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# GRAPHICAL EXAMPLES OF GEOMETRICAL AND WAVE OPTICS*

Władysław Żakowicz

Institute of Physics, Polish Academy of Sciences
Lotników 32/46, 02-668 Warsaw, Poland
e-mail: `zakow@ifpan.edu.pl`

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A simple method of the description of optical pulses of finite dimensions and finite duration time is presented. The pulses are given by superpositions of the known analytical as well as numerical solutions of the Maxwell equations. The examples show optical properties of scattering in detail. In the general case the wide pulses split in the scattering processes. The description of these phenomena requires a full wave approach. The pulses (or their fragments) that do not split but move smoothly through the optical system (or its fragment) may be described with the help of the geometrical optics theory.

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## 1. Introduction

Two theories known as geometrical optics and wave optics describe the propagation of light and other optical effects.

The geometrical optics is older, its beginning can be traced in the Antique times, [1]. With simple laws of light rays propagation: rectilinear in uniform medias, reflection on and refraction laws at interfaces between two medias *etc.*, principles and performance of many optical instruments, *e.g.* magnifying lenses, telescopes, microscopes, optical projectors *etc.*, can be explained and their construction and performance improved.

Beginning from the XVI century new effects, unexplained in the frame of geometrical optics, were recognized. The first observation was the noticing of multiple fringes at the shadow boundary when small objects are illuminated.

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Numerous effects of this type, associated nowadays with the diffraction and interference phenomena, initiated, established and supported the wave picture of light.

Parallel discoveries of various electric and magnetic phenomena and developments in their quantitative descriptions have accumulated in the formulation of a set of equations by Maxwell. Recognizing the electromagnetic nature of light it was realized that Maxwell's equations are the most basic description of all wave properties of light.

The last century discovered a new corpuscular or quantum aspect of light. Pieces of light "photons" exhibit both geometric optics rectilinear propagation and interference effects characteristic for the wave phenomena.

Therefore, it is a natural question to ask about the relations between those two concepts of light. Furthermore, these relations must admit the description of quantum properties of the light.

It has been pointed out by Sommerfeld [2, 3] and described in [4, 5] that starting from the Maxwell equations one can reach asymptotically the geometric optics equations in the limit $\lambda \to 0$.

This approach is based on the concept of an eikonal which can be attributed to the surfaces of a constant phase of the wave. The set of Maxwell equations for the electromagnetic fields are thus replaced by a set of coupled equations for the corresponding phases and amplitudes of the field. These equations are nonlinear and therefore very difficult to solve. So only perturbative methods of their solution are available. The lowest order of the perturbation theory recovers the same equations as are specified by the geometrical optics.

It is worth pointing out that, the theory operating with the phases discovered many points and manifolds of phases singularities. In fact, the concept of the phase singularities appeared to be very general and fruitful. It united many aspects of light propagation [6]. Our discussion attempting to link the geometrical and wave optics is based on the original field equations *i.e.* Maxwell's equations. Considering only linear medias the solution of many source and boundary field problems can be simplified significantly due to validity of the superposition principle.

Basic elements of the wave theory are monochromatic plane waves. Each wave is specified by the frequency, direction of propagation $\boldsymbol{k}$ and polarization.

The wave vector $\boldsymbol{k}$ can be naturally associated with the direction of the geometric optic ray. However, the ray position has no analogical counterpart in the individual plane waves.

The waves as well as the rays interacting with various media perturbations like boundaries, dielectric insertions (passive and active optical elements) are scattered and are transformed into more complicated waves and rays. Figure 1 gives an example of the scattering by a dielectric cylinder, with a radial dependence of the dielectric constant $\varepsilon = \varepsilon(r)$ , according to the geometric and wave optics. As one can see the pictures look very different and one cannot hope that considering the limit $\lambda \to 0$ will make them similar.

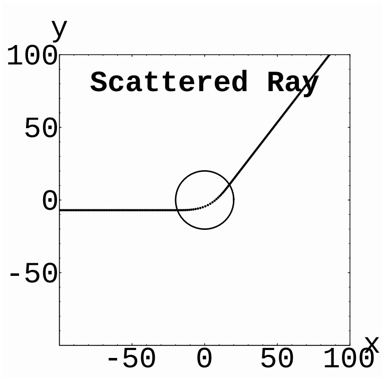


Fig. 1. Optical ray and optical wave scattering by a nonuniform dielectric cylinder, $a = 20, \varepsilon_0 = 3$.

To establish closer analogies between the geometrical and wave optics as well as to find a sufficient criterion for their equivalence we have to consider not only the plane monochromatic waves but also wave beams having finite transverse cross sections and finite duration times.

## 2. Radiation wave packets and their scattering

### *2.1. Stationary wave packets — wave beams*

Starting from an elementary solution of scattered wave problem corresponding to a given incident plane wave specified by $\boldsymbol{k}$ (having the wave frequency $\omega = k = |\boldsymbol{k}|$) and a given scatterer represented by the dielectric constant distribution $\varepsilon = \varepsilon(\boldsymbol{r})$ one can construct the solution corresponding to various wavepackets performing linear operations only. To simplify our discussion let us consider a perturbing dielectric to have a cylindrical symmetry, *i.e.* $\varepsilon$ has only a radial dependence, with the incident waves coming perpendicularly to the cylinder and uniform along its length (the cylinder axis is placed at the origin of a coordinate system and taken as the $z$-axis). These assumptions reduce the scattering problem to a scalar one and the field is completely specified by the $E_z$ component. Assuming furthermore that the dielectric is confined in the cylinder of the radius $a$, the solutions of the scattering problem can be represented in the form of a partial wave expansion

$$E_z^T(\boldsymbol{k},\boldsymbol{r}) = \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \sum_{n=-\infty}^{\infty} \mathrm{e}^{in\phi} \begin{cases} a_n E_{z,n}^{\mathrm{int}}(r), & r < a, \\ c_n H_n^1(kr), & r > a\,, \end{cases} \tag{1}$$

where $H_n^1$ are the Hankel functions of the first kind and order $n$ and $E_{z,n}^{\mathrm{int}}$ represent the internal fields. In the case of the uniform dielectric, considered in detail in [7, 8], the internal fields may be explicitly written in the terms of the Bessel functions $J_n$, in the general case can be determined numerically.

The expansion coefficients $\{a_n, c_n\}$ can be determined from the required field continuity conditions. Knowing these coefficients for the incident wave propagating along the $x$-axis one can get them for the incident wave propagating at the angle $\alpha$ with respect to the $x$-axis.

$$\{a_n^\alpha, c_n^\alpha\} = \{a_n^0, c_n^0\}\mathrm{e}^{-in\alpha}.$$

The superpositions of these scattered waves incoming within selected angular sectors lead to beams of finite cross section. The minimum width of such beams, their waist, occur at the center of the coordinate system.

By introducing the additional phase shifts to the coefficients $\{a_n, c_n\}$

$$\mathcal{S}_{\boldsymbol{R}_0}^\alpha \ = \ \mathrm{e}^{-i\boldsymbol{k}(\alpha)\cdot\boldsymbol{R}_0},$$

the waist of the incident beam can be placed at the point $\boldsymbol{R}_0$.

The above procedure leads to the stationary monochromatic beams which can be compared with the analogical light rays or paths occurring in the geometrical optics.

$$E_{\mathrm{beam}}(p(\alpha), \boldsymbol{R_0}\ ;\ \omega, \boldsymbol{r}) = \sum_\alpha p(\alpha)\ \mathcal{S}_{\boldsymbol{R}_0}^\alpha\ E_z^T(\boldsymbol{k}(\alpha), \boldsymbol{r})\,. \tag{2}$$

The variables proceeding the semicolon ";" indicate their functional dependence in the solution.

Knowing the fields in a given configuration of sources and dielectrics (scatterers) one can introduce wave light rays defined as a set of the lines tangent to the time-averaged value of the Poynting vector field, [7].

Figure 2 presents the case with very close similarities between the corresponding pictures obtained according to the geometrical and wave optics.

To achieve such strong correspondence between the wave and geometric optics it is very important to take into account the full electromagnetic field. This field is expressed as the superposition of the incident wave and scattered wave propagating outwards the scatterer. Restricting the estimation of scattering to the scattered part of the wave leads to incorrect scattering

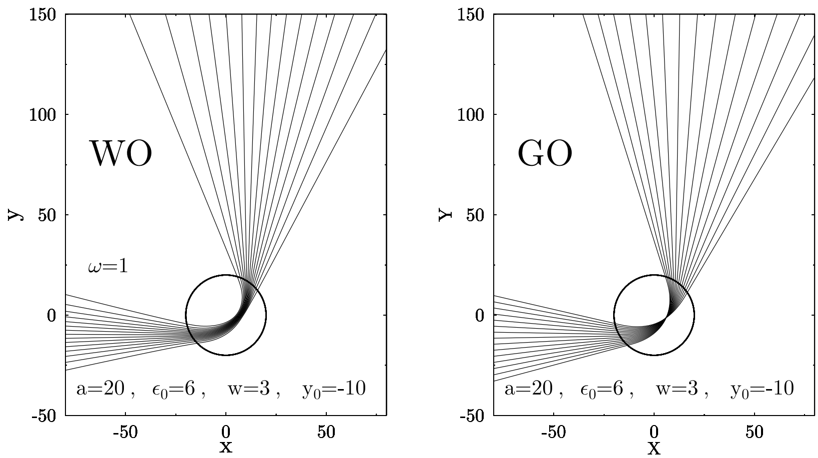


Fig. 2. Bundles of scattering rays in wave and geometrical optics.

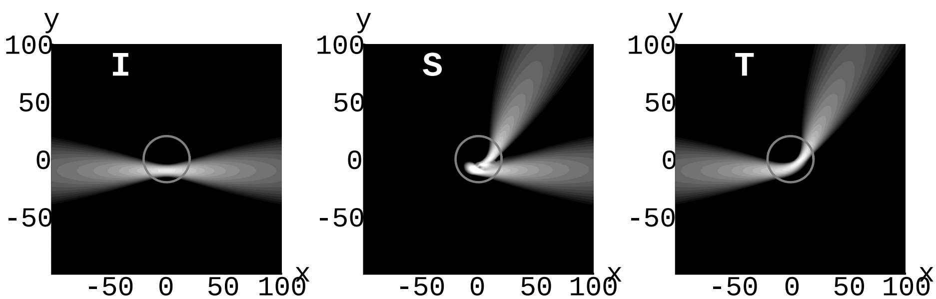


Fig. 3. Incident Gaussian beam { I }, scattered part of the scattering wave { S } and total scattering wave { T }, $a = 20, \omega = 1, \varepsilon_0 = 3, y_0 = -10, w = 3$ .

picture as is shown in figure 3. The scattered part of the wave accompanying the scattering of a narrow beam is composed of two parts, one being bent and one propagating outward the scatterer parallel to the incident beam. This picture is in disagreement with the classical ray behaviour. However, the scattered part of the wave itself has no physical meaning. It is only a subsidiary function which, when, added to the incident wave determines the true electromagnetic field satisfying the proper boundary conditions. As these figures show the total fields representing the narrow beam scattering is in a complete agreement with that provided by the classical ray trajectories.

To illustrate similarities and differences between the wave and geometrical optics in more detail we shall consider optical pulses of finite duration time.

### *2.2. Pulses of wave beams*

Finite duration time pulses can be described by superpositions of finite width beams of different frequencies $\omega$.

$$E_{\rm pulse}(g(\omega), p(\alpha), \boldsymbol{R}_0\ ;\ \boldsymbol{r}, t) = \sum_{\omega} \mathrm{e}^{i\omega t} g(\omega) E_{\rm beam}(p(\alpha), \boldsymbol{R}_0; \omega, \boldsymbol{r}) \tag{3}$$

The pulse shape and duration depends functionally on a spectral amplitude function $g(\omega)$. In the examples we are presenting the $g$ functions have a Gaussian form

$$g(\omega)\ \propto\ \exp\left(-\frac{(\omega-\omega_0)^2}{\Delta\omega^2}\right), \tag{4}$$

where $\omega_0$ is the central wave frequency and $\Delta\omega$ denotes the spectral width of the pulse associated with its duration time.

## 3. Examples

The presented examples illustrate the time evolution of radiation wave packets during the scattering. The graphs are selected from the corresponding films presenting this evolution in an animated form. The examples assume scattering perturbers of the form

$$\varepsilon(r) = \begin{cases} \varepsilon_0(r/a-1)^2+1, & r < a, \\ 1, & r > a. \end{cases} \tag{5}$$

and the radial part of the $E^{\rm int}$ have to be found numerically.

Figure 4 illustrates the scattering of a centrally incoming incident pulse of the width $w = 20$ , comparable with the perturber range $a = 20$. As we can see the incident radiation pulse induces a cylindrical wave propagating outward the scatterer. The outward wave gradually takes a form of separated multiple mini beams that arriving at a distant screen could be recognized as distinct peaks.

It is important to point out that in the forward direction the outward scattered wave constantly interferes with the incident wave, therefore, these two waves are not being distinguishable. Undergoing transient modifications the forward beam reaches eventually the shape similar to that of the incident

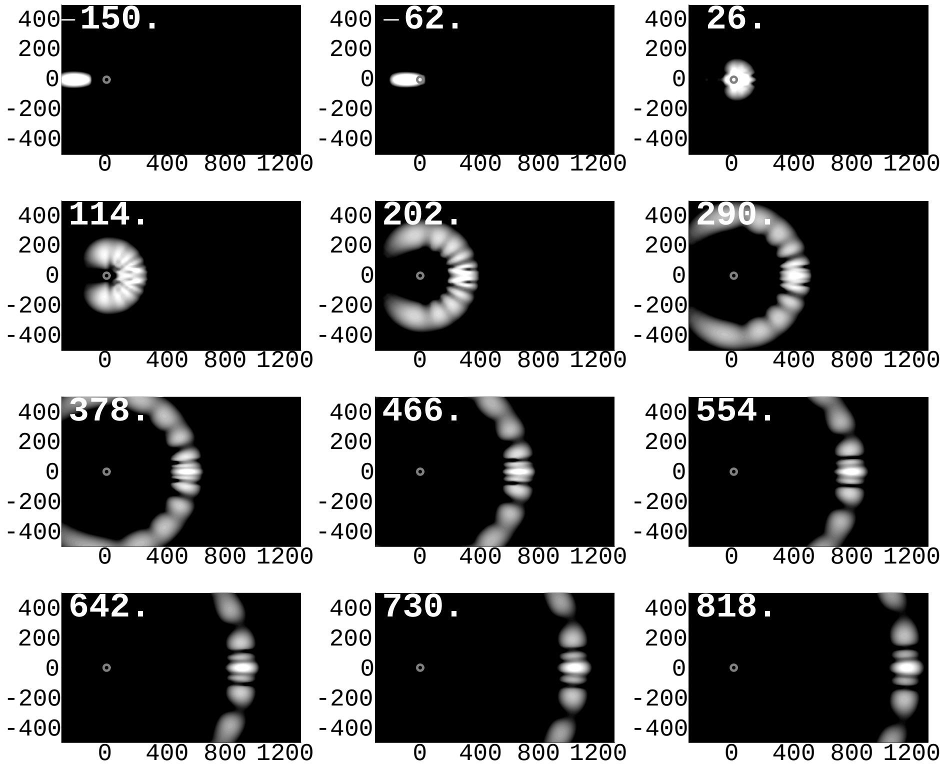


Fig. 4. Time evolution of the light pulse scattering, $a = 20, \varepsilon_0 = 5, w = 20, \omega = 1, \Delta\omega = 0.04$.

pulse. It is clear that its total energy has been reduced and can be recovered in the scattered pulses. This is a kind of the optical theorem valid for the finite width pulses.

Figure 5 shows scattering of a narrow beam by a larger cylinder. The beam is displaced down the center of the cylinder while its waist is kept at the central plane $x_0 = 0$. As we can see these conditions do not cause any splitting of the incident pulse. Although the pulse is spreading, which is caused by a natural transverse spreading of focused pulses in a free propagation and an additional spreading due to the scattering, it propagates as one whole object. One may expect that if the scatterer were larger the beam spreading would be less significant and the pulse would behave as the one described by the geometrical optics.

The next Figure 6 shows the scattering of a similar pulse, the same impact parameter but the waist has been shifted to $x_0 = -250$. This scattering

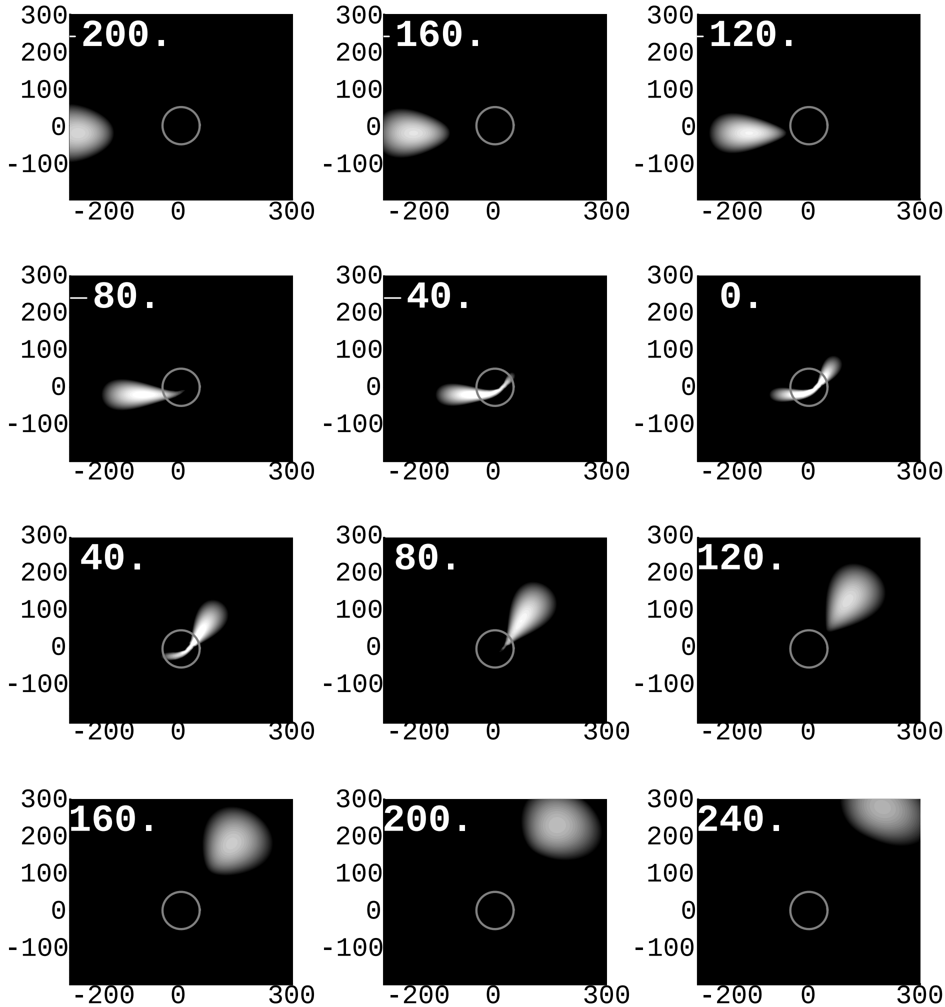


Fig. 5. Time evolution of a light narrow pulse scattering. The pulse is focused at the $x_0 = 0$ plane, $a = 50, \varepsilon_0 = 3, w = 3, y_0 = -20, \omega = 1, \Delta\omega = 0.04$.

looks different. The pulse being a narrow one at $x_0$ upon reaching the scatterer has been spread and passing it was converted to many small pulses. Obviously the description of this process requires a full wave optics analysis.

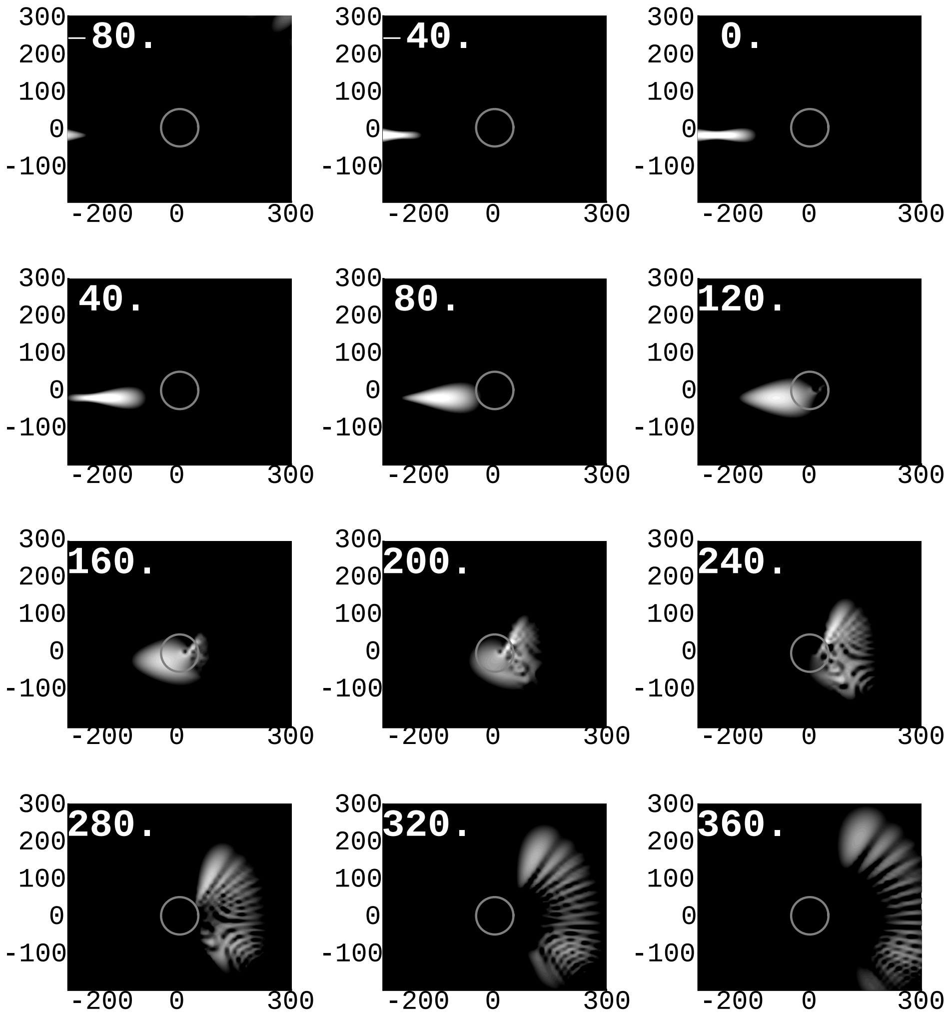


Fig. 6. Time evolution of a light pulse scattering. The pulse is focused at the $x_0 = -250$ plane, $a = 50, \varepsilon_0 = 3, w = 3, y_0 = -20., \omega = 1, \Delta\omega = 0.04$.

## 4. Conclusions

These few examples show that conditions for the validity of the geometric optics approximation is more subtle than the often cited requirement $\lambda \to 0$. This condition can only be treated as a necessary one. In order to find sufficient conditions it is necessary to investigate other light parameters, in particular light beam parameters, in connection with a thorough treatment of the light scattering process.

The examples of the wave scattering show that when the pulses evolve as individual items the description of their evolution can be simplified using geometric optics approach. When the incident pulses split in the scattering process into smaller ones then the analysis has to be based on the wave optics theory.

Very often the fragments of pulses evolution requiring wave optic analysis can be reduced to isolated spots. Away from these spots the geometrical optics provide an adequate description. The best example is the reflection of a light pulse by a flat surface between two dielectrics. Although simple rules of the geometric optics, are sufficient to make drawings of the transmitted (refracted) and reflected pulses one should use the wave theory to determine the amounts of reflected and transmitted light.

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