FOR A FEW SYMMETRIES MORE ... OR HOW TO COMPUTE THE HIGGS MASS*

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Dedicated to Stefan Pokorski on his 60th birthday

The new idea of *deconstruction* allows to realize the physics of extra dimensions in a strictly four dimensional set-up. After a short review of these techniques extended to supersymmetry, I will report on an application to build models in which the low energy spectrum shows no sign of supersymmetry but still the radiative corrections to the mass parameters are weakly dependent on the cutoff scale, if this one remains low enough. As a consequence, the Higgs mass dependence in high energy physics is effectively parametrized by the deconstruction scale which also fixes the gauge boson masses through the Higgs vacuum expectation value. In this regard, deconstruction, somehow as gauge invariance, is a dynamical principle that dictates the interactions of particles and gauge fields at low energy and quantum level.

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1. Electroweak symmetry breaking and the quest for new physics

The structure of Stefan Pokorski's career and works nicely reproduces the structure of the Standard Model of Particle Physics. Indeed an intrusive inspection on Spires (`find a Pokorski,s and topcite 100+`) will easily convince anyone that his papers belong to three different sectors: the gauge sector [1], the flavor sector [2] and the electroweak symmetry breaking sector [3]. As for all of us, his interest in the mechanism of $SU(2)_L \times U(1)_Y$

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breaking is certainly motivated by its potential relevance in the illustrious quest for new physics. I will report here on his last proposal for an electroweak symmetry breaking scenario [5], which I had the honor and the pleasure to think of with Stefan and his youngest collaborator Adam Falkowski. This proposal makes use of the notion of deconstruction theories pioneerly unraveled by Stefan and his collaborators and that I will review in the next Section.

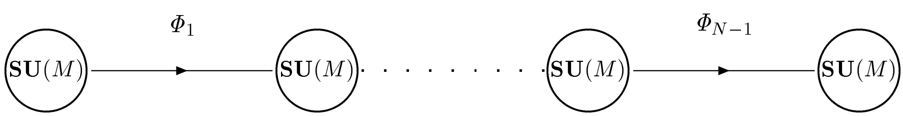
The ElectroWeak Symmetry Breaking (EWSB) sector is essentially characterized by two unknown parameters: the Higgs mass and the cutoff scale of the Standard Model (SM). These two parameters have escaped any direct measurements so far. Yet theoretical consistency constraints like the notorious unitarity, triviality, stability *&* naturality bounds (for a recent review, see for instance [4]) as well as indirect measurements through electroweak precision data jeopardize our current understanding of the SM quantum structure and contrive any extension beyond the SM. In revealing a deep connection between physics at high energies and EWSB, the two papers [6] were of primordial importance: they established that, while supersymmetry fixes the tree level Higgs potential, soft breaking terms induce radiative corrections that trigger EWSB at one loop. The only remaining problem was thus to understand how supersymmetry itself is broken. The standard lore for the last twenty years was that supersymmetry is broken in a hidden sector and this breaking is mediated through gravity or gauge interaction to the visible sector. Unfortunately, the low energy theory contains more than one hundred of parameters that have to be somehow fine-tuned to pass all phenomenological tests. Meanwhile, inspired by string theory, an alternative and more geometrical approach to break supersymmetry was worked out [7] using compactification of extra dimension. The idea is to impose different boundary conditions to the different components of a supersymmetric multiplet. While the low energy theory may break all the supercharges of the high energy theory, it has been realized recently [8] that the local symmetry properties along the extra dimension, even if globally broken, can still dictate the structure of the theory at low energy and then protect it from harmful radiative corrections. The price to pay is to deal with higher dimensional gauge theories plagued by non-renormalizability.

2. Deconstruction

The recent progresses of String Theories have led physicists to a rethinking of the nature of space-time dimensions [9]. First T-duality nullifies the notion of large and small dimension since the spectrum $M_{n,m} = n/R + mR/l_{\rm s}^2$ is invariant under the exchange of Kaluza–Klein and winding modes when $R \leftrightarrow l_{\rm s}^2/R$ ($l_{\rm s}$ is the string length scale). Even more, S

duality teaches us that a space dimension is an emergent phenomena associated to a strongly coupled dynamic: the size of the eleventh dimension of M-theory, $R = g_{\rm s} l_{\rm s}$, really opens up non perturbatively from a ten dimensional strongly coupled string theory. In an extreme step, the Matrix Theory description of M-theory proposes to abolish all space dimensions and to reduce string theory to quantum mechanics over a space of matrices.

Last year Stefan, Hill and Wang [10], and simultaneously Arkani-Hamed, Cohen and Georgi [11], realized that one does not have to rely on string theory to taste the emergence of extra dimensions. Actually even in high school anyone experienced such a phenomena: simple point like balls linked to each other by springs behave, at distances larger than the distance between the balls, as a true one-dimensional string [12]. This mechanical analogy allows to construct 5D gauge theories out of 4D gauge theories. The energy of the spring-ball system, $E = \sum_i \frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} k_i (x_i - x_{i+1})^2$, becomes a Lagrangian describing the dynamics of several copies of interacting 4D gauge fields A_i: $\mathcal{L} = \sum_i -\frac{1}{4} F^i_{\mu\nu} F_i^{\mu\nu} + \frac{1}{2} m^2 (A^i_\mu - A^{i+1}_\mu)^2$. In order to preserve unitarity, the hoping mass term advantageously emerges dynamically in a Higgs phase when link scalars ϕ_i, transforming as bifundamental under two adjacent gauge groups, acquire a vacuum expectation value (note that the link scalars can themselves describe fermionic technicolor-like condensates of asymptotically free theories). This approach is nothing but a latticization of the fifth dimension where the link scalars are Wilson lines connecting two nearest-neighbor gauge groups. In the Higgs phase, the product of gauge groups is broken to the diagonal gauge group and the broken-massive gauge bosons can be identified with the KK modes of a 5D gauge theory. More precisely, the following theory space configuration:



after spontaneous breaking, $\phi_i = v\mathbf{1}$, involves one massless SU(M) gauge boson in interaction with $N-1$ massive gauge bosons $m_{(k)} = 2gv \sin k\pi/2N$, $k = 1 \ldots N-1$. The structure of the interactions exactly matches with the KK modes one resulting from a $S^2/\mathbb{Z}_2$ orbifold compactification and, in the large N limit, the spectra also agree. There is a one to one correspondence between the three parameters defining the *moose* theory (g, N and v) and the three parameters defining the 5D orbifold theory (the gauge coupling g_5, the radius R of the orbifold and the cutoff scale Λ): $g_5 = \sqrt{g/v}, R = N/(\pi g v), \Lambda = \pi g v$. The KK states of the extra dimension arise from having many gauge symmetries in four dimensions.

This approach can be extended to supersymmetric gauge theories [11, 13]. The gauge fields are promoted to full 4D $\mathcal{N} = 1$ vector superfields while the link scalars are promoted to chiral superfields. In order to lift flat directions and to give masses to singlets, a superpotential is introduced: $W = \mu^{2-M}\sum_{i=1}^{N} S_i(\det \Phi_i - v^M)$, where S_i are Lagrange multiplier chiral superfields. The resulting spectrum is

	Scalar	Fermion	Vector
$m = 0$	$2(M^2-1)$ real	$2(M^2-1)$ Weyl	M^2-1
$m_{(k)}$	M^2-1 real	M^2-1 Dirac	M^2-1

and exhibits a 4D $\mathcal{N}=2$ structure which also results from the discrete Lorentz invariance with the same speed of light along the fifth dimension as for the other four.

The introduction of matter requires a doubling (matter and mirror matter) to construct a supersymmetric hopping superpotential:

$$W = \sum_i y_i\, \tilde{Q}_i \Phi_i Q_{i+1} + \sum_i m_i\, \tilde{Q}_i Q_i\,.$$

(Q_i, resp. $\tilde{Q}_i$, transforms as fundamental, resp. antifundamental, representation of $\mathrm{SU}(M)_i$.) The $\mathcal{N}=2$, or 5D Lorentz invariance, is preserved if and only if the Yukawa couplings y_i are equal to the gauge couplings. Furthermore, the resulting $\mathcal{N}=2$ hypermultiplet will be massless when $m_i = -gv$.

As I will describe in the next section, one use of deconstruction (supersymmetric) theories is the breaking of supersymmetry in the low energy effective action. Indeed a hard supersymmetry breaking in theory space can manifest itself softly below the deconstruction scale v in the sense that radiative corrections to the Higgs mass will be sized by v and not the cutoff scale of the theory (provided that this one remains low enough).

3. Soft electroweak breaking from hard supersymmetry breaking

Deconstruction helps to tackle the hierarchy problem in the sense that divergences in non-supersymmetric theories can be considerably softened compared to generic 4D theories[1]. The aim of the following toy model based on replication of SU(2) 4D gauge groups (we will comment later on the introduction of $\mathrm{U}(1)_{\mathrm{Y}}$ interactions) is to illustrate this softening.

We start with $\mathcal{N}=1$ supersymmetric models consisting of a chain of N gauge groups which communicate to each other through $N-1$ bifundamental link superfields Φ_i. To realize the SM matter and Higgs fields in the bulk we need to deconstruct 5D hypermultiplets. To this end, to every gauge group

[1] Recently, another approach to EWSB in deconstruction has been proposed [14].

we attach a set of chiral multiplets: 'Higgs doublets' and 'quark doublets', in the fundamental of the i-th group and their mirror partners with opposite quantum numbers. The superpotential is chosen as[2]:

$$W = \sum_{i=1}^{N-1} y_i^h \tilde{H}_i \Phi_i H_{i+1} - \sum_{i=1}^{N} m_i^h \tilde{H}_i H_i + \sum_{i=1}^{N-1} y_i^q \tilde{Q}_i \Phi_i Q_{i+1} - \sum_{i=1}^{N} m_i^q \tilde{Q}_i Q_i \,. \quad (1)$$

To complete the Standard Model quark spectrum we need to add right-handed quark multiplets U_i and D_i and their mirrors. Since no color or hypercharge group is present in our toy-model these fields are singlets. The superpotential is chosen as $W = \sum_{i=1}^{N-1} y_i^u \tilde{U}_i U_{i+1} - \sum_{i=1}^{N} m_i^u \tilde{U}_i U_i$ and analogously for D_i. In order to get the Yukawa interactions of the Higgs boson with the up-quarks it is sufficient to add to the superpotential the Yukawa term which involves only the superfields from the first site $W = \lambda\, Q_1 H_1 U_1$.

At this point one could proceed towards the phenomenological models in the standard way, that is add soft terms to obtain splittings of the multiplets and to trigger the electroweak symmetry breaking. Deconstruction allows to investigate an alternative road.

3.1. Yukawa loop corrections to the Higgs boson mass

Generically, the dominant contribution to the one-loop Higgs mass is due to Yukawa interactions with the top quarks. In SM this contribution is quadratically divergent, while in the MSSM the quadratic divergence is canceled by the top squark loops. Here, we analyze the set-up where such boson-fermion cancellation occurs when we break supersymmetry *in a hard way* by removing some of the degrees of freedom in a non-supersymmetric way. For the discussion of divergences it is irrelevant what is the precise pattern of the breaking; the only important thing is that the part of the Lagrangian involving the fields of the first site maintains the supersymmetric form. In particular, we assume that all supertraces at the first site are vanishing.

If the link vevs are absent it is clear that at one-loop Yukawa interactions do not feel the supersymmetry breaking on the other end of the chain. Thus, the one-loop radiative correction to the h_1 squared mass proportional to the Yukawa coupling λ are absent. As soon as we switch on the link vevs, the fields living at different sites are allowed to mix and we have to perform an orthogonal transformation to diagonalize the mass matrix. Since supersymmetry is broken, generically the spectrum is completely non-supersymmetric (boson and fermion masses will be different and there can

[2] From now, x_i and ψ_{X_i} will denote respectively the scalar and the chiral fermionic components of the chiral superfield X_i. The mass eigenstates will be denoted with parenthesized subscripts: $x_{(m)}$ and $\psi_{X_{(m)}}$ with masses $m^x_{(m)}$ and $m^X_{(m)}$.

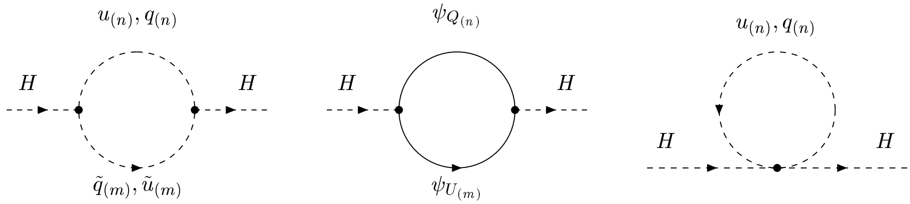


Fig. 1. One-loop diagrams involving the top Yukawa coupling and contributing to the squared mass of the Higgs boson.

be a different number of bosonic and fermionic degrees of freedom). However, the λ-proportional corrections to the Higgs mass are still controlled by the first site and, as a consequence, they are finite! To see this we need to perform an orthogonal transformation to express the original fields in terms of the mass eigenstates: $q_i = \sum_n a^q_{i\,n} q_{(n)}$, $\psi_{Q_i} = \sum_n b^Q_{i\,n}\psi_{Q_{(n)}}$. The zero mode Higgs mass receives one-loop radiative corrections proportional to the Yukawa coupling through the diagrams depicted in Fig. 1. The divergences in δm^2 are found to be proportional to:

$$\delta m^2 \sim \Lambda^2 \left(\sum_n |a^q_{1\,n}|^2 - \big(\sum_n |b^Q_{1n}|^2\big)^2\right) + 2\,\ln\Lambda^2 \left(m^Q m^{Q\dagger} - m^{q2}\right)_{11} . \quad (2)$$

The coefficient of the quadratic divergence vanishes by the fact, that $a^q_{i\,n}$, $b^Q_{i\,n}$ are coefficients of the orthogonal transformation diagonalizing the squark and quark squared mass matrices, respectively (similar relations have also been used to simplify the logarithmically divergent part). This leads to the conclusion that the Higgs mass gets logarithmically divergent contribution proportional to the supertrace in the quark sector at the first site, which we assumed to vanish. Thus, in spite of the fact that the theory is non-supersymmetric, the Higgs mass (in fact the same holds for the squarks) gets, from the Yukawa interactions, only a finite one-loop correction to its mass. These conclusions hold even if the model has a different number of bosonic and fermionic degrees of freedom!

To illustrate this discussion we present a construction inspired by the five-dimensional of the BHN model [8]. Arriving at the spectrum of [8] involves some tunings of the parameters. But we stress that these tunings are by no means important for the cancellation of divergences; they serve only to obtain simple mass matrices, so that formulae for the Higgs boson mass can be evaluated explicitly. So we tune the parameters as (g_0 is the common gauge coupling):

$$y^h_i = y^q_i = y^d_i = y^u_i = g_0 \quad \text{and} \quad m^h_i = m^q_i = m^d_i = m^u_i = g_0 v\,. \qquad (3)$$

We also add Φ_N link-Higgs, as in Fig. 2, which we need to avoid a massless gaugino mode.

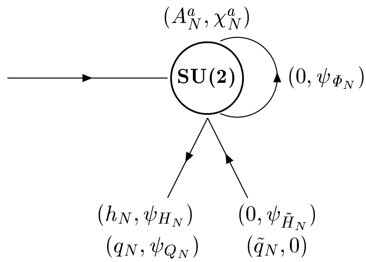


Fig. 2. The magnifying glass view of the N-th site of the model. The fields ϕ_N, $\tilde{h}_N$ and $\psi_{\tilde{Q}_N}$ have been removed in order to break supersymmetry and induce a mass splitting in the low-energy theory.

We break supersymmetry by setting ϕ_N, $\tilde{h}_N$ and $\psi_{\tilde{Q}_N}$ — the mirror components of the SM fields — to zero in the Lagrangian (see Fig. 2). This is, of course, hard breaking of supersymmetry, as some of the fields at the last site lose their superpartners (a similar supersymmetry breaking has also been proposed in Ref. [15]). At the massless level we have only the gauge field, quarks and the Higgs boson. Their lightest superpartners have masses $\tilde{m}_{(1)} \sim g_0 v/(2N+1)$ and include a Dirac gaugino, *two* squarks for every quark and a Dirac Higgsino.

The one-loop radiative correction to the Higgs mass can now be computed explicitly and after some algebra, we arrive at:

$$\delta m^2 = -\lambda_t^2\, g^2 v^2\, F(\Lambda, N)\,, \tag{4}$$

where $F(\Lambda, N)$ is a pure numerical factor given by:

$$F(\Lambda, N) = \frac{N^3}{\pi^2} \int\limits_0^X dx\, \mathrm{ch}\, x\, \mathrm{sh}^3\, x\, \frac{(\mathrm{ch}\, 2x + 1)(\mathrm{ch}\, 2x + 2\mathrm{ch}\, 4Nx - 1)}{\mathrm{sh}^2\, 2Nx\, \mathrm{ch}^2\, (2N+1)x} \tag{5}$$

the cutoff X being related to the cutoff scale of the theory by $\Lambda = 2g_0 v\, \mathrm{sh} X$. The normalized coupling, $\lambda_t = \lambda/N^{3/2}$, is the Yukawa coupling of the zero mode Higgs to the zero mode quarks, *i.e.*, the Yukawa coupling of the effective SM; similarly, $g = g_0/\sqrt{N}$ is the zero mode SU(2) gauge coupling. Notice that according to our general discussion, $F(\Lambda, N)$ is finite when Λ goes to infinity.

We have shown in general that one-loop corrections in certain non-supersymmetric theories can be surprisingly softened. What about two- and higher-loop corrections? The one-loop cancellation of quadratic divergences depends crucially on the tree-level equality of the Yukawa and 4-scalar couplings of the Higgs field on the first site. However, due to the mass splitting between quarks and squarks these couplings are renormalized differently. Thus we expect quadratic divergences to reappear at the two-loop level.

3.2. Gauge loop corrections to the Higgs boson mass

Except for the top-Yukawa couplings there are other sources of quadratic divergences which are proportional to the gauge coupling or to the Yukawa couplings to the link fields. Following the discussion of the top-Yukawa contributions we analyze the general conditions to avoid any quadratic divergences. The first potential source originates from the couplings of the Higgs field to the gauge multiplet and to itself in the D-term scalar potential. The second source comes from the F-term of the superpotential. The situation is qualitatively different than in the case of top-Yukawa contribution, as interactions occur at all sites. To avoid quadratic divergences proportional to g_0 we have to ensure that at every site the full Higgs multiplet interacts with the full gauge multiplet.

Similarly, for quadratic divergences proportional to y_i to be absent, we need full link and mirror multiplets to be present at the i-th site. Note that, since $y_N \equiv 0$, adding or removing scalar link and mirror degrees of freedom at the N-th site has no consequence for the divergence of the Higgs mass. We used this fact in our model and placed the hard supersymmetry breaking sector at the N-th site.

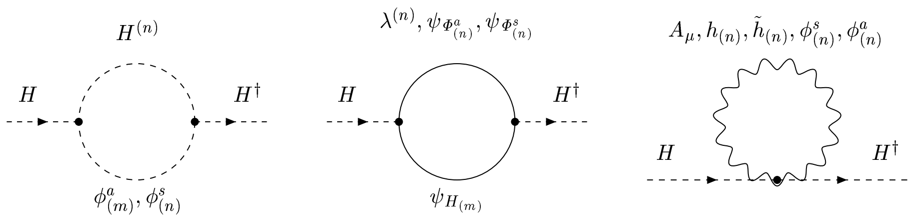


Fig. 3. One-loop diagrams involving gauge interactions and contributing to the squared mass of the Higgs boson.

In our specific model these interactions are all proportional to the gauge coupling (as we tuned the link-Yukawa couplings with the gauge coupling). The diagrams that contribute to the one-loop zero mode Higgs mass are depicted in Fig 3. They give:

$$\delta_{m^2} = -g^4 v^2 \, G(\Lambda, N) \,, \tag{6}$$

where g is the zero mode SU(2) gauge coupling and v is the deconstruction scale. The numerical factor $G(\Lambda, N)$ is given in terms of a complicated integral explicitly written in [5]. As it can be checked and following our general discussion, this integral is free of quadratic divergence. However, it exhibits a logarithmic divergence in the cutoff scale:

$$\delta m^2 = \frac{6}{8\pi^2}\, g^4 v^2\, N \ln \frac{\Lambda}{2gv\sqrt{N}}\,. \tag{7}$$

The cutoff dependence is similar as in the softly broken supersymmetry, but the $M_{\rm SUSY}$ scale is replaced here by the deconstruction scale v. If v is close to the weak scale (which is the case as long as N is not too large) then the one-loop sensitivity to the cutoff is weak and the only dependence in high energy physics to the Higgs mass is parametrized by the deconstruction scale, v.

3.3. One-loop effective potential and EWSB

The previous evaluation of the Yukawa and gauge radiative corrections to the zero mode Higgs mass suggests that they will trigger the electroweak symmetry breaking. To study in full details this breaking, we need now to compute the one-loop effective potential given by:

$$V = \frac{1}{2}\,{\rm Tr} \int \frac{d^4 k_E}{(2\pi)^4}\, \ln \frac{k_E^2 + m_b^2(v_{\rm H})}{k_E^2 + m_f^2(v_{\rm H})}\,, \tag{8}$$

where m_b and m_f are respectively the bosonic and fermionic mass matrices as functions of the vev of the Higgs field, $v_{\rm H}$[3]. We consider only the top-multiplet contribution and the dependence on the Higgs vev is coming from the Yukawa interactions localized on the first site only.

For the *stop sector*, we obtain the following squared mass matrix (m, n, p, $q = 1 \ldots N$):

$$\begin{pmatrix} \tilde{m}^2_{(m)} \delta_{mp} + a_b\, c(m)\, c(p) & b_b\, \tilde{m}_{(m)}\, c(m)\, c(q) \\[2ex] b_b\, \tilde{m}_{(p)}\, c(n)\, c(p) & \tilde{m}^2_{(n)} \delta_{nq} \end{pmatrix}, \tag{9}$$

where we have defined: $c(m) = \cos \frac{(2m-1)\pi}{4N+2}$, and the two coefficients a_b and b_b are related to the Yukawa coupling as:

$$a_b = \frac{4\lambda^2 v_{\rm H}^2}{(2N+1)N} \quad \text{and} \quad b_b = -\frac{4\lambda v_{\rm H}}{(2N+1)\sqrt{N}}\,. \tag{10}$$

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[3] In this paper, our convention is to define $v_{\rm H}$ as the vev of the complex Higgs field, *i.e.*, $v_{\rm H} \sim 174$ GeV.

Note that squarks mix with mirror squarks once the electroweak symmetry is broken.

Similarly in the *top sector*, the squared mass matrix reads ($m,p = 0\dots N-1$, $n,q = 1\dots N-1$):

$$\begin{pmatrix} m^2_{(m)}\delta_{mp} + a_f\,\eta_m\eta_p\,d(m)\,d(p) & b_f\,\eta_m\,m_{(q)}\,d(m)\,d(q) \\ b_f\,\eta_p m_{(n)}\,d(n)\,d(p) & m^2_{(n)}\delta_{nq} \end{pmatrix}, \tag{11}$$

where we have now defined $d(m) = \cos\frac{m\pi}{2N}$, and the two coefficients a_f and b_f are given by:

$$a_f = \frac{2\lambda^2 v_{\mathrm{H}}^2}{N^2} \quad \text{and} \quad b_f = -\frac{2\lambda v_{\mathrm{H}}}{N\sqrt{N}}\,. \tag{12}$$

Let us mention that two important supertraces are independent of the vev of the Higgs:

$$\mathrm{S\,Tr}\,M^2 = 2\,g_0^2 v^2 \quad \text{and} \quad \mathrm{S\,Tr}\,M^4 = 6\,g_0^4 v^4\,. \tag{13}$$

This ensures that the one-loop potential for v_{H} has no divergent dependence on the cutoff of the theory: the EWSB is triggered by the low energy physics and is not dominated by unknown physics that will be revealed at or above the cutoff scale. Adding the tree-level Higgs self-coupling originating from the D-terms, we get:

$$V(v_{\mathrm{H}}) = \frac{1}{8}g^2 v_{\mathrm{H}}^4 + \frac{3}{16\pi^2}\,\mathrm{S\,Tr}\,m^4 \ln\left(\frac{m^2}{2g_0 v}\right)\,, \tag{14}$$

where the supertrace is over the $2N$ bosonic and $2N-1$ fermionic eigenvalues of the matrices (9) and (11).

Let us now turn to the determination of the spectrum. The secular equation of the stop squared mass matrix is given by:

$$1 - \frac{16\lambda^2 v_{\mathrm{H}}^2}{N(2N+1)^2}\rho^2 \left(\sum_{m=1}^{N} \frac{\cos^2\frac{(2m-1)\pi}{4N+2}}{\tilde{m}^2_{(m)} - \rho^2}\right)^2 = 0 \tag{15}$$

which, using some remarkable identities, can be rewritten as a polynomial equation of degree $2N$:

$$RT_N(1-x^2) = \tau\, x\, RU_{N-1}(1-x^2)\,, \tag{16}$$

where x is the dimensionless eigenvalue $x = \rho/(2g_0 v)$, $\tau = \lambda_t v_{\rm H} N/(g_0 v)$ characterizes the Higgs vev in units of $g_0 v$, and RT_N and RU_{N-1} are the reduced Chebyshev polynomials:

$$RT_N(X) = \frac{T_{2N+1}(\sqrt{X})}{\sqrt{X}} \quad \text{and} \quad RU_{N-1}(X) = \frac{U_{2N-1}(\sqrt{X})}{\sqrt{X}}\,. \tag{17}$$

Similarly, the fermionic secular equation is

$$1 - \frac{4\lambda^2 v_{\rm H}^2}{N^3}\rho^2 \left(\sum_{m=0}^{N-1} \eta_m^2 \, \frac{\cos^2 \frac{m\pi}{2N}}{m_{(m)}^2 - \rho^2} \right)^2 = 0 \tag{18}$$

and it can be written in the form of a polynomial equation of degree $2N-1$:

$$RT_{N-1}(1-x^2) = \tau^{-1}\, x \, RU_{N-1}(1-x^2)\,. \tag{19}$$

All the pieces are now set to numerically evaluate the potential $V(v_{\rm H})$ and find its minimum. The results are plotted in Fig. 4 for different values of the replication number N. The Higgs mass after EWSB becomes a function of low energy parameters only: the top Yukawa coupling, the Higgs vev, the

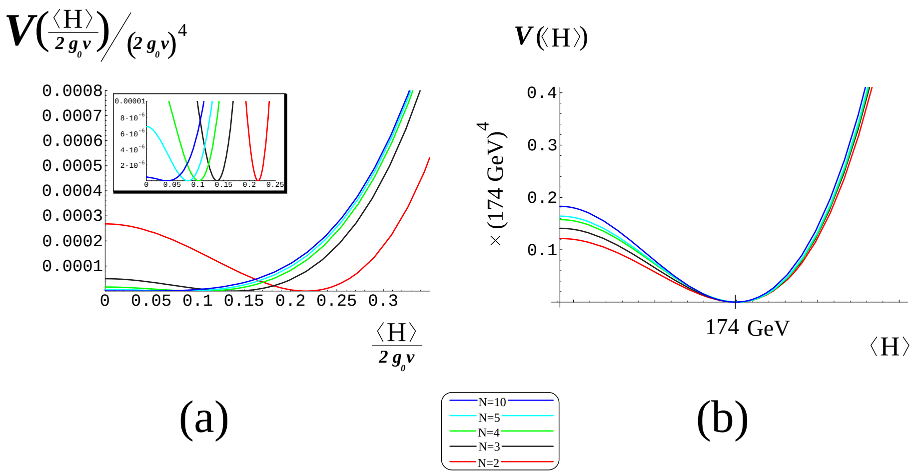

Fig. 4. One-loop effective potential for the Higgs scalar field for different value of the replication number N. As in softly broken supersymmetric theories, the tree level Higgs potential is fixed by symmetry and radiative corrections trigger EWSB. The Higgs vev is a function of the top Yukawa coupling, the SM gauge coupling constants and the deconstruction scale. Figure (a) plots the potential in units of the deconstruction scale. When the Higgs vev is fitted to its phenomenological value (figure (b)), a prediction for the Higgs mass is obtained.

electroweak gauge coupling and the replication number. The deconstruction scale, v is indeed fixed in terms of the low energy parameters once the Higgs vev is fitted to its phenomenological value. Some numerical results are given in Table I for the Higss mass, the stop mass, the first KK excitation of the top and the deconstruction scale, v.

TABLE I

Spectrum after EWSB for different values of the replication number, N.

	Higgs mass (GeV)	stop mass (GeV)	top first KK (GeV)	v (GeV)
$N = 2$	158	142	502	437
$N = 3$	166	158	532	565
$N = 4$	170	161	533	664
$N = 5$	172	167	537	745
$N = 10$	178	164	517	1051

3.4. Towards a more realistic model

Finally, we comment on how the toy-model presented here can be extended to match the Standard Model phenomenology. The obvious missing pieces are:

- **SU(3) color group** In the real world quarks transform in the **3** or $\bar{\mathbf{3}}$ representation of the color group. Replicating SU(3) gauge group so as to obtain only one octet of massless gluon and superpartners separated by a mass gap does not bring any complication. One introduces a set of link-Higgs superfields Γ_i transforming as $(\mathbf{3_i}, \bar{\mathbf{3}}_{\mathbf{i+1}})$ and the rest of the construction is analogous to the SU(2) case. The problem appears when we want to obtain the KK-tower for the quark doublets; the gauge invariant superpotential must now have the form $W = g_0 \sum(\frac{1}{v}\tilde{Q}_i \Phi_i \Gamma_i Q_{i+1} - v\tilde{Q}_i Q_i)$ and leads to non-renormalizable interactions. A more satisfactory alternative which allows to maintain renormalizability is *not* to replicate the color gauge group and assume that all quark superfields Q_i, U_i and D_i are charged under a single SU(3). Then the superpotential for these superfields has the same form as in the pure SU(2) case. Of course, then the model has no extra-dimensional interpretation but this does not change any conclusions about softness of the radiative corrections. It is nothing but a deconstructed version of a brane-world scenario where QCD interactions are localized on the brane while weak interactions are free to propagate in the bulk.

- **U(1) hypercharge group** Similar problems as in the SU(3) case arise when we replicate the hypercharge group: writing an invariant superpotential so as to get the KK-tower of quarks and leptons implies non-renormalizable interactions. In addition, one must worry about anomalies, which must be compensated, *e.g.*, by deconstructed Wess–Zumino terms [16]. Therefore, the more plausible alternative is not to replicate the hypercharge group. One avoids non-renormalizable interactions and as a byproduct the anomalies automatically cancel. Indeed, the fermion spectrum at all sites but the last one is vector-like: every fermion is accompanied by the mirror fermion with opposite quantum numbers. At the N-th site the fermion spectrum is the same as in the MSSM. However, the scalar content is different and tr$Y$ does not vanish, which is a source of quadratic divergences[4]. Moreover since the $\mathrm{U(1)_Y}$ interactions break translational invariance along the fifth dimension, exact cancellation between boson and fermion loops does not hold. Actually, the $\mathrm{U(1)_Y}$ interactions give a quadratic divergent contribution to the Higgs zero mode mass:

$$\delta m^2 = \frac{3}{2}\frac{{g'}^2\Lambda^2}{16\pi^2}\,.$$

  Requiring that this contribution remains subdominant requires a relatively low cutoff scale around 5 TeV.

4. Conclusion

Deconstruction, somehow as gauge invariance, is a dynamical principle that contributes to dictate the interactions of particles and gauge fields. As advocated in this proceeding, deconstruction can be a useful tool to break supersymmetry without relying on an unknown and highly constrained soft sector: indeed even if supersymmetry is hardly broken locally in theory space, the structure of interactions as resulting from deconstruction is such that the Higgs potential at one loop remains governed by the symmetry of the theory (as long as the cutoff scale remains below around 5 TeV) and does not depend in the details of the physics at high energy that effectively manifests itself at low energy through the deconstruction scale. Then the top Yukawa couplings trigger EWSB and this breaking is dictated by low energy parameters only.

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