

ON CASIMIR ENERGY CONTRIBUTION TO OBSERVABLE VALUE OF THE COSMOLOGICAL CONSTANT

IGOR O. CHEREDNIKOV

Bogolyubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Russia
Institute for Theoretical Problems of Microphysics
Moscow State University
119899 Moscow, Russia
e-mail: igorch@thsun1.jinr.ru

(Received February 12, 2002)

The contribution of the ground state energy of quantum fields to the cosmological constant is estimated from the point of view of the standard Casimir energy calculation scheme. It is shown that the requirement of the renormalization group invariance leads to the value of the effective Λ -term which is of 11 orders higher than the result extracted from the experimental data.

PACS numbers: 98.80.Es, 11.10.Gh

1. Introduction

One of the most intriguing and challenging problems of the modern physics is the enormously large difference between the experimentally extracted cosmological constant and its value estimated within the convenient quantum field theory [1]. This is so-called old cosmological constant problem, while the new one is why it is comparable to the present mass density [2, 3]. Here we will study only the old one. This problem attracts a great interest of the physical community (just mention that the SPIRES-SLAC database gives more than 800 references for “cosmological constant”), and a lot of sophisticated approaches have been proposed to its solution [1, 4]. However, the present situation seems to be far from satisfactory.

The Einstein’s equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = g_{\mu\nu}\Lambda - 8\pi GT_{\mu\nu} , \quad (1)$$

(1973)

where G is the gravitational constant and other symbols are standard [1], it contains the classical Λ -term as well as the contribution of the vacuum energy density due to the quantum fluctuations, which as shown by Zeldovich [6], have the same structure: $\langle T_{\mu\nu} \rangle = -g_{\mu\nu} \langle \rho \rangle$. It means that (1) can be written in the form:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = g_{\mu\nu}\Lambda_{\text{eff}}, \quad (2)$$

where

$$\Lambda_{\text{eff}} = 8\pi G\rho_{\text{eff}} = \Lambda + 8\pi G\langle \rho \rangle. \quad (3)$$

It is known from the experimental data that the effective energy density of the Universe ρ_{eff} defined in (3), is of order 10^{-47} GeV^4 [5]. In the same time, the directly evaluated vacuum energy with the UV cutoff at the Planck scale $M = (8\pi)^{-1/2}m_{\text{P}}$, $m_{\text{P}} \approx 1.2210(9) \cdot 10^{19} \text{ GeV}$ [7] reads

$$\langle E_{\text{vac}} \rangle = \frac{1}{2} \int_0^M \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2} \approx \frac{M^4}{16\pi^2}, \quad (4)$$

which is about 10^{71} GeV^4 , *i.e.*, 118 orders higher. Even if one takes the cutoff at the supersymmetry breaking scale $\rho_{\text{eff}}^{\text{susy}} \sim 10^{12} \text{ GeV}^4$, the discrepancy will remain to be of 59 orders [1, 3].

In the present paper, we propose to calculate the vacuum energy $\langle \rho \rangle$ within the framework of the standard Casimir energy computations for various geometrical configurations with quantized fields under boundary conditions. This approach allows one to obtain the finite value for the Casimir energy by means of absorption of the singularities into the definitions of the corresponding classical contact terms which characterize the total energy of the analogous classical configuration [8]. In our case, the divergences will be absorbed into the definition of the single “classical” parameter $\Lambda \rightarrow \Lambda_0$ which is treated, therefore, as a “bare” constant from the beginning.

2. Renormalization of the Casimir energy contribution

Let us consider the “toy” Universe filled with the free neutrino field with the mass m_ν of order 10^{-9} GeV . The crucial role of light neutrinos in generating the small non-vanishing value of the cosmological term has been proposed and discussed in [9] for the case of Spontaneous Symmetry Breaking (SSB). The total energy of the vacuum fluctuations of this field reads

$$E_\nu = - \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_\nu^2}. \quad (5)$$

This integral diverges and then must be regularized before any calculation will be done. We use the ζ -regularization which seems to be one of the most

convenient for this situation. Besides this, we need to introduce the additional mass parameter μ (such an arbitrary mass scale emerges unavoidably in any regularization scheme) in order to restore the correct dimension for the regularized quantities. Then we have

$$\langle \rho \rangle = E_\nu \rightarrow E_\nu(\varepsilon) = -\mu^{2\varepsilon} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2 + m_\nu^2)^{\varepsilon - \frac{1}{2}}}. \quad (6)$$

The regularization will be removed by taking the limit $\varepsilon \rightarrow 0$. After the simple calculations, we get

$$E_\nu(\varepsilon) = -\frac{m_\nu^4}{8\pi^{3/2}} \left(\frac{\mu^2}{m_\nu^2} \right)^\varepsilon \frac{\Gamma(\varepsilon - 2)}{\Gamma(\varepsilon - \frac{1}{2})}. \quad (7)$$

Taking into account the well-known relations for the Γ -function [10]

$$\Gamma(\varepsilon - 2) = \frac{\Gamma(1 + \varepsilon)}{\varepsilon(\varepsilon - 1)(\varepsilon - 2)}, \quad \Gamma(\varepsilon - 1/2) = \frac{\Gamma(\frac{1}{2} + \varepsilon)}{\varepsilon - \frac{1}{2}}, \quad (8)$$

and the expansions for small ε

$$\begin{aligned} \Gamma(1 + \varepsilon) &= 1 - \gamma_E \varepsilon + O(\varepsilon^2), \quad x^\varepsilon = 1 + \varepsilon \ln x + O(\varepsilon^2), \\ \Gamma\left(\frac{1}{2} + \varepsilon\right) &= \Gamma\left(\frac{1}{2}\right) - \varepsilon \Gamma\left(\frac{1}{2}\right) (\gamma_E + 2 \ln 2) + O(\varepsilon^2), \end{aligned} \quad (9)$$

we find:

$$E_\nu(\varepsilon) = \frac{m_\nu^4}{32\pi^2} \left(\frac{1}{\varepsilon} + 2 \ln 2 - \frac{1}{2} \right) \ln \left(\frac{\mu^2}{m_\nu^2} \right) + O(\varepsilon). \quad (10)$$

The part singular in the limit $\varepsilon \rightarrow 0$ can be extracted as

$$E_{\text{div}}(\varepsilon) = \frac{1}{\varepsilon} \frac{m_\nu^4}{32\pi^2}. \quad (11)$$

The absence of the leading divergence scaled as m_P^4 is the generic feature of the ζ -function regularization method, and is well-known in the Casimir energy calculations [8, 11, 13]. Thus, the renormalization is performed via the absorption of this singularity into the re-definition of the bare classical constant Λ :

$$\Lambda \rightarrow \Lambda_0 - \frac{1}{\varepsilon} \frac{m_\nu^4}{32\pi^2}. \quad (12)$$

Therefore, the remaining finite value for the effective energy density reads

$$\rho_{\text{eff}} = \frac{\Lambda_0}{8\pi G} + \frac{m_\nu^4}{32\pi^2} \left(\ln \left(\frac{\mu^2}{m_\nu^2} \right) + 2 \ln 2 - \frac{1}{2} \right). \quad (13)$$

This quantity depends on the arbitrary mass scale μ . It is natural to demand it to be unchanged under any variations of this parameter. The role of such a condition in the Casimir energy calculations have been studied in [8,13], and investigated in detail in [9,12]. This requirement leads to the renormalization group equation

$$\mu \frac{d}{d\mu} \rho_{\text{eff}} = \frac{1}{8\pi G} \mu \frac{\partial}{\partial \mu} \Lambda_0(\mu) + \frac{m_\nu^4}{16\pi^2} = 0. \quad (14)$$

Solving it we find that the renormalized constant Λ_0 should be treated as a “running” one in the sense that it varies provided that the scale μ is changing:

$$\Lambda_0(\mu) = -\frac{Gm_\nu^4}{2\pi} \ln \frac{\mu}{\mu_0}, \quad (15)$$

where μ_0 can be called the normalization point, determined by the condition

$$\Lambda_0(\mu_0) = 0. \quad (16)$$

Substituting (15) into (13) we find

$$\rho_{\text{eff}} = \frac{m_\nu^4}{16\pi^2} \left(\ln \frac{\mu_0}{m_\nu} + 2 \ln 2 - \frac{1}{2} \right). \quad (17)$$

If we assume that the normalization point μ_0 coincides with the Planck scale, *i.e.*, $\mu_0 = m_{\text{P}} \sim 10^{19}$ GeV, and take the light neutrino mass to be $m_\nu \sim 10^{-9}$ GeV, the estimate for the total effective renormalized cosmological constant will read:

$$\rho_{\text{eff}} \sim 10^{-36} \text{ GeV}^4. \quad (18)$$

3. Conclusion

We see that by virtue of the normalization condition (16), one obtains now the model of the Universe with $\Lambda_0 = 0$ at the energy scale compared to that in the first moments of its existence. The other possible normalization — $\Lambda_0 = 0$ in the very far IR region is used and discussed in [9].

This value (18) is still far from the experimentally observed one, but is much better than a straightforward evaluation of the ground state energy of quantum field based on the direct UV cutoff. It should be mentioned that this result is close in order to the one obtained by Zeldovich ($\rho_{\text{Zel}} \sim 10^{-38} \text{ GeV}^4$) by means of the quite different considerations [14].

One can see that the value of the cosmological constant depends crucially from the chosen mass of the elementary fermion m_ν and, in contrast, the

dependence from the normalization point μ_0 is only logarithmic and may be neglected, in contrast to the direct evaluation based on the UV cutoff of the high frequency contributions of the quantum field fluctuations.

The author thanks Prof. I. L. Shapiro for drawing attention to his recent papers on the similar topics, and Dr. H. Stefancic for the discussion. This work is partially supported by RFBR under grants nos. 01-02-16431 and 00-15-96577, and INTAS under grant no. 00-00-366. The warm hospitality of the Abdus Salam ICTP in Trieste during October–December 2001 is also acknowledged.

REFERENCES

- [1] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
- [2] S. Weinberg, [astro-ph/0005265](#).
- [3] A. Vilenkin, [hep-th/0106083](#).
- [4] V. Sahni, A. Starobinsky, *Int. J. Mod. Phys.* **D9**, 373 (2000).
- [5] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).
- [6] Ya.B. Zeldovich, *Sov. Phys. Uspekhi* **11**, 381 (1968).
- [7] Particle Data Group, *Eur. Phys. J.* **C15**, 1 (2000).
- [8] S.K. Blau, M. Visser, A. Wipf, *Nucl. Phys.* **B310**, 163 (1988); E. Elizalde, M. Bordag, K. Kirsten, *J. Phys. A* **31**, 1743 (1998).
- [9] I.L. Shapiro, J. Sola, *Phys. Lett.* **B475**, 236 (2000); *J. High Energy Phys.* 0202 (2002) 006, [hep-th/0012227](#).
- [10] I.S. Gradshteyn, I.M. Ryzhik, *Tables of Integrals, Series, and Products*, Acad. Press, 1980.
- [11] M. Bordag, U. Mohideen, V.M. Mostepanenko, *Phys. Rep.* **353**, 1 (2001).
- [12] A. Babic, B. Guberina, R. Horvat, H. Stefancic, [hep-ph/0111207](#).
- [13] I. Brevik, L.N. Granda, S.O. Odintsov, *Phys. Lett.* **B367**, 206 (1996); I. Cherednikov, *Phys. Lett.* **B498**, 40 (2001); I. Cherednikov, *Int. J. Mod. Phys.* **A17** (2002), in print; [hep-th/0111037](#).
- [14] Ya.B. Zeldovich, *JETP Lett.* **6**, 316 (1967).