BRANES AND ORBIFOLDS ARE OPAQUE

Marcela Carena[a], Tim T.M. Tait[b] Carlos E.M. Wagner[b,c]

[a]Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA

[b]HEP Division, Argonne National Laboratory
9700 Cass Ave., Argonne, IL 60439, USA

[c]Enrico Fermi Institute, University of Chicago
5640 Ellis Ave., Chicago, IL 60637, USA

(Received July 3, 2002)

Dedicated to Stefan Pokorski on his 60th birthday

We examine localized kinetic terms for gauge fields which can propagate into compact extra dimensions. We find that such terms are generated by radiative corrections in both theories with matter fields confined to branes and in theories imposing orbifold boundary conditions on bulk matter. In both cases, the radiative corrections are logarithmically divergent, indicating that from an effective field theory point of view they cannot be predicted in terms of other parameters, and should be treated as independent leading order parameters of the theory. Specializing to the five dimensional case, we show that these terms may result in gross distortions of the Kaluza–Klein gauge field masses, wave functions, and couplings to brane and bulk matter. The resulting phenomenological implications are discussed.

PACS numbers: 04.50.+h, 11.25.Mj, 11.25.Sq

1. Introduction

Particle physics currently finds itself in the perplexing situation in which most experimental results conform to the expectations of the Standard Model (SM), but leave many theoretical questions unanswered. For example, the mechanism of electroweak symmetry breaking is currently unverified, and the mystery as to why the weak scale is so much smaller than the Planck scale remains unanswered. The spectrum of quark masses and mixings has been experimentally determined, and progress is being made on the corresponding quantities for the leptons, but no clues as to why the pattern observed shows large hierarchies in masses and mixings have been established.

Extra compact dimensions allow for novel solutions to these (and other) mysteries. By diluting gravity in a (relatively) large volume which gauge fields and matter cannot enter, one can lower the fundamental Planck scale to just above the weak scale, ameliorating the hierarchy problem [1]. There also exist compelling reasons to consider gauge fields which may propagate into extra dimensions. Having the gauge fields in the bulk may allow one to address questions as to why low-scale gravitational effects do not cause unacceptably fast proton decay [2], pursue a geometric origin for the observed spectrum of fermion masses [2–4], naturally break the electroweak symmetry through strong dynamics [5–7], identify the Higgs as an extra-dimensional component of the gauge field thus protecting its mass from large corrections [8], achieve gauge coupling unification at high scales [9,10], provide a viable dark matter candidate [11], and can provide interesting alternatives to GUT symmetry-breaking and associated problems such as the Higgs doublet-triplet splitting problem [12–14]. Provided the compactification scale, related to the size of the extra dimension by $M_{\rm c} \equiv 1/2\pi R$, is not much larger than 1 TeV, interesting collider signatures involving production of Kaluza–Klein (KK) modes of the gauge fields through the scattering of either brane-localized matter [15–17] or bulk matter [18–21] fields may be obtained.

Models with gauge fields in more than four dimensions are not renormalizable in the classic sense, and must be regarded as effective theories which break down at some scale Λ. In fact, because of the rapid classical evolution of the coupling constant in more than four dimensions, the gauge coupling becomes strong at energy scales on the order of ten times the compactification scale, and thus the scale Λ is expected to be relatively close to $M_{\rm c}$. Since the nature of the UV completion is unknown, these theories should be understood as an expansion in the energy of the process at hand, with effects of the unknown physics beyond Λ reflected in the (infinite number of) undetermined coefficients which must be treated as theoretical inputs. The theory can be predictive at energies much below Λ, when effects of order E^n/Λ^n may be neglected, and only a finite number of terms in the effective Lagrangian contribute to measurable quantities.

In this article we attempt to rigorously treat gauge fields which can propagate in compact extra dimensions from an effective field theory point of view. We find that in addition to the bulk kinetic terms for the gauge field generally considered in the literature, there is also a kinetic term for the gauge field localized on branes or at the boundaries of an orbifolded compact space. Such a term was recently considered by Dvali, Gabadadze, and Shifman [22]. The original motivation was to have the fifth dimension *infinite* in size, with the brane term allowing one to recover four dimensional behavior at short distances, but in this article we will show that, as happens in the analogous case of gravity [23], it has interesting implications for compact spaces as well. The brane kinetic term is not suppressed by any power of Λ compared to the 5d couplings which result in the apparently renormalizable 4d interactions at low energies, and is consistent with all symmetries of the theory. Thus, from general renormalizability arguments, one expects that the term *must* be included in any consistent description of the theory[1]. In fact, one can show from explicit computation that such a term is required to cancel divergences in the five dimensional (5d) theory. Thus, its magnitude should be treated as an input to the theory, and one might expect it to be sizeable.

In this article, while we have chosen to illustrate the physics with the specific example of bulk gauge fields, we recognize that by no means is this the only possibility. *Any* bulk field will experience renormalizations on branes or boundaries of the type we are describing. Previous work [23,26–29] has focused on the case of gravity in various background geometries and numbers of dimensions, as is motivated by solutions of the hierarchy problem. We choose to work with gauge theories in five dimensions because, aside from being well-motivated for the reasons outlined above, they are under better theoretical control than theories with quantum gravity or larger numbers of extra dimensions. We find that some of the qualitative results seen in the gravitational case, such as the appearance of "collective" Kaluza–Klein modes with small masses and strong couplings, may also be explored in our framework.

The article is organized as follows. In Section 2 we briefly discuss the existence of such a brane kinetic term, and argue that from an effective theory point of view it should be included. In Section 3, we compute the resulting spectrum of KK modes of the gauge fields and examine the masses and couplings to brane and bulk fields. In Section 4 we examine some of the phenomenological implications of the modifications to masses and couplings. We reserve Section 5 for our conclusions.

2. Framework and brane kinetic terms

We now discuss the existence of local gauge kinetic terms from the point of view of effective field theory. To illustrate our discussion, we consider a five dimensional (5d) theory of gauge fields $\mathcal{A}^M$

$$S \; = \; \int d^5x \left\{ -\frac{1}{4g_5^2}\mathcal{F}^{MN}\mathcal{F}_{MN} - \delta(x_5)\frac{1}{4g_a^2}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} \right\} , \tag{1}$$

[1] For an example of a string theory model in which gauge kinetic terms on boundaries occur at tree level with calculable magnitude, see [24, 25].

where capital latin letters refer to the full 5d coordinates, $M = 0, 1, 2, 3, 5$, and lower case greek letters refer only to the four uncompactified dimensions, $\mu = 0, 1, 2, 3$. Note that written this way, the bulk gauge coupling g_5 has mass dimension $-1/2$ and the gauge field $\mathcal{A}^M(x^\mu, x_5)$ has dimension 1. The brane coupling g_a is dimensionless and characterizes the "opacity" of the brane. Analyses which neglect the brane term ($1/g_a^2 \to 0$) can be understood as the "transparent brane" approximation. One may rescale the bulk term to its canonical normalization by absorbing $1/g_5$ into $\mathcal{A}_M$, in which case the brane term has a coefficient with dimensions of length, $r_{\rm c} = g_5^2/g_a^2$ and $\mathcal{A}_M$ has dimension $3/2$, as usual for a boson in five dimensions. $\mathcal{F}$ is the usual field-strength functional of the gauge fields,

$$\mathcal{F}^a_{MN} \; = \; \partial_M \mathcal{A}^a_N - \partial_N \mathcal{A}^a_M + f^{abc} \mathcal{A}^b_M \mathcal{A}^c_N \,, \tag{2}$$

for a non-Abelian Yang–Mills theory, with the final term omitted in the Abelian case. We generally omit the group index on the gauge fields wherever we may do so without confusion.

The 5th coordinate x_5 corresponds to a compactified dimension $S^1/\mathcal{Z}_2$, with $-\pi R \leq x_5 \leq \pi R$. Under the orbifold $\mathcal{Z}_2$, the points $-x_5$ and x_5 are identified, and the fields transform as

$$\begin{aligned} \mathcal{A}^\mu(x^\mu, -x_5) &= \;\; \mathcal{A}^\mu(x^\mu, x_5) \,, \\ \mathcal{A}^5(x^\mu, -x_5) &= -\mathcal{A}^5(x^\mu, x_5) \,. \end{aligned} \tag{3}$$

The action and orbifold are compatible with the 5d subset of gauge transformations ($\mathcal{A}^M \to \mathcal{A}^M - \partial^M \lambda(x^M)$ for a U(1) theory) with transformation function $\lambda(x^M)$ chosen to be an even function of x_5.

It is important to note that we have added only the four-dimensional part of the gauge field kinetic term on the brane. If our theory was invariant under the full 5d set of gauge and Lorentz transformations, these symmetries would have forced us to include the full 5d gauge kinetic term. However, the 5d Lorentz invariance is broken firstly by the fact that one of the dimensions is compact, secondly by the orbifold boundary conditions, and finally by choosing $x_5 = 0$ as a special point with different physics from the rest of the extra dimension. As discussed above, five dimensional gauge invariance is similarly present only in a restricted sense. Thus, one could also consider including the remaining terms on the brane with a different coefficient

$$\int dx_5 \delta(x_5) \left\{ \frac{1}{2\tilde{g}_a^2} \left[\partial_\mu \mathcal{A}_5 \partial^\mu \mathcal{A}_5 - 2 \partial_\mu \mathcal{A}_5 \partial_5 \mathcal{A}_\mu + \partial_5 \mathcal{A}_\mu \partial_5 \mathcal{A}_\mu \right] + \text{Interactions} \right\} . \tag{4}$$

In the thin brane approximation under which we work, all of these terms may be neglected. The first two terms vanish because the orbifold boundary conditions require $\mathcal{A}_5$ to vanish at the orbifold fixed points (and in fact we will impose $\mathcal{A}_5 = 0$ everywhere as a convenient gauge choice). The last term is somewhat more subtle. Naively the orbifold conditions seem to require $\partial_5 \mathcal{A}_\mu$, as an odd function of x_5, to vanish at the fixed points. However, as we will see below the effect of the brane term is to force the slope of the KK wave functions to be discontinuous at $x_5 = 0$, implying the derivative is not well defined in the thin brane approximation. However, it is clear that when the derivative is understood in terms of the difference between the wave function of $\mathcal{A}_\mu$ around $x_5 = 0$, the term vanishes.

One can attempt to consider this problem more carefully by introducing a finite brane thickness. For example, one can replace the δ-function with any smooth function sharply peaked about the orbifold fixed point. In a "fat brane" model, which represents the brane and its attendant localized fermions as a scalar field whose VEV has a domain wall profile along the extra dimension, this function is related to the scalar potential which generates the domain wall, and the brane width and shape can be adjusted. This finite thickness for the brane will smooth our KK wave function solutions such that their derivatives will become well-defined, and in fact the terms in Eq. (4) will vanish at $x_5 = 0$. However, the terms still have some non-zero effect in the region away from $x_5 = 0$, but still inside the brane. Thus, their effect must be proportional to the thickness of the brane, and we are justified in dropping the terms of Eq. (4) in the limit in which we treat the brane as infinitely thin.

2.1. Branes are opaque

Many theories in which gauge fields exist in extra compact dimensions introduce submanifolds, or branes on which fields may be confined. A simple application is to have chiral fermions living on a 3-brane. This allows one to have an effective 4d theory which is chiral, despite the fact that five (or more) dimensional theories generally produce mirror fermions after compactification to 4d, and thus are vector-like.

The existence of a brane violates the 5d Poincare invariance, and thus one would generically expect terms living on the brane would be invariant only under the 4d Poincare invariance of the brane itself. Thus, it would be quite plausible to consider a separate gauge kinetic term on the brane at tree level. In fact, the existence of charged matter on the brane *demands* such a term. Loops of the brane matter fields result in log divergent contributions to the gauge field 2-point function, localized on the brane itself [22]. In this case, if the brane is approximated as infinitely thin the computation becomes effectively four-dimensional because the fields running around in the loops are four dimensional. In fact, the log divergence is nothing more than the familiar renormalization of the gauge coupling by the brane fields.

The cancellation of the divergence invokes a local term in x_5 of the form of the gauge field kinetic term, and indicates that the bare theory without such a term is inconsistent. After canceling the divergence, what is left behind is a term whose coefficient cannot be computed in terms of other quantities of the theory, but must instead be determined by experiment. As usual, a log term appears in conjunction with the divergence, and its resummation dictates that even if one were to imagine a UV completion which resulted in the local term being zero at some energy scale, a non-zero term will evolve at other energy scales through renormalization group evolution. One can, of course, choose the coefficient $1/g_a^2$ to be very small, but unless one can derive the small value of $1/g_a^2$ within the framework of a more fundamental theory, this choice can be regarded as a fine-tuning.

One may also invoke the fat brane picture, allowing the brane to have a non-zero thickness corresponding to the width of the transition region between the two limiting values of the VEV. In this case, fermions can be localized with wave functions whose widths are related to the thickness of the fat brane. Loops of such fermions will also lead to localized renormalization of the gauge fields, but now with a profile proportional to the fermion wave functions, and not to the δ-function one obtains in the thin brane case. The detailed shape of the local term is thus model-dependent in general fat brane cases.

2.2. Orbifolds are opaque

It is somewhat counter-intuitive that local terms exist for orbifold theories even in the absence of localized fields. However, in theories with an orbifold, the identification of x_5 with $-x_5$ indicates that the sign of momentum along the fifth dimension is not meaningful, and singles out the orbifold fixed points as special points where translation invariance is violated. These effects may be cast into a particularly convenient form by using technology developed in [32], in which one writes the bulk fields obeying orbifold boundary conditions as a combination of fields which are unconstrained. So, for example, a 5d bulk scalar Φ which is odd under the orbifold $\mathcal{Z}_2$ is written,

$$\Phi(x^\mu, x_5) \;=\; \frac{1}{2}\left[\phi(x^\mu, x_5) - \phi(x^\mu, -x_5)\right] , \tag{5}$$

where $\phi(x^\mu, x_5)$ is a 5d scalar field without orbifold boundary conditions, and thus may be treated conventionally. Φ, by construction, obeys the orbifold boundary conditions. The Φ propagator in momentum space now contains terms which flip the sign of the momentum in the extra dimension

$$\langle \Phi\Phi^* \rangle(q, q') \;=\; \frac{i}{2}\left\{\frac{\delta_{q_5, q'_5} - \delta_{-q_5, q'_5}}{q^2 - q_5^2}\right\}\delta^4(q_\mu - q'_\mu) \, . \tag{6}$$

The first term conserves q_5 whereas the second induces the sign flip. If this scalar is now coupled to a bulk gauge field $\mathcal{A}^M$, its loop contributions to the gauge field two-point function will also contain a term which conserves the gauge field momentum, and a term which conserves its magnitude but flips its sign [32]. Transforming back to position space, one has the operator

$$-\frac{r_{\mathrm{c}}}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\left[\delta(x_5)+\delta(x_5-\pi R)\right] , \tag{7}$$

where r_{c} contains the loop integrals, and is log divergent. Again, this signals that the term is in actuality tree-level, and all we have divined is the running from the cut-off scale to the energy scales of interest to us. Therefore, even universal extra dimensions, with no fields living on the boundaries, will generally have kinetic terms which do live on the boundaries. Note that the same term is induced on both boundaries, which is important if a KK parity is to be a self-consistent symmetry of the low energy dynamics.

2.3. Naive dimensional analysis

While the effective field theory perspective strictly demands the coefficient of the brane kinetic term to be a free parameter of the theory, it is interesting to see how large one might expect this term to be if one makes further assumptions. In particular, Naive Dimensional Analysis (NDA) determines the size of various couplings under the assumption that all couplings are strong at the scale Λ [33]. While NDA estimates are interesting (and sometimes useful in order to judge the applicability of perturbation theory), we do not wish to consider them as predictions — we take the more practical view that the brane kinetic terms are remnants of the unknown physics beyond the cut-off, and must be included irrespective of their size in any valid effective field theory description.

The techniques of [34] allow us to simply determine the values of the couplings g_5 and r_{c} at Λ, and we may use the renormalization group to examine their magnitudes at other energy scales of interest. The NDA estimate for r_{c} at Λ is given by

$$r_{\mathrm{c}} \sim \frac{6\pi}{\Lambda} \tag{8}$$

and thus $r_{\mathrm{c}}/R \sim 6\pi M_{\mathrm{c}}/\Lambda$. If Λ is as low as roughly 20 times M_{c}, we have $r_{\mathrm{c}}/R \sim 1$. At lower energy scales r_{c} will receive additional logarithmic corrections under the renormalization group. Since these corrections are suppressed by loop factors, they can be considered subdominant corrections to the NDA estimates.

3. Kaluza–Klein decomposition

We now derive the KK decomposition for the gauge fields in the presence of the brane kinetic term. Before starting, it is worthwhile to recall the results for transparent branes ($r_{\rm c} \to 0$). In the transparent brane case, the action Eq. (1) can be decomposed into

$$\frac{1}{g_5^2} \int dx_5 \left\{ -\frac{1}{4} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} + \frac{1}{2} \partial_5 \mathcal{A}_\mu \, \partial_5 \mathcal{A}^\mu \right\} , \tag{9}$$

where we have chosen a gauge in which $\mathcal{A}_5 = 0$, corresponding to a unitary gauge in which the fifth components of the gauge field are eaten by the 4d components to provide longitudinal degrees of freedom to the massive modes [35]. We expand the gauge field in a KK tower

$$\mathcal{A}^\mu(x^\mu, x_5) = \sum_n f_n(x_5) \; A_n^\mu(x^\mu) \, , \tag{10}$$

where $f_n(x_5)$ is a set of complete functions which we choose by requiring the KK masses to be diagonal

$$\partial_5^2 f_n(x_5) = m_n^2 \; f_n(x_5) \, . \tag{11}$$

The solution to this equation consistent with the orbifold boundary conditions are cosines, with frequencies (masses) $m_n = n/R$, $n = 0, 1, 2, 3\ldots$. There is a zero mode, whose wave function is a constant in x_5, and thus properly normalized is $1/\sqrt{2\pi R}$. The normalization for the cosine functions is given by $1/\sqrt{\pi R}$, and this difference in normalization results in a $\sqrt{2}$ enhancement of the KK gauge boson coupling to brane matter compared to the coupling of the gauge boson zero mode (*i.e.,* see Ref. [36]).

3.1. One opaque brane

In this section we derive the KK decomposition for a five dimensional theory in the presence of a single non-zero localized gauge kinetic term. For small choices of g_5^2/g_a^2, the brane term is a perturbation on the KK spectrum, introducing a small amount of mixing between the KK modes of various levels. For $g_5^2/g_a^2 \sim R$, these mixing effects drastically affect the KK decomposition, and one is no longer justified in treating the brane term as a perturbation, but should instead include its effect on the KK spectrum *ab initio.*

In the presence of the brane term, the KK decomposition will be diagonal if the wave functions $f_n(x_5)$ satisfy

$$\begin{aligned} \frac{1}{g_5^2}\int dx_5\ [1+r_{\rm c}\,\delta(x_5)]\ f_n(x_5)\ f_m(x_5) &= Z_n\,\delta_{nm}\,, \\ \frac{1}{g_5^2}\int dx_5\ f_n'(x_5)\ f_m'(x_5) &= Z_n\,m_n^2\,\delta_{nm}\,, \end{aligned} \tag{12}$$

where the prime represents partial differentiation with respect to x_5. In order to solve these equations simultaneously, we follow a variant of the procedure used in Ref. [30] to handle scalar fields. We begin with the relevant 5d linearized equation of motion for the gauge field

$$\partial_M\partial^M\mathcal{A}_\mu - \partial_\mu\left(\partial_M\mathcal{A}^M\right) + r_{\rm c}\ \delta(x_5)\left\{\partial_\nu\partial^\nu\mathcal{A}_\mu - \partial_\mu\left(\partial_\nu\mathcal{A}^\nu\right)\right\} = 0\,, \tag{13}$$

where we have dropped the group index on $\mathcal{A}$ for convenience. We now expand the gauge field in a KK tower as in Eq. (10), and determine the $f_n(x_5)$ by requiring the A_n^μ to satisfy the linearized equation of motion of a 4d massive gauge field

$$\partial_\nu\partial^\nu A_n^\mu - \partial^\mu\left(\partial_\nu A_n^\nu\right) + m_n^2 A_n^\mu = 0\,. \tag{14}$$

This procedure becomes particularly simple if we make the 5d gauge choice $\mathcal{A}^5 = 0$. In that case one obtains the equation for the f_n

$$\left[\partial_5^2 + m_n^2 + r_{\rm c}\ m_n^2\ \delta(x_5)\right] f_n = 0\,. \tag{15}$$

This equation, which embodies the diagonalization conditions in Eq. (12), is the same as the equation found in [30] for a scalar field. It bears a strong resemblance to the nonrelativistic Schrödinger equation for a δ-function potential whose strength is energy-dependent, and its spectrum is thus guaranteed to have real eigenvalues. Away from $x_5 = 0$, the solutions are sums of sine and cosine functions. We thus write solutions piece-wise in the regions $x_5 < 0$ and $x_5 > 0$ and impose periodicity and continuity at $x_5 = 0$

$$\begin{aligned} f_n(x_5 - 2\pi R) &= f_n(x_5)\,, \\ f_n(0^+) &= f_n(0^-)\,, \\ f_n'(0^+) - f_n'(0^-) &= -r_{\rm c} m_n^2 f_n(0)\,, \end{aligned} \tag{16}$$

where 0^+ and 0^- denote the limit as x_5 approaches zero from above or below.

The resulting solutions have quantized masses which are solutions of the transcendental equation,

$$\frac{r_{\rm c}\ m_n}{2} = -\tan[\pi\ R\,m_n] \qquad\qquad (m_n \geq 0)\ , \tag{17}$$

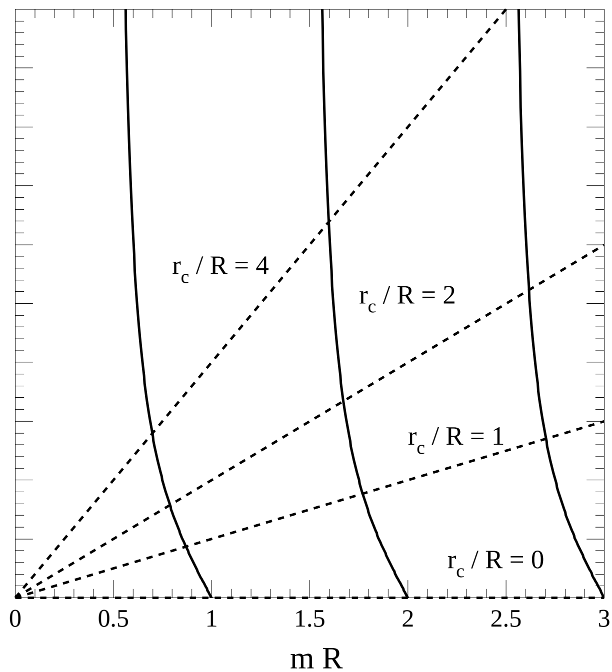


Fig. 1. Graphical solution of the eigenmass equation, $-\tan[\pi mR] = (r_{\rm c}/2R) \times mR$ for several values of $r_{\rm c}/R$.

which may be solved graphically as in Fig. 1. The corresponding wave functions are

$$f_n(x_5) = \mathcal{N}_n \begin{cases} \cos[m_n x_5] + (m_n \frac{r_{\rm c}}{2}) \sin[m_n x_5] & x_5 < 0 \\ \cos[m_n x_5] - (m_n \frac{r_{\rm c}}{2}) \sin[m_n x_5] & x_5 \geq 0 \end{cases} . \tag{18}$$

We define the constant $\mathcal{N}_n$ by normalizing f_n such that

$$\int_{-\pi R}^{+\pi R} dx_5 \, f_n^2(x_5) \; = \; 1 \, , \tag{19}$$

which results in

$$\frac{1}{\mathcal{N}_n^2} \; = \; \pi R \left(1 + \frac{m_n^2 \, r_{\rm c}^2}{4} - \frac{r_{\rm c}}{2\pi R} \right) , \tag{20}$$

for $n \geq 1$ and $\mathcal{N}_0 = 1/\sqrt{2\pi R}$. Inserting this KK decomposition into our original 5d action, Eq. (1), and performing the integration over x_5 results in kinetic terms for the gauge fields which are diagonal:

$$\mathcal{L}_4^0 = \sum_n \left\{ -\frac{1}{4} Z_n \left(\partial_\mu A_\nu^n - \partial_\nu A_\mu^n\right) \left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right) + Z_n \frac{m_n^2}{2} A_n^\mu A_\mu^n \right\}, \quad (21)$$

where Z_n is a normalization factor with dimensions of mass. This equation is consistent with Eq. (12), indicating that we have successfully diagonalized the KK decomposition.

Note that Eq. (17) always has a solution for $m_n = 0$, and that the corresponding $f_0(x_5)$ is always a constant. Thus, there is always a zero mode gauge field whose profile does not depend on the extra dimension. In the limit $r_c \to 0$, in which the brane kinetic term is negligible, we reproduce the standard KK spectrum with masses n/R. In Fig. 2, we present the masses of the first four KK modes as a function of r_c/R. Clearly, for $r_c/R \sim 1$, the spectrum shows some distortion in the spacing between the lowest modes. For any r_c/R, the higher modes asymptote to equal spacing of $1/R$ as expected, though the spectrum still shows an over-all shift dependent on r_c/R. For $r_c \gg R$, the masses asymptotically approach $n/2R$.

It is also instructive to examine the couplings of the KK tower to various types of fields, either confined to branes or living in the bulk. Some representative interaction terms in the 5d theory are

$$\begin{aligned}\mathcal{L} = \int dx_5 \Bigg\{ & \delta(x_5 - x_\psi) \ \left[\overline{\psi} \mathcal{A}_\mu \gamma^\mu \psi\right] \\ & + \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2}\right) \left[2(\partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a) f^{abc} \mathcal{A}_b^\mu \mathcal{A}_c^\nu\right] \\ & + \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2}\right) \left[f^{abc} f^{ade} \mathcal{A}_b^\mu \mathcal{A}_c^\nu \mathcal{A}_\mu^d \mathcal{A}_\nu^e\right] \Bigg\}. \end{aligned} \quad (22)$$

The first term represents coupling to a fermion on a brane at x_ψ (for a bulk fermion mode with wave function $f_\psi(x_5)$ one replaces $\delta(x_5 - x_\psi) \to |f_\psi(x_5)|^2$) and the later two terms are the interactions among the bulk gauge fields for a non-Abelian theory. In order to derive the effective interactions between various KK modes, one inserts the KK decomposition into this equation, and then rescales each A^n by $Z_n^{-1/2}$ in order to canonically normalize its kinetic terms. Given our convention to normalize the $f_n(x_5)$, the n-mode gauge field has Z_n

$$Z_n = \left(\frac{1}{g_5^2} + \frac{f_n^2(0)}{g_a^2}\right), \quad (23)$$

where $f_n(0) = \mathcal{N}_n$ is the wave function of the n-th mode evaluated at the origin.

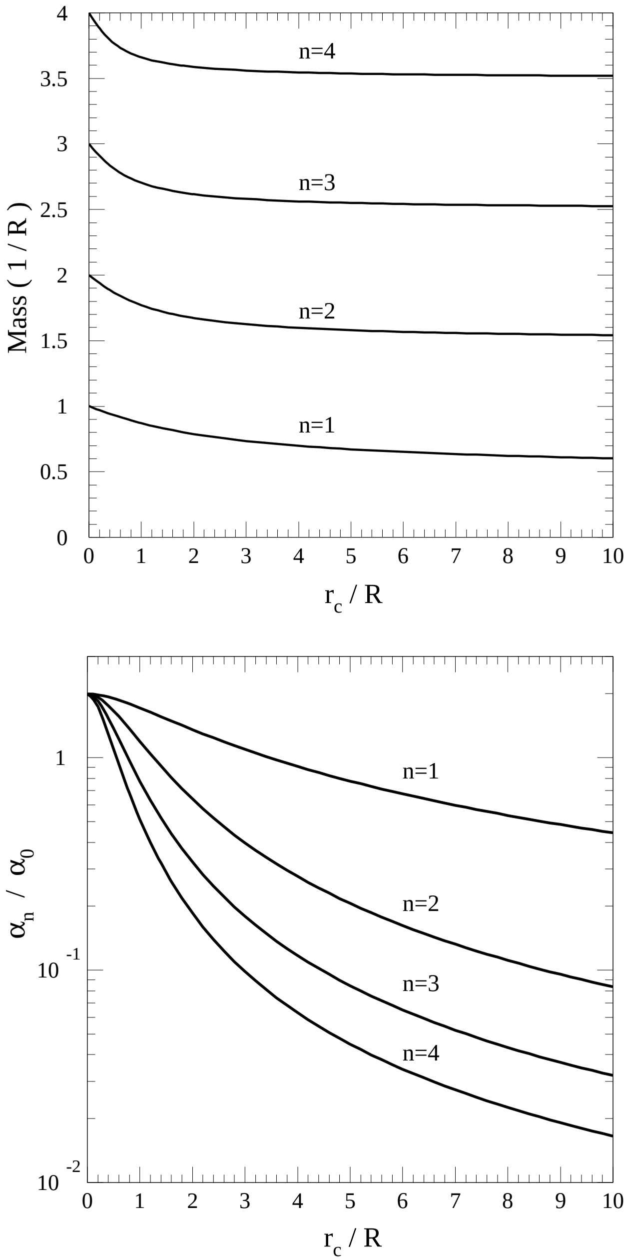


Fig. 2. The $n = 1, 2, 3, 4$ (bottom to top) KK mode masses in units of $1/R$ and couplings relative to the zero mode coupling for the one-brane case, as a function of r_c/R.

For the brane field at x_ψ this results in coupling to the nth KK mode,

$$\frac{f_n(x_\psi)}{\sqrt{Z_n}}. \tag{24}$$

The wave functions $f_n(x_5)$ for the first two modes are presented in Fig. 3. Note that this implies that the zero mode gauge field, whose wave function is constant, couples universally to all brane matter with coupling

$$\frac{1}{g_0^2} = \frac{2\pi R}{g_5^2} + \frac{1}{g_a^2} \tag{25}$$

irrespective of the location of the brane. Of course, in principle a brane containing charged matter located away from $x_5 = 0$ would also be opaque, and should be included in our derivation of the $f_n(x_5)$. We analyze this case in the next sections.

For fields localized on the opaque brane itself, the relevant coupling to the higher modes may also be expressed

$$\frac{1}{g_n^2} = \frac{1}{f_n^2(0) g_5^2} + \frac{1}{g_a^2}\,. \tag{26}$$

In Fig. 2 we show the ratio of g_n^2 to g_0^2 as a function of r_c/R. Evident from the figure is the usual $r_c/R = 0$ expectation that all modes couple equally strongly to the brane fields, with coupling $g_n = \sqrt{2} g_0$. However, once $m_n \sim 1/r_c$ the brane no longer seems transparent, and the KK modes have difficulty penetrating it, decoupling from its fields. This is evident from the wave functions (see Fig. 3) themselves, which show significant distortion away from the brane once the masses are greater than $1/r_c$.

Couplings of the higher KK modes of bulk fields are model-dependent, being given by integrals of products of several of the wave functions. For example, if there are bulk fermions, the coupling of the n-mode gauge field to two bulk fermion modes with wave functions $f_\psi^i(x_5)$ and $f_\psi^j(x_5)$ is

$$\frac{1}{\sqrt{Z_n}} \int dx_5\; f_n(x_5)\; f_\psi^i(x_5)^*\, f_\psi^j(x_5)\,. \tag{27}$$

Thanks to the orthonormality of the fermion KK decomposition, the zero mode gauge field couples only to two fermions of the same mode number,

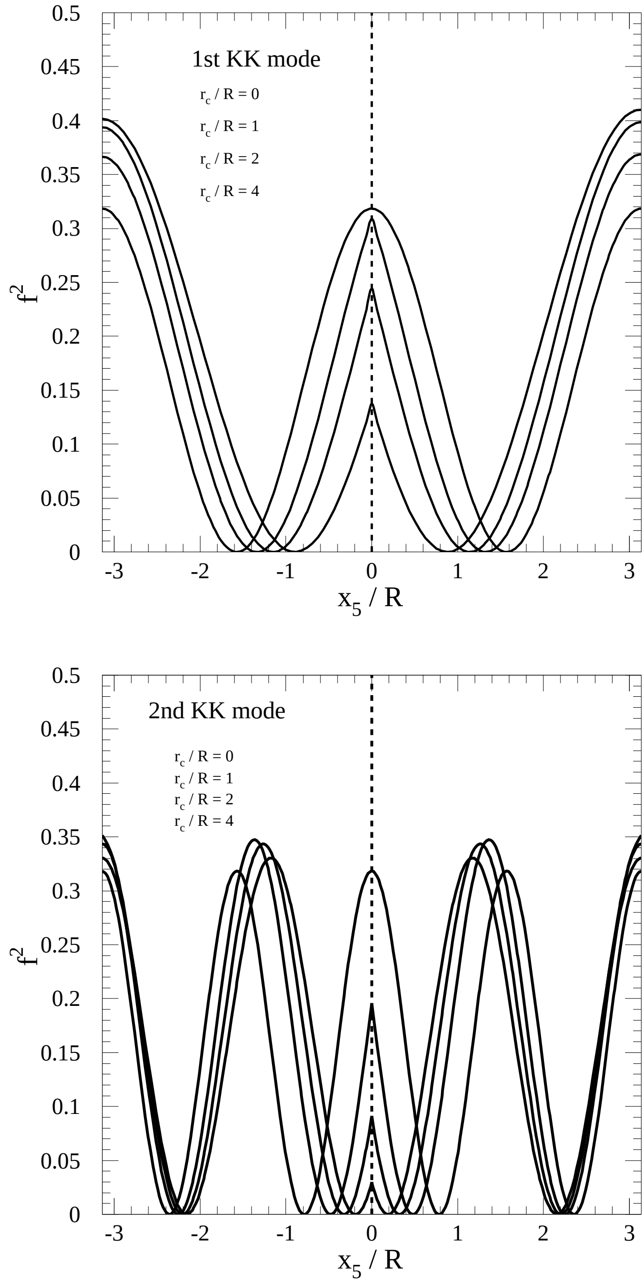


Fig. 3. The $n = 1$ and $n = 2$ KK mode wave functions for one brane with (from top to bottom at $x_5/R = 0$) $r_c/R = 0, 1, 2,$ and 4.

with universal coupling g_0. For the three- and four-point gauge field vertices, we have:

$$g_{nml} \;=\; \frac{1}{\sqrt{Z_n Z_m Z_l}} \int dx_5 \, \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2} \right) \; f_n(x_5) \, f_m(x_5) \, f_l(x_5) \, , \tag{28}$$

$$g_{nmlk} \;=\; \frac{1}{\sqrt{Z_n Z_m Z_l Z_k}} \int dx_5 \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2} \right) \; f_n(x_5) f_m(x_5) f_l(x_5) f_k(x_5) \tag{29}$$

between the A_n-A_m-A_l and A_n-A_m-A_l-A_k modes, respectively. We have suppressed the vector and color indices, but these are simply restored.

The above results can be simplified for the vertices involving the zero mode, because its wave function is independent of x_5 and thus drops out of the integration. In the three-point vertex, we find that setting $l = 0$ reduces the integration to the same one which diagonalized the kinetic energy term; thus using Eq. (12) the integration gives $Z_n \delta_{mn}$ and the vertex factor is $f_0(0)\delta_{mn}/\sqrt{Z_0} \;=\; g_0 \delta_{mn}$ for all modes. In the four-point vertex, setting $l = k = 0$ results in the same integral, $Z_n \delta_{mn}$, and the vertex factor is thus $f_0^2(0)\delta_{mn}/Z_0 \;=\; g_0^2 \delta_{mn}$. Together, these results demonstrate the fact that the zero mode gauge field's couplings take a universal form as dictated by its unbroken gauge invariance, resulting in the same coupling to both bulk and brane fields.

3.2. Two opaque branes

It is relatively simple to generalize our results to include two branes at the orbifold fixed points, one at $x_5 = 0$ and one at $x_5 = \pi R$. The action thus contains,

$$S = \int d^5x \left\{ -\frac{1}{4g_5^2} \mathcal{F}^{MN} \mathcal{F}_{MN} - \delta(x_5) \frac{1}{4g_a^2} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} - \delta(x_5 - \pi R) \frac{1}{4g_b^2} \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu} \right\} . \tag{30}$$

The wave functions satisfy the eigenvalue equation in terms of $r_a \equiv g_5^2/g_a^2$ and $r_b \equiv g_5^2/g_b^2$

$$\left[\partial_5^2 + m_n^2 + r_a m_n^2 \delta(x_5) + r_b m_n^2 \delta(x_5 - \pi R) \right] f_n \;=\; 0 \, . \tag{31}$$

Wave functions can be constructed by the same technique used in the previous section to match solutions across the branes. The resulting solutions are,

$$f_n(x_5) = \mathcal{N}_n \left\{ \begin{array}{ll} \cos[m_n x_5] + (m_n \frac{r_a}{2}) \sin[m_n x_5] & \quad x_5 < 0 \\ \cos[m_n x_5] - (m_n \frac{r_a}{2}) \sin[m_n x_5] & \quad x_5 \geq 0 \end{array} \right. , \tag{32}$$

in which the dependence on r_b is hidden inside the quantized masses, which satisfy the relation

$$0 = \left(r_a r_b m_n^2 - 4\right) \tan[m_n \pi R] - 2\,(r_a + r_b)\, m_n \,. \tag{33}$$

This equation may be reduced to the form $m_n =$ a function of $\tan[m_n \pi R]$ using the quadratic equation, and solved numerically as before. This determines the mass spectrum and couplings of the KK gauge bosons. Continuing to normalize the integral of f_n^2 to one, the Z_n normalization factors are

$$Z_n = \left(\frac{1}{g_5^2} + \frac{f_n^2(0)}{g_a^2} + \frac{f_n^2(\pi R)}{g_b^2}\right) . \tag{34}$$

The couplings of given modes or combinations of modes are derived from these results. For brane field couplings to a single KK mode, one finds the same equation (24) as before in terms of the new wave functions. The couplings among gauge modes are

$$g_{nml} = \frac{1}{\sqrt{Z_n Z_m Z_l}} \int dx_5 \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2} + \frac{\delta(x_5 - \pi R)}{g_b^2}\right) \times f_n(x_5)\, f_m(x_5)\, f_l(x_5) \,, \tag{35}$$

$$g_{nmlk} = \frac{1}{\sqrt{Z_n Z_m Z_l Z_k}} \int dx_5 \left(\frac{1}{g_5^2} + \frac{\delta(x_5)}{g_a^2} + \frac{\delta(x_5 - \pi R)}{g_b^2}\right) \times f_n(x_5) f_m(x_5) f_l(x_5) f_k(x_5) \,. \tag{36}$$

In particular, the zero mode coupling is

$$\frac{1}{g_0^2} = \frac{2\pi R}{g_5^2} + \frac{1}{g_a^2} + \frac{1}{g_b^2}. \tag{37}$$

Before considering specific two-brane configurations, we note that many of these formulae are easy to generalize. In particular, the Z_n generalize into an obvious sum of $1/g_5^2$ plus $f_n^2(x_i)/g_i^2$ for each opaque brane at x_i with coefficient $1/g_i^2$. The coupling to brane matter fields always takes the same form, and the bulk couplings generalize to an integral over the same product of the f_n times $1/g_5^2$ plus $\delta(x_5 - x_i)/g_i^2$. What remains is to determine the eigenmass equation and associated wave functions for a given set of branes, a straight forward (but in the case of many branes, tedious) exercise.

3.2.1. Symmetric branes

For our first example, consider equal brane kinetic terms, $r_a = r_b \equiv r_c$. This is the case induced by radiative corrections to 5d theories with orbifold boundary conditions and no brane fields. Note that this set-up preserves a $\mathcal{Z}_2$ symmetry under which even number KK modes are even and odd number modes are odd. This KK parity forces couplings involving an odd number of odd-mode fields to vanish. The resulting mass spectrum satisfies the equation,

$$\frac{r_c\, m_n}{2} = \frac{\cos[m_n \pi R] \pm 1}{\sin[m_n \pi R]} \qquad (m_n \geq 0)\ , \tag{38}$$

and is shown for the first few KK modes in Fig. 4. The + sign in the equation is realized for the first KK mode, and higher modes are realized for alternating signs. An interesting feature of the two brane case is evident for $r_c/R \gg 1$, in which the mass of the first KK mode approaches zero and the remaining modes approach their canonical values of $(n-1)/R$. This contrasts with the one brane case, for which the first KK mode mass was always greater than $1/2R$. One can solve exactly for a solution with $m_1 R \ll 1$ by expanding the sin and cos in the eigenmass equation. The result is

$$m_1^2 = \frac{4}{r_c^2}\left(1 + \frac{r_c}{\pi R}\right) \approx \frac{4}{\pi\, r_c\, R}\,. \tag{39}$$

This lightest KK mode can be understood to be a sort of "collective mode", composed of a tiny amount of every wave function in the otherwise unperturbed tower. The coupling of the n-mode gauge fields to fields confined on either opaque brane may be expressed as,

$$\frac{1}{g_n^2} = \frac{1}{g_5^2 f_n^2(0)} + \frac{2}{g_a^2} \tag{40}$$

and is plotted for the first few modes in Fig. 4. Unlike the rest of the tower, which exhibits similar decoupling from the branes seen in the one brane case, the collective mode's coupling approaches the zero mode coupling in the limit of large r_c/R. We have chosen to extend Fig. 4 up to $r_c/R = 100$ in order to display the asymptotic behavior as a function of r_c/R. For completeness, in Fig. 5 we show the behaviour of the first and second KK mode wave functions.

One can gain intuition into the reason why this feature appears in the case with two opaque branes by considering an observer living on the brane at $x_5 = 0$ and measuring the gauge coupling at that brane by scattering

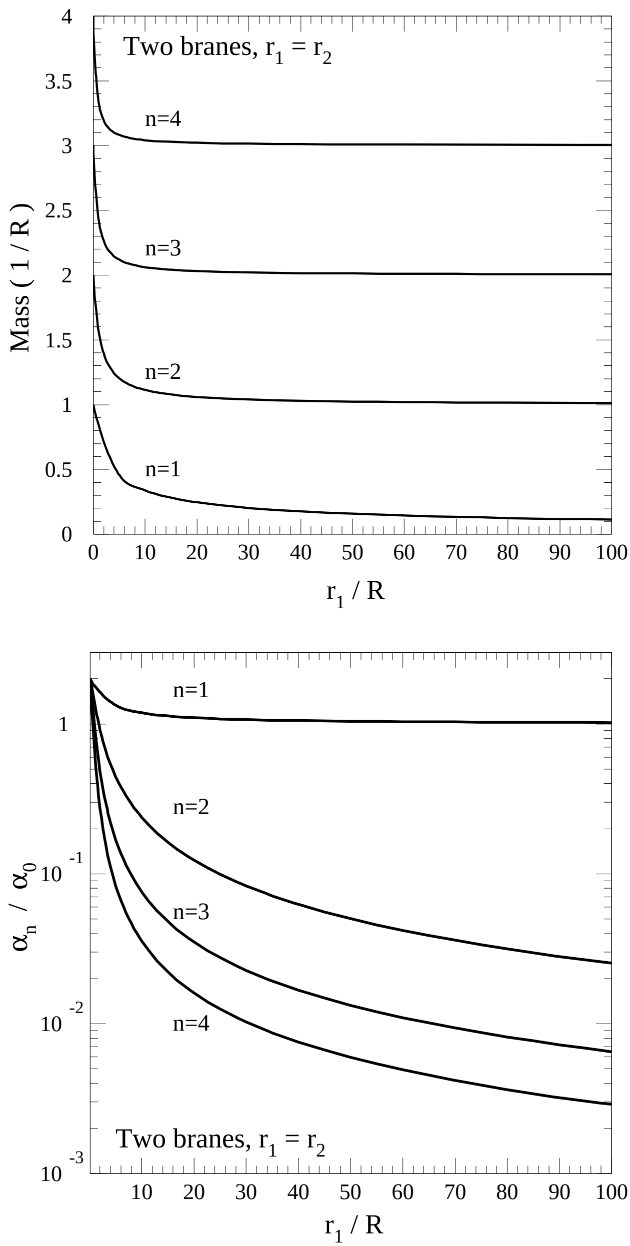
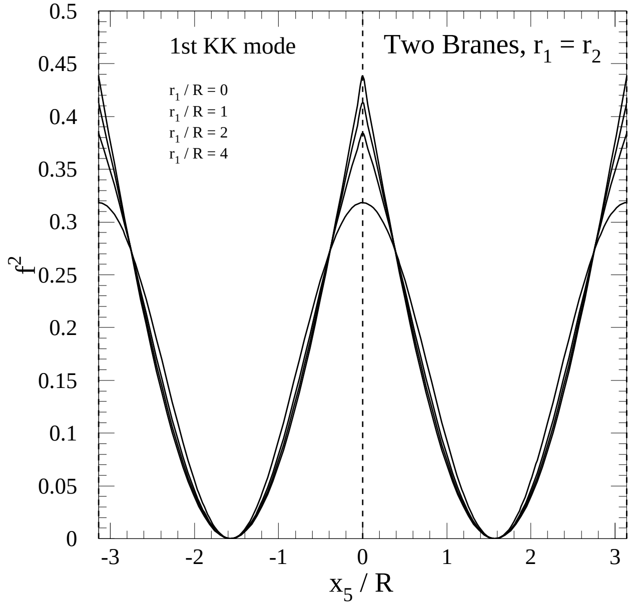


Fig. 4. The $n = 1, 2, 3, 4$ (bottom to top) KK mode masses in units of $1/R$ and KK mode couplings relative to the zero mode coupling as a function of r_c/R for two branes with equal terms.



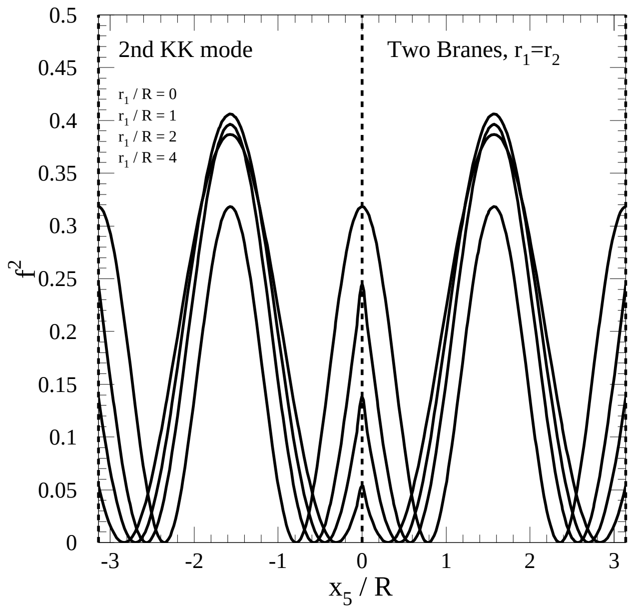


Fig. 5. The $n = 1$ and $n = 2$ KK mode wave functions for two branes with $r_1 = r_2$ and (from bottom to top at $x_5/R = 0$ for $n = 1$, and from top to bottom at $x_5/R = 0$ for $n = 2$) $r_1/R = 0, 1, 2$, and 4.

various types of charged matter at various energies. At distances somewhat shorter than the size of the extra dimension, the observer fails to realize that the other brane is present, and scattering between matter localized on different branes should cease. Furthermore, at these energies the distances probed are too short to realize that there is a second opaque brane at all, and the observed coupling should not depend on g_b. At very high energies, the distances probed are short enough that the fact that there is an extra dimension becomes irrelevant and the coupling should be dominated by the coupling present on the brane, and thus must approach g_a. However, in contrast to the one brane case, the zero mode coupling does not approach g_a, but to a combination of g_a and g_b. Thus, something is needed to restore the correct behavior, and the higher KK modes will not serve because they decouple from the brane. Thus, the collective mode's couplings must approach the zero mode coupling (for $g_a = g_b$) with an appropriate relative sign in order for the net force to be described by g_a alone.

One can further explore this intuitive picture by examining the effective coupling between fields either on the same brane or on different branes. In the KK description of the theory, the net force between them is a sum over all of the KK modes

$$\frac{1}{4\pi}\sum_{n\geq 0}\frac{g_n^i g_n^j}{Q^2+m_n^2} = \frac{\alpha_0}{Q^2}\left(1+\sum_{n\geq 1}\frac{g_n^i g_n^j/\alpha_0}{1+m_n^2/Q^2}\right) \to \frac{\alpha_{ij}(Q^2)}{Q^2}\,, \tag{41}$$

where Q^2 is the momentum transfer, g_n^i is the coupling of the n-th KK mode to the i-th brane, and $\alpha_{ij}(Q^2)$ is an effective coupling which includes the exchange of all KK modes in the interaction of brane field i with brane field j. Using our numerical solution for the symmetric two brane case, we can explicitly compute the effective intra- and inter-brane couplings. The result is shown in Fig. 6, and illustrates the physics described above. At low Q^2 the exchange is dominated by the zero mode, and the two effective couplings are equal to the zero mode coupling. The effect of the collective mode appears rather early, thanks to its small mass, and the couplings begin to differ. For $Q^2 \gtrsim 1/\pi^2 R^2$, interactions occur at distance scales smaller than the separation between the two branes, and the intra-brane coupling vanishes as each brane fails to realize that the other is there. Finally, at very large momentum transfer the brane field fails to realize that there is an extra dimension, and the physics is described entirely by its own (four dimensional) gauge term with coupling g_a.

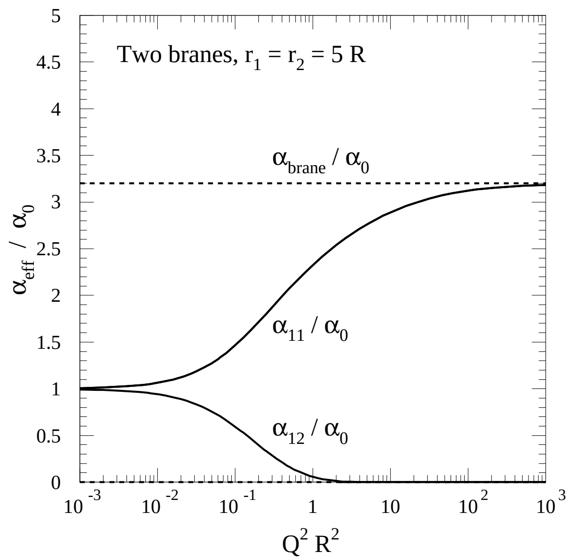


Fig. 6. The effective coupling between two fields on the same brane (upper line) and two fields on different branes (lower line) as a function of momentum transfer, Q^2.

3.2.2. Asymmetric branes

Another interesting case has two branes with different terms. For simplicity, we fix the term on the first brane (at $x_5 = 0$) to $r_a = r_c$ and allow the term on the second brane (at $x_5 = \pi R$) to vary as $r_b = z r_a$. For $z \neq 1$, this configuration explicitly violates KK parity. As motivation, one might imagine a construction in which some fermions (for example, the leptons) are confined to the brane at $x_5 = 0$ while some others (for instance, the quarks) are confined to the other brane at $x_5 = \pi R$. This configuration can suppress local operators leading to unacceptably fast proton decay because of a low Planck scale [2]. Given the asymmetry between the two branes, it would be somewhat contrived if the gauge kinetic terms living on them were the same. In addition, the much larger number of quark degrees of freedom will imply very different quantum corrections to both terms, so the choice of equal couplings at the two branes is only justified in some particular cases.

The KK masses are now solutions of the general two brane eigenmass equation (33). In terms of z and r_1 its solutions can be expressed

$$m_n r_1 \;=\; \frac{(1+z) \pm \sqrt{(1+z)^2 + 4\tan^2\left[m_n \pi R\right]}}{z \tan\left[m_n \pi R\right]}\,, \qquad (42)$$

where again the + sign is realized for the first KK mode solution, and successive modes are realized for alternating signs. Results are plotted for $r_1 = R$ and $r_2 = zR$ in Fig. 7. The features are roughly similar to those evident in

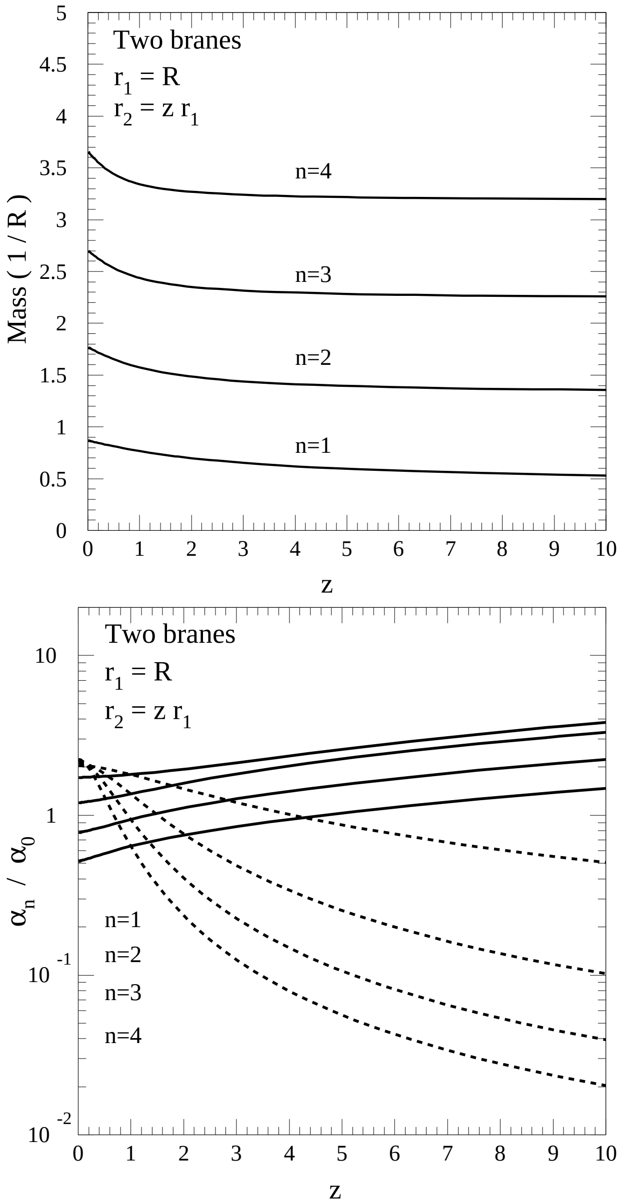


Fig. 7. The $n = 1, 2, 3, 4$ (bottom to top) KK mode masses in units of $1/R$ and the corresponding KK mode couplings (top to bottom) relative to the zero mode coupling (solid lines are the couplings to brane fields at $x_5 = 0$ and dashed lines are couplings to brane fields at $x_5 = \pi R$) as a function of z for two branes with kinetic terms $r_1 = R$ and $r_2 = zr_1$.

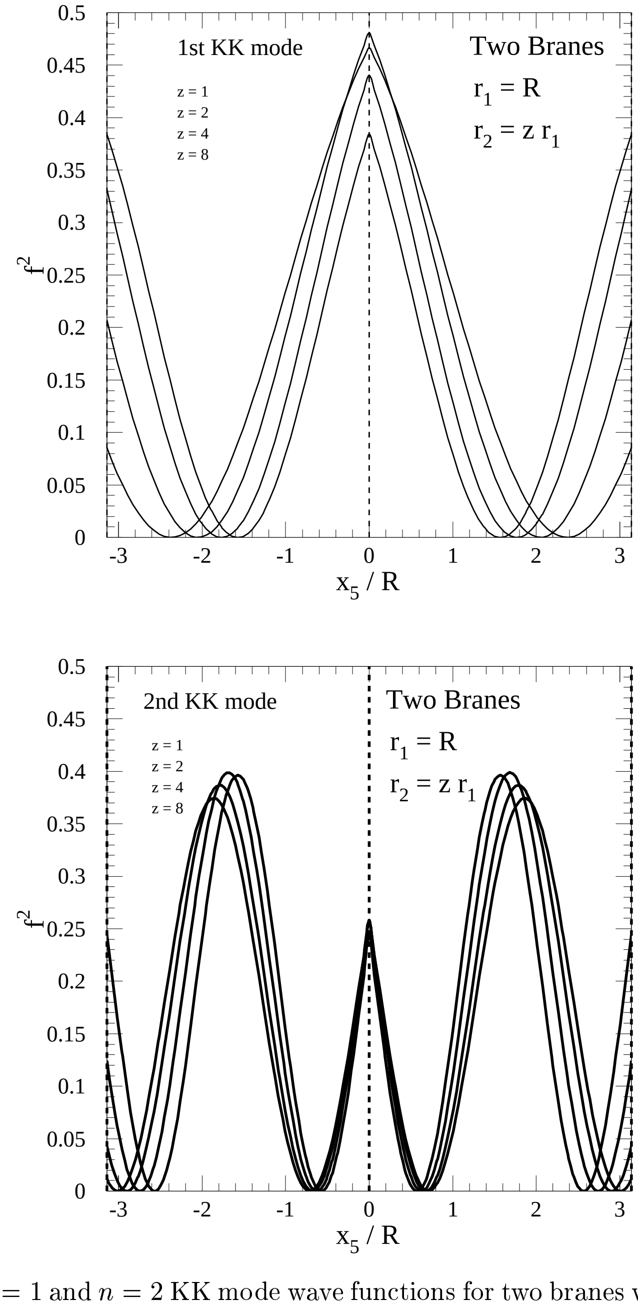


Fig. 8. The $n = 1$ and $n = 2$ KK mode wave functions for two branes with $r_1/R = 1$ and $r_2 = zr_1$ for (from top to bottom at $x_5/R = \pm\pi$) $z = 1$, 2, 4, 8.

the symmetric two brane case, including the existence of a collective mode in the limit of $r_1, r_2 \gg R$ with very small mass

$$m_1^2 \;=\; \frac{2}{zr_1^2}\left(2+\frac{r_1(1+z)}{\pi R}\right) \approx \frac{2(1+z)}{z\pi\, r_1\, R}\,. \tag{43}$$

Having found the masses, the next step is to examine the wave functions. We expect that for asymmetric branes, the larger brane term will dominate, pushing the wave functions further away from that brane. This implies that the higher KK modes couple more weakly to brane fields on one of the opaque branes than to those fields on the other. This is evident in the wave functions, plotted for the first two modes in Fig. 8. For $z = 1$ we see, apart from an over-all sign, equal coupling at both branes, whereas for $z \gg 1$ the wave functions become very small on the brane at $x_5 = \pi R$. This has the implication that the KK modes couple much more strongly to fields located on the less opaque brane than to fields localized on the more opaque brane. We plot these couplings in Fig. 7, and find that the effect is quite striking for $z \gg 1$. One can understand this feature in terms of the effective couplings on each brane. $z \gg 1$ implies that g_b^2 is very small, and thus dominates the zero mode coupling. This in turn implies that, since the effective coupling in the ultra-violet should converge to the local brane terms, KK modes should rapidly decouple from the second brane, while they must couple relatively strongly to the first one in order to make up the difference between the zero mode coupling and the local term on that brane.

4. Implications for phenomenology

Our results can have profound implications for the phenomenology of models in which gauge fields propagate in the bulk. The standard picture for this situation is that high energy colliders can identify an extra dimension by discovering the tower of KK modes with masses n/R and couplings $\sqrt{2}$ times greater than the zero mode coupling to brane-localized matter [15,36]. In the limit of a very small gauge-kinetic term on the brane these results will approximately hold for a large number of KK modes, and this phenomenological picture will remain valid. However, the results above indicate that these theories have, in addition to the parameter R which controls the size of the extra dimension and thus the masses of the KK modes, at least one other, "hidden" parameter, r_c, which will distort the KK spectrum and modify the couplings to brane fields. Since it is effectively a tree level coupling, it is somewhat arbitrary to set it equal to zero.

4.1. One brane case

As a simple example of what the local brane couplings may do to limits from colliders, let us consider the effect of virtual KK photon and Z exchange on the process $e^+e^- \to f\overline{f}$. This was discussed for the transparent brane case in Ref. [36]. At energies far below the mass of the first KK mode, the effect of the virtual KK photon exchange can be included model-independently as a four-fermion operator

$$\mathcal{O}_\gamma \;=\; -e^2 Q_e Q_f \frac{V}{m_W^2}\left[\overline{e}\gamma^\mu e\right]\left[\overline{f}\gamma_\mu f\right] \,, \tag{44}$$

where the dependence on R is hidden inside the coefficient V

$$V \;=\; m_W^2 \sum_{n\geq 1} \frac{\alpha_n/\alpha_0}{M_n^2}\,, \tag{45}$$

where M_n is the mass of the nth KK gauge boson and α_n the product of its couplings to the e and f fields.

In the transparent brane case

$$V = m_W^2 R^2 \sum_{n\geq 1} \frac{2}{n^2} \;=\; 2m_W^2 R^2 \zeta(2) \qquad (r_{\rm c}=0)\,, \tag{46}$$

with the factor of 2 a result of the KK mode couplings being $\sqrt{2}$ times the zero mode couplings, and the sum over n includes all KK states, whose masses are n/R. In the five dimensional case we consider the sum is convergent to $\zeta(2)$ as indicated (though the result changes only by about 5% if truncated at $n=10$), but in higher dimensions it would have to be cut-off in some fashion, introducing dependence on the UV completion. Similar operators can be written for KK modes of the $W^\pm$ and Z (and in the case of $\mathcal{O}_Z$ will generally interfere with $\mathcal{O}_\gamma$ in physical processes). In the $r_{\rm c}\to 0$ limit, the quantity V will be the same for all of the bulk gauge fields. Thus, including all relevant operators, Ref. [36] deduces that with $200\,{\rm pb}^{-1}$ of $\sqrt{s}=195\,{\rm GeV}$ LEP data the reach extends up to $V \lesssim 4.5\times 10^{-3}$ which corresponds to $R^{-1}\lesssim 2.2$ TeV. At the NLC with $\sqrt{s}=500$ GeV and collecting 500 fb^{-1} of data the bound becomes $V\lesssim 1.2\times 10^{-4}$ or $R^{-1}\lesssim 13$ TeV.

The situation with $r_{\rm c}\sim R$ can be quite different. For example, we consider the one-brane case with both e and f fields living on the opaque brane. To compute V one must return to the definition, Eq. (45) where α_n and M_n are now complicated functions of R and $r_{\rm c}$, as shown in Fig. 2. The leading term (from the first KK mode) may be somewhat enhanced by the mass of that mode being lighter than $1/R$, but is also somewhat decreased by the suppressed coupling to brane fields. The suppression dominates the enhancement. The higher KK number states are still approximately equally spaced in mass, but their couplings to brane fields become highly suppressed, and the sum in Eq. (45) converges much more quickly than the $r_c = 0$ expression in Eq. (46). Using the new expression for V, we translate the LEP and NLC bounds on V (which are independent of the new physics) into the plane of $1/R$ and r_c/R. For $r_c/R \gtrsim 1$ the limits on $1/R$ can be substantially modified; for the LEP (projected NLC) limits derived above, we have, for $r_c/R \sim 1$, $R^{-1} \gtrsim 2\,\mathrm{TeV}$ ($12\,\mathrm{TeV}$) and for $r_c/R \sim 10$, $R^{-1} \gtrsim 1.3\,\mathrm{TeV}$ ($8.1\,\mathrm{TeV}$).

Another interesting possibility is the fact that each gauge group in the bulk may have a separate r_c on the brane. This would allow mass splittings between the KK modes much larger than one would normally consider from radiative corrections. This allows each gauge boson to have its own V, which would not be expected from the simple extra-dimensional picture with transparent branes. It further has the effect that the KK modes for the neutral weak boson sector can have a different Weinberg angle than the one observed for the zero modes, and thus may be poorly approximated as KK modes of the ordinary photon and Z, and better represented as different mixtures of heavy copies of the SU(2) and U(1) neutral bosons. A similar effect can also occur when some of the weak gauge groups and/or Higgs fields are confined to a brane [37].

4.2. Two branes and split fermions

When there are two or more branes, the lightest KK mode is generally a collective mode which does not decouple from the brane, and whose mass is not characterized by R^{-1}, but instead by $1/\sqrt{R r_c}$. This leads to the interesting possibility in which a first KK mode can be discovered, but higher modes (whose masses are characterized by R^{-1} and whose couplings to brane fields are suppressed) remain out of reach. This is very different from the transparent brane case in which one expects evenly spaced KK modes, so that the second mode has mass twice as large as the first mode and the same coupling strength. The discovery potential for the collective mode will be similar to existing searches for standard W' and Z' bosons, colorons [38], and so forth.

In models with low-scale quantum gravity, there is motivation to consider the possibility that quarks and leptons live on separate branes, in order to prevent dangerous operators which would mediate proton decay in conflict with existing bounds [2]. The simplest implementation of such a picture has two branes, one containing the quarks (and possibly the gluons) and the other containing the leptons. One is thus forced to consider the weak gauge bosons propagating in the bulk, and loops of the brane fermions should induce kinetic terms for the weak gauge fields localized on each brane.

Given the obvious asymmetry in the underlying dynamics which localized the quarks on one brane but the leptons on the other, it seems natural that one brane (*i.e.* the quark brane) could have a larger kinetic term than the other one (*i.e.* the lepton brane), and the results of Section 3.2.2 could be relevant to the phenomenology of the KK modes. This leads to two interesting variations on the usual phenomenology of bulk gauge fields. The first is that, owing to the larger repulsion from the quark brane than from the lepton brane, the KK modes may couple more weakly to quarks than to leptons. This would alter the expected relative production cross sections at, say high energy e^+e^- colliders and hadron colliders, and would further affect the branching ratios into a given species of fermion. Furthermore, at large momentum transfers, the two branes lose sight of each other, and at very high energies, quarks and leptons *miss* each other because of their separation in the extra dimension [39]. This is evident in our two-brane results for the effective coupling between fields located one on each of the two branes (see Fig. 6) which approaches zero at high Q^2. In contrast, quark–quark and lepton–lepton interactions remain appreciable even at large Q^2.

Models which separate not only quarks from leptons, but also left- and right-handed quarks and leptons from each other, may naturally explain the observed hierarchy of fermion masses [2–4] by generating small Yukawa couplings for the zero modes as the tiny overlap in fermion wave functions. Each localized fermion demands a renormalization of the gauge field whose shape is related to the profile of the fermion KK modes, and a full theory of flavor could have as many separate contributions as there are fermions. The resulting picture is therefore rather complicated and model-dependent, and is beyond the scope of this work, but we can divine some general features from the simple cases we have studied.

First, we would see the high energy suppression of cross sections outlined above for split quarks and leptons for any two different fermions, including same flavor fermions with different helicities! Since the induced localized gauge kinetic terms are sensitive to the shape of the fermion zero mode wave functions, production cross sections and decay branching ratios could be flavor-dependent in a complicated way. The properties of the KK gauge bosons thus provide one with a powerful test of extra-dimensional flavor dynamics, exploring the cartography of the extra dimension [40].

Second, models with split fermions have strong flavor-changing neutral current (FCNC) constraints from Kaluza–Klein modes of gauge fields [4, 41, 42], because while the gauge fields couple flavor-diagonally, the KK modes couple flavor-dependently, inducing FCNC's after the CKM rotation from the gauge to mass basis is performed. The limits derived in this way on R^{-1} from the Kaon system are quite strong, of order $R^{-1} \gtrsim 100$–1000 TeV. However, these limits could be relaxed quite substantially if appreciable r_c

terms are included. Such terms will force the KK modes of the gauge fields to try to avoid the places where fermions are localized, and would limit the strength of the FCNC's.

5. Conclusions

Theories with extra dimensions offer both unique solutions to the puzzles of particle physics as well as unique theoretical challenges. To date, all known descriptions must be regarded as effective theories, and without a deeper model to describe the underlying physics responsible for the compactification of dimensions, generation of branes and boundary conditions, and confinement of fields to brane world-volumes, the best one can do is to write down effective descriptions which are self-consistent. The theoretical motivations are many, and the resulting phenomenology intriguing.

We have explored a simple consequence of any theory with gauge fields in the bulk of the extra dimensions, and charged matter either in the bulk subject to orbifold boundary conditions, or confined to a brane. Radiative corrections to these theories mandate that such branes or boundaries are not transparent to the gauge fields — instead they are opaque. While the opacity of the brane, parameterized by the size of a kinetic term for the gauge field living on the brane or boundary, is not calculable in terms of other parameters in the theory without introducing assumptions about the nature of the UV completion, this does not justify ignoring it. Such terms may very well be large, and comprise an important part of relating a theory with extra dimensions to the real world. The effect on phenomenology can be sizable, and the result qualitatively different from the situation in which they are neglected. For example, charged fields confined to an opaque brane will decouple from the high Kaluza–Klein modes of the gauge field, contrasting with the standard picture under with all KK modes couple equally to brane fields. This decoupling of the higher KK modes is somewhat similar to the effect of dimensional deconstruction [43, 44], which replaces an extra dimension with a chain of 4d gauge theories linked by scalar fields. In the deconstructed case, the analogues of the KK modes of bulk fields naturally distort and terminate at some high energy scale. In the case of the brane kinetic term, the KK spectrum remains infinite, but nevertheless the coupling to brane fields becomes arbitrarily small for arbitrarily heavy modes.

In addition, the existence of local gauge kinetic terms implies there may be collective KK modes whose masses are not related to the size of the extra dimension, but instead to the size of the brane kinetic term. These collective modes typically do not decouple the way the higher KK modes do, and thus have unique phenomenology compared with the typical expectations of extra dimensional theories. Finally, theories which have different types of matter living at different locations in an extra dimension can show the interesting behavior that at very high energies interactions between different particles are suppressed. At very high energies, the particles *miss* each other because of their extra-dimensional separation.

Our framework has been five dimensional theories with gauge fields living in all five dimensions. We have chosen this framework because it is simple and predictive, but there are many alternatives to explore. Our results are representative for *any* bulk field, and suggest that a complete, self-consistent effective theory including compact dimensions has a few more parameters than one might naively guess. The appearance of divergences, which must be renormalized, implies that it is more generic to treat these effects as tree-level couplings, in contrast to the naive expectation that they arise as loop effects. It would also be interesting to explore larger numbers of compact dimensions, to see the results in theories with six or more dimensions. In addition, it would be exciting to see if our results could be exploited in model-building, allowing new extra dimensional theories to better explain the puzzles of the Standard Model.

The authors are grateful for conversations with I. Antoniadis, N. Arkani-Hamed, C. Csaki, E. Dudas, G. Gabadadze, D.E. Kaplan, M. Peskin, A. Pilaftsis, and S. Pokorski. Work at ANL is supported in part by the US DOE, Div. of HEP, Contract W-31-109-ENG-38.

REFERENCES

[1] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, *Phys. Lett.* **B429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, *Phys. Lett.* **B436**, 257 (1998).

[2] N. Arkani-Hamed, M. Schmaltz, *Phys. Rev.* **D61**, 033005 (2000).

[3] G.R. Dvali, M.A. Shifman, *Phys. Lett.* **B475**, 295 (2000).

[4] D.E. Kaplan, T.M.P. Tait, *J. High Energy Phys.* **0006**, 020 (2000); D.E. Kaplan, T.M.P. Tait, *J. High Energy Phys.* **0111**, 051 (2001).

[5] H.C. Cheng, B.A. Dobrescu, C.T. Hill, *Nucl. Phys.* **B589**, 249 (2000).

[6] N. Arkani-Hamed, H.C. Cheng, B.A. Dobrescu, L.J. Hall, *Phys. Rev.* **D62**, 096006 (2000).

[7] H.J. He, C.T. Hill, T.M.P. Tait, *Phys. Rev.* **D65**, 055006 (2002).

[8] L.J. Hall, Y. Nomura, D.R. Smith, `arXiv:hep-ph/0107331`.

[9] K.R. Dienes, E. Dudas, T. Gherghetta, *Phys. Lett.* **B436**, 55 (1998).

[10] K.R. Dienes, E. Dudas, T. Gherghetta, *Nucl. Phys.* **B537**, 47 (1999).

[11] G. Servant, T.M.P. Tait, `arXiv:hep-ph/0206071`.

[12] Y. Kawamura, *Prog. Theor. Phys.* **105**, 999 (2001).

[13] L.J. Hall, Y. Nomura, *Phys. Rev.* **D64**, 055003 (2001).

[14] L.J. Hall, Y. Nomura, T. Okui, D.R. Smith, *Phys. Rev.* **D65**, 035008 (2002).

[15] J. Lykken , S. Nandi, *Phys. Lett.* **B485**, 224 (2000).

[16] D.A. Dicus, C.D. McMullen, S. Nandi, `arXiv:hep-ph/0012259`.

[17] C.D. McMullen, S. Nandi, `arXiv:hep-ph/0110275`.

[18] T. Appelquist, H.C. Cheng, B.A. Dobrescu, *Phys. Rev.* **D64**, 035002 (2001).

[19] T.G. Rizzo, *Phys. Rev.* **D64**, 095010 (2001).

[20] C. Macesanu, C.D. McMullen, S. Nandi, `arXiv:hep-ph/0201300`.

[21] H.C. Cheng, K.T. Matchev, M. Schmaltz, `arXiv:hep-ph/0205314`.

[22] G.R. Dvali, G. Gabadadze, M. A. Shifman, *Phys. Lett.* **B497**, 271 (2001).

[23] M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros, C.E.M. Wagner, *Nucl. Phys.* **B609**, 499 (2001).

[24] I. Antoniadis, C. Bachas, E. Dudas, *Nucl. Phys.* **B560**, 93 (1999).

[25] L.E. Ibanez, R. Rabadan, A.M. Uranga, *Nucl. Phys.* **B542**, 112 (1999).

[26] C. Csaki, J. Erlich, T.J. Hollowood, Y. Shirman, *Nucl. Phys.* **B581**, 309 (2000).

[27] C. Csaki, J. Erlich, T.J. Hollowood, *Phys. Rev. Lett.* **84**, 5932 (2000).

[28] G.R. Dvali, G. Gabadadze, M. Porrati, *Phys. Lett.* **B485**, 208 (2000).

[29] G.R. Dvali, G. Gabadadze, *Phys. Rev.* **63**, 065007 (2001).

[30] G R. Dvali, G. Gabadadze, M. Kolanovic, F. Nitti, *Phys. Rev.* **D64**, 084004 (2001),

[31] H.C. Cheng, K.T. Matchev, M. Schmaltz, `arXiv:hep-ph/0204342`.

[32] H. Georgi, A.K. Grant, G. Hailu, *Phys. Lett.* **B506**, 207 (2001).

[33] A. Manohar, H. Georgi, *Nucl. Phys.* **B234**, 189 (1984).

[34] Z. Chacko, M.A. Luty, E. Ponton, *J. High Energy Phys.* **0007**, 036 (2000).

[35] R. Sekhar Chivukula, D.A. Dicus, H.J. He, *Phys. Lett.* **B525**, 175 (2002).

[36] M. Masip, A. Pomarol, *Phys. Rev.* **D60**, 096005 (1999); T.G. Rizzo, J.D. Wells, *Phys. Rev.* **D61**, 016007 (2000).

[37] A. Muck, A. Pilaftsis, R. Ruckl, *Phys. Rev.* **D65**, 085037 (2002).

[38] M.K. Pillai [CDF Collaboration], `arXiv:hep-ex/9608013`; K.M. Cheung, R.M. Harris, `arXiv:hep-ph/9610382`.

[39] N. Arkani-Hamed, Y. Grossman, M. Schmaltz, *Phys. Rev.* **D61**, 115004 (2000).

[40] T.G. Rizzo, *Phys. Rev.* **D64**, 015003 (2001).

[41] C.D. Carone, *Phys. Rev.* **D61**, 015008 (2000).

[42] A. Delgado, A. Pomarol, M. Quiros, *J. High Energy Phys.* **0001**, 030 (2000).

[43] N. Arkani-Hamed, A.G. Cohen, H. Georgi, *Phys. Rev. Lett.* **86**, 4757 (2001)

[44] C.T. Hill, S. Pokorski, J. Wang, *Phys. Rev.* **D64**, 105005 (2001).