FRINGE CONTRAST IN MACH–ZEHNDER ATOM INTERFEROMETERS*

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(Received June 6, 2002)

In the present paper, we analyze several factors which limit the fringe contrast in atom interferometers of the Mach–Zehnder type. We consider only the case of interferometers operating with thermal atoms, as there are very specific problems in this case. All the effects considered here are already known to reduce the fringe contrast but the quantitative analysis was not complete. In particular, vibrations play a very important role: a static description of the grating motions is not sufficient and dynamical effects must be taken into account. Such a description has been already made by Schmiedmayer *et al.* in their contribution to the book "Atom Interferometry" (1997). We recall this description and we discuss further some results of this calculation.

PACS numbers: 03.75.Dg, 32.80.Lg, 39.20.+q

1. Introduction

Atom interferometry has developed very rapidly since 1991 and an excellent overview of this field and of its applications can be found in the book "Atom Interferometry" [1]. With thermal atoms, the interferometers are usually of the Mach–Zehnder or of the Ramsey–Bordé types. In the present paper, we discuss the fringe contrast $\mathcal{C}$ (also called visibility and defined by $\mathcal{C} = [I_{\max} - I_{\min}]/[I_{\max} + I_{\min}]$) and we consider only the particular case of Mach–Zehnder interferometers using an elastic diffraction process and thermal atoms. Several such interferometers have been built and operated:

* Presented at the Photons, Atoms and All That, PAAT 2002 Conference, Cracow Poland, May 31–June 1, 2002.

- in 1991, an interferometer was built by Pritchard and co-workers using sodium atom and diffraction on material gratings [2]; the contrast initial value was 13% value and it has been improved up to 49% [3];
- in 1995, Zeilinger and co-workers [4] operated an interferometer, using metastable argon and laser diffraction in the Raman–Nath regime, with a 10% contrast;
- also in 1995, Lee and co-workers [5] built an interferometer, using metastable neon and laser diffraction in the Bragg regime, with a 62% contrast;
- in 2001, Toennies and co-workers [6] have operated an interferometer with material gratings, using ground state helium atom, with a 71 % contrast;
- also in 2001, our group [7] has built an interferometer, using lithium atom and laser diffraction in the Bragg regime, with a 74% contrast.

The quest for improving the contrast and reaching a value close to its theoretical maximum $\mathcal{C} = 1$ is now more than 10 years long and we think that elucidating the factors which limit the contrast is a very interesting task for the following reasons. One important use of an interferometer is to make phase measurements and, in this case, assuming a Poisson statistics for the noise, as it is usually the case, the measurement accuracy increases with the figure of merit given by $I\mathcal{C}^2$ where I is the output flux. An interferometer can also be used in a different way, as in the case of the measurement of the index of refraction of gases for atomic waves [3, 8, 9]. In this case, the contrast decreases when one of the two interfering beams is attenuated as a result of the existence of an imaginary part of the index of refraction. The relation between the contrast and the imaginary part of the index is simple to analyze only if the initial value of the contrast is well understood. Finally, several authors [10–12] have developed calculations of the quantum decoherence of the atomic wave during its propagation through an atomic interferometer. This decoherence may have various fundamental origins (gravitational waves, space-time foam, *etc.*) and the main experimental consequence is the reduction of the fringe contrast. If this effect is not vanishingly small, its observation will be very difficult as the source of decoherence cannot be switched on and off. Obviously, such observation will be feasible only if an excellent understanding of all the other factors limiting the contrast is available.

In the present paper, we will consider separately two important factors which limit the fringe contrast. Some aspects of this question have already been discussed: previous works by Turchette *et al.* [13] and by our group [14] were made to evaluate the fringe contrast in Mach–Zehnder interferometers in the presence of alignment defects. Here, we will assume that these alignment defects are negligible and we consider other effects inducing a contrast reduction. Starting from a simplified calculation of the wave propagation (Section 2), we evaluate the contrast loss due to imperfect separation of the exit beams (Section 3), the general effects of phase averaging (Section 4), the role of interferometer vibrations (Section 5). The effects of vibrations were already discussed in a review paper written by Schmiedmayer *et al.* [3] and our goal here is to complete some aspects of their discussion.

2. Simple calculation of the fringe contrast

We consider a Mach–Zehnder atom interferometer schematically represented in figure 1 and we will use a simplified description in which each beam is described by a plane wave. Obviously, the plane waves must be truncated in the transverse direction (direction x in figure 1), so that the various beams do not overlap everywhere. The paper by Turchette *et al.* [13] and our previous paper [14] used a detailed calculation of the wave propagation in such interferometer: this rather complex analysis, involving Fresnel diffraction, is necessary to discuss several alignment defects. For instance, when the distances L_{12} and L_{23} between consecutive gratings are slightly different, the waves which interfere on the detector present similar diffraction patterns slightly displaced in the x direction and, obviously, a complete calculation

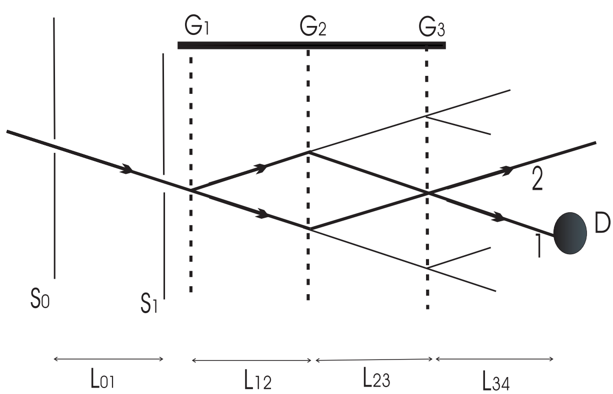

Fig. 1. Schematic drawing of a three grating Mach–Zehnder atom interferometer, in the Bragg diffraction geometry. The atomic beam is collimated by two slits and is diffracted by the three gratings. The main exit beams, labeled 1 and 2, carry complementary signals. The notations for the distances are defined and the x, y, z axes are represented.

involving diffraction theory is necessary to get the spatial dependence of the relative phase of these two waves. In the present discussion, we consider that this type of alignment defects are fully negligible and we focus our attention on simpler but practically important effects. A schematic drawing of the rays inside the interferometer appears in figure 1. The incident atomic wave of vector $\boldsymbol{k}$ is written as

$$\Psi = \exp(i\boldsymbol{k}\boldsymbol{r})\,. \tag{1}$$

The beam produced by diffraction of order p_j by grating G_j is described by a plane wave

$$\Psi \;=\; \exp(i\boldsymbol{k}\boldsymbol{r})\alpha_j(p_j)\exp\left(ip_j\boldsymbol{k}_{Gj}\left(\boldsymbol{r}-\boldsymbol{r}_j\right)\right)\,. \tag{2}$$

This equation is exact in the case of Bragg diffraction geometry [15–17]. It is a first order approximation in power of k_{Gj}/k when the wavevectors $\boldsymbol{k}$ and $\boldsymbol{k}_{Gj}$ are almost perpendicular. In this equation, $\alpha_j(p_j)$ is the diffraction amplitude of order p_j by grating G_j. The wavevector $\boldsymbol{k}_{Gj}$ of grating j is in the plane of the grating, perpendicular to the grating lines and of modulus $k_{Gj} = 2\pi/a$, where a is the grating period, the same for the three gratings. In the case of diffraction by a laser of wavelength λ, $a_j = \lambda/2$. Finally, $\boldsymbol{r}_j$ is a coordinate which measures the position of a reference point in grating G_j. The dependence of the phase of the diffracted beam with the position of the grating in its plane is not pointed out in most textbooks on diffraction but it has very important consequences. Because the grating is a periodic structure, this phase factor must be periodic function of $\boldsymbol{r}_j$, with a period equal to the grating period. We can then calculate the two waves exiting from the interferometer by the exit labeled 1 in figure 1. The wave following the upper path (corresponding to the diffraction orders p, $-p$ and 0 by grating G_1, G_2 and G_3, respectively) is given by

$$\begin{aligned}\Psi_{\mathrm{u}} \;=\;& \alpha_1(p)\alpha_2(-p)\alpha_3(0)\\ &\times\,\exp\left[i\left(\boldsymbol{k}+p\boldsymbol{k}_{G1}-p\boldsymbol{k}_{G2}\right)\boldsymbol{r}\right]\exp\left[ip\left(\boldsymbol{k}_{G2}\boldsymbol{r}_2-\boldsymbol{k}_{G1}\boldsymbol{r}_1\right)\right]\end{aligned} \tag{3}$$

and the wave following the lower path (corresponding to the diffraction orders 0, p and $-p$ by grating G_1, G_2 and G_3, respectively) is given by

$$\begin{aligned}\Psi_{\mathrm{l}} \;=\;& \alpha_1(0)\alpha_2(p)\alpha_3(-p)\\ &\times\,\exp\left[i\left(\boldsymbol{k}+p\boldsymbol{k}_{G2}-p\boldsymbol{k}_{G3}\right)\boldsymbol{r}\right]\exp\left[ip\left(\boldsymbol{k}_{G3}\boldsymbol{r}_3-\boldsymbol{k}_{G2}\boldsymbol{r}_2\right)\right]\,.\end{aligned} \tag{4}$$

These two waves interfere on the detector and the resulting intensity is given by

$$I_1 = \int d^2\boldsymbol{r}\left|\Psi_{\mathrm{u}}+\Psi_{\mathrm{l}}\right|^2, \tag{5}$$

where the integral is carried over the detector surface. Fringes will appear over the detector area if the condition

$$\boldsymbol{k}_{G1} + \boldsymbol{k}_{G3} = 2\boldsymbol{k}_{G2} \tag{6}$$

is not fulfilled. In the experiments, this condition is verified thanks to a fine tuning of the orientation of one grating in its plane. As any small deviation induces a large contrast loss, we assume that this condition is well fulfilled. Then, the integration appearing in equation (5) becomes trivial, as $|\Psi_{\rm u} + \Psi_{\rm l}|$ is independent of $\boldsymbol{r}$. We can thus write for the two waves in a simplified form, $\Psi_{\rm u} = a_{\rm u} \exp{(i\varphi_{\rm u})}$ and $\Psi_{\rm l} = a_{\rm l} \exp{(i\varphi_{\rm l})}$, assuming the amplitudes $a_{\rm u}$ and $a_{\rm l}$ as real and positive. In the phases $\varphi_{\rm u}$ and $\varphi_{\rm l}$, we may distinguish three contributions

- the phases of the products of diffraction amplitudes $\alpha_j(p_j)$. These phases are not negligible and they may present some dispersion, resulting from the dispersion of some parameters (for instance, the velocity of the atomic wave). The analysis of these phases requires a complete modeling of the diffraction process. For material gratings, one must take into account atom-grating van der Waals interaction [14, 18] and for laser diffraction, one must calculate the propagation inside the laser standing waves [19]. This analysis is beyond the scope of the present paper;

- the phases associated to the grating positions come from the arguments of the last exponential in equations (3) and (4). The dependence of these phases with the grating positions are used to sweep the interference pattern and to measure the fringe contrast. But these phases are also sensitive to the vibrations of the interferometer and this effect may reduce the fringe contrast, as discussed below in part V;

- another effect has been forgotten up to now, because we have implicitly assumed that the atomic waves propagate in free space. Inside the interferometer, the two atomic paths are separated in space and therefore submitted to slightly different environments, the dominant effect being due to the gradient of the magnetic field. The propagation phases for the two paths are slightly different and the phase difference is usually a function of the atom internal sublevels. This last effect, which has been discussed by Giltner in his thesis [20] and in the review paper written by Schmiedmayer *et al.* [8], will not be studied in the present paper.

The intensity I_1 of the beam labeled 1 in figure 1 is given by

$$I_1 = a_{\rm u}^2 + a_{\rm l}^2 + 2a_{\rm u}a_{\rm l}\cos(\varphi_{\rm u} - \varphi_{\rm l}) = I_{1\,\rm mean}\left[1 + \mathcal{C}\cos(\varphi_{\rm u} - \varphi_{\rm l})\right] , \tag{7}$$

where the fringe contrast $\mathcal{C}$ is given by

$$\mathcal{C} = \frac{2a_{\rm u}a_{\rm l}}{a_{\rm u}^2 + a_{\rm l}^2} = \frac{2\sqrt{\rho}}{1+\rho}\,. \tag{8}$$

Here ρ is the ratio of the intensities carried by the two interfering beams, $\rho = a_{\rm u}^2/a_{\rm l}^2$. The contrast as a function of ρ is plotted in figure 2. Because the contrast $\mathcal{C}$ is a symmetric function of $a_{\rm u}$ and $a_{\rm l}$, the contrast has the same value when the value of ρ is replaced by its inverse. Any amplitude mismatch reduces the contrast, but this effect is surprisingly slow. For ρ close to 1, $\rho = 1 + \varepsilon$, then $\mathcal{C} \simeq 1 - (\varepsilon^2/8)$. Even if the intensities differ by factor 4 ($\rho = 0.25$ or 4), the contrast remains large, $\mathcal{C} = 0.8$.

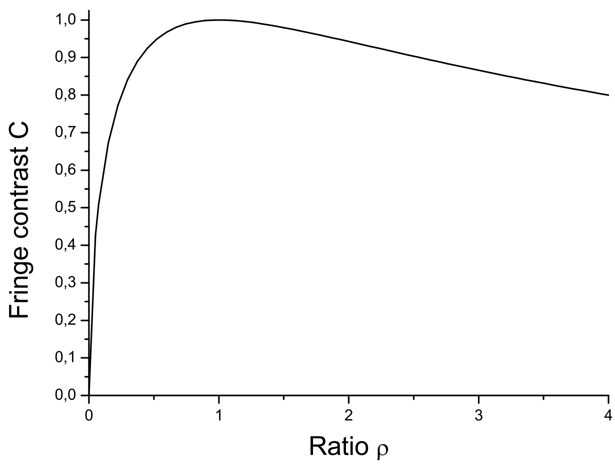

Fig. 2. Fringe contrast $\mathcal{C}$ for a two beam interference as a function of the intensity ratio ρ. Near $\rho = 1$, the contrast goes through a very flat maximum $\mathcal{C}_{\rm max} = 1$.

Because the Mach–Zehnder interferometer is highly symmetric, if the first and third gratings have the same diffraction efficiency, no amplitude mismatch is theoretically expected for the beams interfering at exit 1. In an experiment, a small amplitude mismatch will usually result from some minor defects of the interferometer but the present result proves that a small mismatch does not induce a noticeable contrast loss.

3. Contrast loss due to imperfect separation of exit beams

This effect is typical of atom interferometers using thermal atoms and elastic diffraction. Because of the use of elastic diffraction, exit beams are distinguished only by their position in space. Moreover, because of the very small values of the diffraction angle which is a direct consequence of the very small value of the de Broglie wavelength of thermal atoms, it is difficult to prevent any stray beam from reaching the detector. This effect may reduce considerably the fringe contrast.

As discussed in our previous work [14], this effect is stronger with phase gratings than with amplitude gratings. With amplitude gratings, a large contrast can be obtained even if the detector is located just behind the third grating G_3. This result has been explained by the Moiré filtering by the third grating of the atomic standing wave produced in its plane by the two-beam interference [8] and this remark can be used to make a quantitative calculation of the contrast when the detector is in the plane of the grating G_3. With phase gratings, the total intensity of the various beams exiting from grating G_3 is obviously independent of its position and the contrast vanishes if the detector is in the G_3 plane. The interference signals carried by the exit beams labeled 1 and 2 are complementary and to observe fringes with a good contrast, one must put the detector in a region where these two beams do not overlap. Assuming that the dominant contribution to the stray intensity is due to the beam labeled 2 in figure 1, we can write the total detected intensity due to stray beams in a form very similar to equation (7), but with an opposite contrast

$$I_{\rm stray} = I_{\rm s\,mean}\left[1 - \mathcal{C}_{\rm s}\cos(\varphi_{\rm u} - \varphi_{\rm l})\right] . \tag{9}$$

The total signal is the sum of I_1 and $I_{\rm stray}$ and the associated contrast $\mathcal{C}_{\rm tot}$ is smaller than $\mathcal{C}$

$$\mathcal{C}_{\rm tot} = \mathcal{C}\frac{I_{1\,\rm mean}}{I_{1\,\rm mean} + I_{\rm s\,mean}} - \mathcal{C}_{\rm s}\frac{I_{\rm s\,mean}}{I_{1\,\rm mean} + I_{\rm s\,mean}} . \tag{10}$$

A small admixture of stray beams may strongly reduce the contrast and, as expected, this effect is larger if the stray signal comes from a complementary beam with a large contrast $\mathcal{C}_{\rm s}$. For example, with an intensity ratio $I_{\rm s\,mean}/I_{1\,\rm mean} = 0.1$, the contrast is multiplied by 0.91 if the contrast carried by the stray beam vanishes ($\mathcal{C}_{\rm s} = 0$) and by 0.82 if the contrast of the stray beam is equal to the contrast of the main beam (*i.e.* $\mathcal{C}_{\rm s} = \mathcal{C}$). The optimization of the total contrast $\mathcal{C}_{\rm tot}$ requires the best possible rejection of the stray beams and this optimization induces a large intensity loss. However, the best phase sensitivity, corresponding to the largest value the product $I_{\rm tot}\mathcal{C}_{\rm tot}^2$, is obtained with very different conditions: such an optimization has been realized by Pritchard and co-workers when they used their interferometer as a gyrometer [21].

We have made a calculation of the role of stray beams in our Bragg diffraction interferometer, assuming that the diffraction gratings are equivalent to 50% beam splitters and 100% reflective mirrors. In this case, the interferometer produces only the two exit beams, labeled 1 and 2 on figure 1, and no other stray beams. Both beams carry the same total intensity with opposite contrast equal to 100%. Neglecting completely diffraction by the collimating slits, we describe these two exit beams as in Ramsey's book "*Molecular Beams*" [22], with a trapezoidal intensity profile depending on the widths and separation of the slits and of the distance. Assuming that the detector slit is centered on the axis of the exit beam 1, we have calculated the contrast as a function of the distance of the detector slit to the third grating G_3, for various choices of the slit widths. These results are compared in figure 3 to the results of the full calculation involving diffraction, developed in our previous work [14]. The agreement is good, the largest differences being of the order of a few %, and this simple calculation may be useful for optimizing the design of an interferometer.

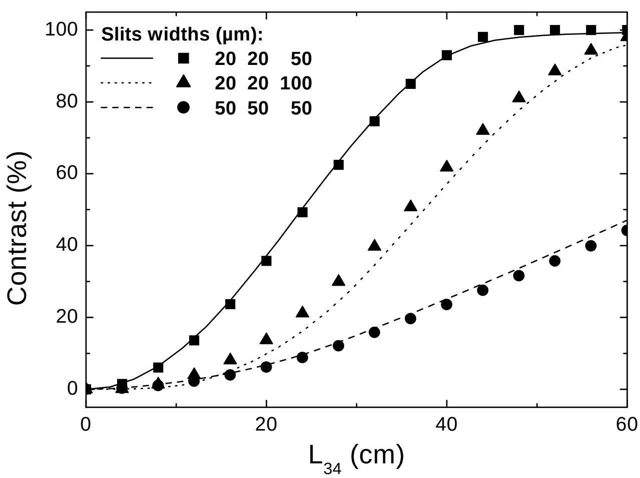

Fig. 3. Fringe contrast $\mathcal{C}$ in a Bragg atom interferometer as represented in figure 1: the contrast is plotted as a function of the distance L_{34} from third grating to the slit S_{D} defining the effective detector width. Three cases are considered corresponding to three choices of the slits widths (given in the following order: collimating slits S_0 and S_1, detector slit S_{D}). The symbols represent the results of the simple calculation neglecting diffraction while the curves represent the results of the complete calculation.

4. Contrast reduction due to phase averaging

An important effect explaining contrast reduction is the existence of some phase averaging. A phase averaging may be due either to temporal averaging (this will be illustrated in the next section by the vibrations of the interferometer) or to an internal state averaging (for instance, due to the effect of a stray magnetic field) or finally to wavefront distortions, corresponding to a spatial dependence of the propagation phases (the analogous effect is well known in optics and here it may take its origin in the diffraction phases due to the gratings). If the phase difference $\delta = \varphi_{\mathrm{u}} - \varphi_{\mathrm{l}}$ appearing in equation (7) is randomly distributed around a mean value δ_{mean}, the averaging effect induces a contrast loss. We assume that the phase distribution is Gaussian with a variance σ

$$P(\delta)d\delta = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\delta - \delta_{\mathrm{mean}})^2}{2\sigma^2}\right] d\delta\,. \tag{11}$$

Then, after averaging the intensity given by equation (7) over this phase distribution, we get

$$\bar{I}_1 = I_{1\,\mathrm{mean}} \left[1 + \mathcal{C}\cos(\delta_{\mathrm{mean}}) \exp\left(-\frac{\sigma^2}{2}\right)\right]\,. \tag{12}$$

The resulting contrast $\bar{\mathcal{C}}$ is related to the contrast $\mathcal{C}$ given by equation (8)

$$\bar{\mathcal{C}} = \mathcal{C} \exp\left(-\frac{\sigma^2}{2}\right)\,. \tag{13}$$

This equation will be applied to evaluate the effect of the vibrations of the grating positions.

5. Effects of vibrations on the fringe contrast

When the manuscript of this paper was almost completed, we found that the equations written below appear already in the review paper by Schmiedmayer *et al.* [3]. We have decided to keep a brief derivation here and to insist on a few remarks which may be useful.

5.1. A naive view

We use equation (7) involving the phases φ_{u} and φ_{l}, considering only the part of these phases which depends on the grating positions (see equations (3) and (4)). This part of the phase difference is given by

$$\varphi_{\mathrm{u}} - \varphi_{\mathrm{l}} = pk_G\,(2x_2 - x_1 - x_3) \tag{14}$$

the wavevectors of the three gratings being exactly parallel to the x-axis. We will call bending of the interferometer bar the quantity $b = 2x_2 - x_1 - x_3$. It seems reasonable to consider that, as a result of vibrations, the bending b

is randomly distributed around its mean value with a variance σ_b (and during the experiments, the mean value of b is swept to record fringes). The phase difference $(\varphi_{\rm u} - \varphi_{\rm l})$ is also randomly distributed, with a variance σ

$$\sigma = pk_G\sigma_b\,. \tag{15}$$

Therefore, the contrast depends on the diffraction order p in the following manner

$$\mathcal{C}_{\rm final} = \mathcal{C}_0 \exp\left(-\gamma p^2\right)\,, \tag{16}$$

where the factor γ is given by

$$\gamma = k_G^2\sigma_b^2/2\,. \tag{17}$$

This simple calculation gives a practically important result, the contrast is a Gaussian function of the diffraction order p. However, this simple calculation does not explicitly consider the motions of the three gratings. As atom interferometers are very sensitive to inertial effects, *i.e.* to rotations (through Sagnac effect) and to accelerations [3, 23], a complete calculation should also consider these effects.

5.2. The role of inertial effects

Equations (7) and (14) are valid provided that, in the phase, we take for the origins of the grating j their values at the times t_j, at which the atomic wavepacket goes through the corresponding grating j. We then get the response of the interferometer to one atomic wavepacket

$$I_1 = I_{1\,\rm mean}\Big[1 + \mathcal{C}\cos\Big(pk_G\left(2x_2(t_2) - x_1(t_1) - x_3(t_3)\right)\Big)\Big]\,. \tag{18}$$

A real experimental signal is obtained by averaging over many wavepackets collected during a time period which is usually long with respect to the characteristic vibrational periods. To give a simpler form to equation (18), we expand the quantity $(2x_2(t_2) - x_1(t_1) - x_3(t_3))$ in powers of the time difference $T = t_2 - t_1 = t_3 - t_2 = L_{12}/v$ up to the term in T^2. In the equation defining T, $L_{12} = L_{23}$ is the distance between consecutive gratings (see figure 1) and v is the atom velocity. We first express $x_1(t_1)$ and $x_3(t_3)$ as a function of their value at time t_2

$$x_1(t_1) = x_1(t_2) - v_{1x}T + \frac{a_{1x}T^2}{2}\,, \tag{19}$$

$$x_3(t_3) = x_3(t_2) + v_{3x}T + \frac{a_{3x}T^2}{2}\,. \tag{20}$$

The velocities v_{jx} and the accelerations a_{jx} are measured by reference to a Galilean frame and, although they are fluctuating functions of time, we assume that they can be considered as constant over the time interval T. We thus get

$$[2x_2(t_2) - x_1(t_1) - x_3(t_3)] = [2x_2(t_2) - x_1(t_2) - x_3(t_2)] - [v_{3x} - v_{1x}]\,T - \frac{[a_{1x} + a_{3x}]\,T^2}{2}\,. \tag{21}$$

In this equation, we recognize three contributions:

- the first term is the instantaneous bending of the interferometer bar $b(t_2) = 2x_2(t_2) - x_1(t_2) - x_3(t_2)$, evaluated at time t_2, *i.e.* at the center of the time interval spent by the atomic wavepacket in the interferometer;

- the second term corresponds to the usual Sagnac effect. The velocity difference $(v_{3x} - v_{1x})$ is equal to $(v_{3x} - v_{1x}) = 2\Omega_y L_{12}$, where Ω_y is the y-component of the angular velocity of the interferometer bar. Following equation (18), the associated phase term is

$$\Delta\Phi_{\mathrm{Sagnac}} = 2pk_G \Omega_y T L_{12}\ . \tag{22}$$

  It is very easy to write this result in the classic form of the Sagnac phase shift $\Delta\Phi_{\mathrm{Sagnac}} = 2mA\Omega_y/\hbar$ where A is the area enclosed by the two atomic paths in the interferometer, $A = pk_G L_{12}^2/k$;

- the third term describes the sensitivity to acceleration and is classic too. A small difference with the usual form of this term comes from the fact that we have considered different accelerations for the two extreme gratings. The phase shift is equal to

$$\Delta\Phi_{\mathrm{acc.}} = \frac{1}{2} pk_G\,(a_{1x} + a_{3x})\,T^2 = pk_G a_{\mathrm{x\,mean}} T^2\,, \tag{23}$$

  where $a_{\mathrm{x\,mean}}$ is the mean value of the acceleration of grating G_1 and G_3.

We can calculate the contrast reduction if we assume that each of the three quantities $b(t_2)$, Ω_y and $a_{\mathrm{x\,mean}}$ are independently distributed with a Gaussian probability distribution, with the associated variance σ_b, $\sigma_{\Omega y}$ and σ_{ax}. For the angular velocity and for the acceleration, the mean values are not equal to zero as a result of Earth motion with respect to a Galilean frame. If the atomic wave is not monochromatic (*i.e.* if the velocity distribution is not very narrow), these nonzero values introduce a further reduction of the contrast because these phases depend on the atom velocity. This effect, which has been observed by Kasevich and co-workers [24, 25] in the case of the Sagnac phase, is small and will be neglected here. The contrast is still given by equation (16) with a generalized value of the coefficient γ

$$\gamma = \frac{k_G^2}{2}\left(\sigma_b^2 + (2\sigma_{\Omega y} L_{12} T)^2 + \left(\sigma_{ax} T^2\right)^2\right) = \frac{k_G^2}{2}\sigma_{\text{eff}}^2 \,. \tag{24}$$

5.3. *Experimental test of this contrast loss*

Up to now, only one Mach–Zehnder interferometer was run with different diffraction orders. This was done by Giltner, McGowan and Lee [5], who measured the following fringe contrasts 62% for order $p = 1$, 22% for order $p = 2$ and 7% for order $p = 3$. In a separate study [20], they applied a magnetic field gradient and they observed the contrast reduction due to the dependence of the propagation phases with the magnetic quantum number M. From this study, it appears that, in the absence of an applied gradient, the contrast loss due to the gradient of the stray magnetic field is fully negligible. Therefore, we may think that equations (16) and (24) explain the variation of the contrast with the order p. Figure 4 presents a fit of equation (16) to this data. The fit is very good and this success supports the idea that the dependence of fringe contrast with the diffraction order are due to vibrations. The values of the fitted parameters are interesting too:

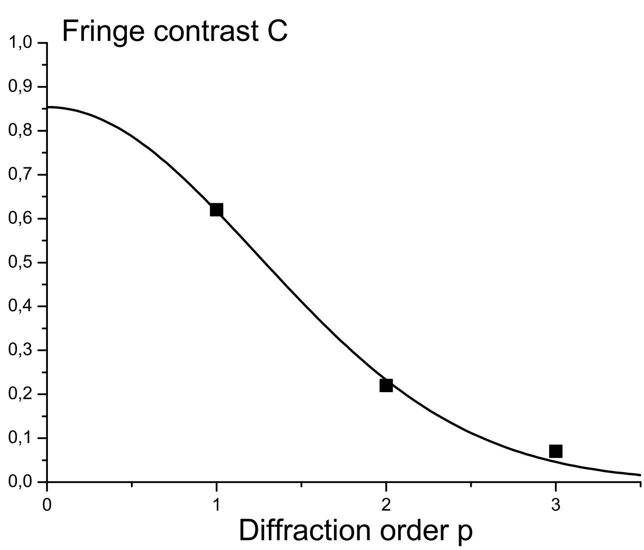

Fig. 4. Fringe contrast C measured by Giltner, McGowan and Lee ([5]) as a function of the diffraction order p. The curve is the best fit following equation (16).

- the fitted value of $\mathcal{C}_0$ is equal to $86 \pm 6\%$: this excellent value of the contrast would have been reached in the absence of vibrations;

- the value of γ is equal to $\gamma = 0.325 \pm 0.037$ corresponding to a value of $\sigma_{\text{eff}} \approx 41$ nm (in this experiment, the wavelength of the laser standing waves is $\lambda_{\text{L}} = 640$ nm, corresponding to grating wavevectors $k_G = 4\pi/\lambda_{\text{L}}$). In this apparatus, as done previously by the group of Pritchard [2], an optical Mach–Zehnder interferometer linked to the gratings of the atom interferometer is used to measure the instantaneous value of the bending b (in this case, the gratings being laser standing waves, the positions are those of the mirrors). This measurement has been used to reduce the vibration noise on the bending b, by acting on one of the mirror x-position with a piezoelectric actuator, thanks to a servo-loop. From the residual error signal of the servo-loop, S.A. Lee and co-workers [5] have estimated that they "were able to hold the relative positions of the three mirrors within 20 nm". It is difficult to convert this information in the variance of a Gaussian distribution, but it is likely that the variance σ_b is substantially smaller than 41 nm. This result suggests that the two other terms contribute very substantially to σ_{eff}.

A final comment concerns equation (21). The interference phase appears to be sensitive to the velocities and accelerations of the two extreme gratings G_1 and G_3, but not to the same quantities for the central grating G_2. This surprising result is related to the fact that this grating plays a similar role for both atomic paths and this double role induces a cancellation effect. Therefore, provided that the approximations made in this derivation are good, a very interesting consequence is that, to reduce the bending vibrations, one should act on the central grating G_2 and not on the extreme gratings G_1 and G_3. This action will reduce the value of σ_b without increasing the two other terms appearing in equation (24), which will remain unaffected.

6. Conclusion

We have given a simple discussion of several important effects which reduce the fringe contrast in Mach–Zehnder atom interferometers, using elastic atom diffraction. We think that an excellent fringe contrast is useful for accurate measurements and also for more fundamental studies of decoherence processes and that the present analysis will be useful for further improvements of atom interferometers.

The first effect discussed is the overlap of exit beams and this effect has been illustrated by a calculation of the contrast as a for various collimating slits: we have shown that a simple calculation neglecting completely diffraction by the slits is sufficient to predict accurately the contrast when the complementary exits of the interferometer are not fully separated.

The second effect discussed here is the role of vibrations of the grating positions. Since the development of three grating interferometers with neutrons, it is clear that an excellent stability of the relative positions of the three gratings is necessary. As done before by Schmiedmayer *et al.* [3], we have completed this discussion by taking into account the fact that the atom wavepackets sample the positions of the three gratings at different times, thus showing that the atom interferometers are sensitive not only to the instantaneous bending of the interferometer bar but also to its rotation and its acceleration, measured at the position of the two extreme gratings.

The predicted dependence of the fringe contrast with the order of diffraction explains very well the values measured by Lee and co-workers: this result suggests that, in the absence of vibrations, the contrast would have been excellent in this apparatus and also that, in the observed contrast reduction, the rotation and acceleration terms were probably important.

We thank C. Champenois for her contribution to an early phase of this work and Région Midi Pyrénées for financial support.

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