \end{pmatrix} \begin{pmatrix} \cos\theta_{\odot} & \sin\theta_{\odot} & 0 \\ -\sin\theta_{\odot} & \cos\theta_{\odot} & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{23}$$

This form explains the smallness of U_{e3} in agreement with CHOOZ limit [10] and SuperKamiokande data on atmospheric ν_e's [1, 3]. Moreover, if the present picture is correct there is a relation between μ_1, μ_2 and $\tan^2\theta_{\odot}$. Let us consider the 1-2 block of $P_{23}\mathcal{N}P_{23}$. For small off-diagonal elements in $m^{(\nu)}$, *cf.* Eq. (10) this 2×2 matrix is proportional to

$$\begin{pmatrix} * & 1 \\ 1 & a \end{pmatrix}$$

with $|a| < 1$ and the element 1-1 small as a consequence of mass hierarchy in $m^{(\nu)}$. Diagonalization of this sub-matrix produces a unitary transformation in the 1-2 plane which is reflected in U_{MNS}, see Eq. (23). Thus the masses of ν_1 and ν_2 are related to $\tan^2\theta_{\odot}$:

$$\frac{\mu_1}{\mu_2} \approx \tan^2\theta_{\odot} . \tag{24}$$

As a final remark let us note that the mass scale of the Majorana masses is between 10^{10} and 10^{11} GeV if $m_2 \sim m_c$ is assumed. This range of Majorana masses may be quite interesting for baryogenesis; see [11] and references therein.

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