# BRANE WORLD SUPERSYMMETRY, DETUNING, FLIPPING AND ORBIFOLDING

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*Dedicated to Stefan Pokorski on his 60th birthday*

We emphasize the necessity of a delicate interplay between the gauge and gravitational sectors of five-dimensional brane worlds in creating phenomenologically relevant vacua. We discuss locally supersymmetric brane worlds with unflipped and flipped fermionic boundary conditions and with matter on the branes. We point out that a natural separation between the gauge and gravity sectors, very difficult in models with true extra dimensions, may be achieved in 4d models with deconstructed dimensions.

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## 1. Introduction

There exist locally supersymmetric theories in five dimensions that include nontrivial physics localized on four-dimensional branes [1–9]. The brane sectors may contain arbitrary four-dimensional gauge theories as well as localized interactions between bulk fields. In the bulk one has a gauged 5d supergravity, which can be further coupled to five-dimensional gauge sectors. Such five-dimensional models with branes are likely to lead to interesting extensions of the Standard Model, pertaining to novel approaches to the hierarchy problem. In general it is easier to separate (super)gravity from fields which are charged under the gauge group, and to study globally supersymmetric five-dimensional gauge models with branes, instead of working with the complete matter-supergravity Lagrangian. However, in the context

of brane worlds, where the geometry of the extra dimension may play a non-trivial role, this approach is not sufficient. The potentials that appear in the bulk and on the branes due to gauge sector interactions couple to moduli fields which serve as sources in the Einstein equations, hence they back-react on the geometry. In fact, the gauge sector potential should be studied simultaneously with the issue of the stability of the orbifold, otherwise the conclusions drawn from the simplified models with decoupled gravity may turn out to be misleading. The issue of constructing an explicit model with a general bulk and brane non-Abelian gauge sectors coupled consistently to supergravity is a fairly complex one, and at present one has to rely on simpler constructions. In this paper we summarize the attempts made in this direction in published work, and in forthcoming publications.

## 2. Five-dimensional supergravities on $S^1/Z_2$

### *2.1. Supergravity Lagrangian: detuning and flipping*

Let us summarize the basic features of pure 5d $N = 2$ gauged supergravity on $S^1/Z_2$. The gravity multiplet $(e_\alpha^m, \psi_\alpha^A, \mathcal{A}_\alpha)$ consists of the vielbein, a pair of symplectic Majorana gravitini, and a vector field called the graviphoton. There is a global SU(2) $R$-symmetry which rotates the two supercharges into each other. Making use of the graviphoton we can gauge a U(1) subgroup of the $R$-symmetry group. Such gauging can be described by an SU(2) algebra valued matrix $\mathcal{P} = \vec{P} \cdot i\vec{\sigma}$ (prepotential). We do not give the complete form of the action and supersymmetry transformation laws in gauged supergravity (see [10] for details), but only the relevant terms. The gravitino transformation law gets the following correction due to gauging (we use the normalization of [11])

$$\delta\psi_\alpha^A = -i\frac{\sqrt{2}}{3}\gamma_\alpha g \mathcal{P}_B^A \varepsilon^B \,, \tag{1}$$

where $g$ is the U(1) gauge charge. Gauging introduces also the potential term into the action

$$V = \frac{8}{3} g^2 \mathrm{Tr}(\mathcal{P}^2) \,. \tag{2}$$

Without bulk matter fields the prepotential is just a constant matrix so the potential term corresponds to a (negative) cosmological constant.

The distinguishing feature of the brane-bulk scenario is that the fifth dimension is an orbifold $S_1/Z_2$ with branes located at the fixed points. It is equivalent to work on a smooth circle $S_1$ with the fifth coordinate ranging from $-\pi\rho$ to $\pi\rho$ and impose $Z_2$ symmetry on the fields of the Lagrangian. The $Z_2$ symmetry acts by $x^5 \to -x^5$. Under its action the bosonic fields

$g_{\mu\nu}$, $g_{55}$ and $\mathcal{A}_5$ are even, while $g_{\mu 5}$ and $\mathcal{A}_\mu$ are odd. The $Z_2$ action on the gravitino is defined as follows

$$\psi_\mu^A(-x^5) = \gamma_5 \mathcal{Q}_B^A \psi_\mu^B(x^5) \;\; \psi_5^A(-x^5) = -\gamma_5 \mathcal{Q}_B^A \psi_5^B(x^5)\,, \tag{3}$$

where $\mathcal{Q} = \vec{Q}\cdot\vec{\sigma}$ and $\vec{Q}^2 = 1$. The $Z_2$ action on the supersymmetry generating parameter $\varepsilon$ must be the same as that on the 4d components of the gravitino.

Further we need to define the $Z_2$ symmetry under reflection around the second fixed point at $x^5 = \pi\rho$

$$\begin{aligned} \psi_\mu^A(\pi\rho - x^5) &= \alpha\gamma_5 \mathcal{Q}_B^A \psi_\mu^B(\pi\rho + x^5)\,, \\ \psi_5^A(\pi\rho - x^5) &= -\alpha\gamma_5 \mathcal{Q}_B^A \psi_5^B(\pi\rho + x^5)\,. \end{aligned} \tag{4}$$

Apart from the conventional case $\alpha = 1$, in this letter we also consider the 'flipped' supersymmetry with $\alpha = -1$. In the latter case supersymmetry is always broken globally, as different spinors survive the orbifold projection on each wall. Note also, that in the flipped case we have $\psi_\alpha^A(x^5 + 2\pi\rho) = -\psi_\alpha^A(x^5)$.

It is straightforward to check that the 5d *ungauged* supergravity action is invariant under transformations (3) and (4) but the *gauged* supergravity action is not invariant if the prepotential $\mathcal{P}$ is a general one. The action and the supersymmetry transformation laws are $Z_2$ invariant if we choose the prepotential in the form:

$$g\mathcal{P} = g_1 \varepsilon(x^5)\mathcal{R} + g_2 \mathcal{S}\,, \tag{5}$$

where $\mathcal{R} = \vec{R}\cdot i\vec{\sigma}$ commutes with $\mathcal{Q}$ and $\mathcal{S} = \vec{S}\cdot i\vec{\sigma}$ anticommutes with $\mathcal{Q}$. Equivalently, $\mathcal{R} = i\sqrt{\vec{R}^2}\,\mathcal{Q}$ and $\mathcal{S} = (\vec{Q}\times\vec{U})\cdot i\vec{\sigma}$ with some arbitrary vector $\vec{U}$. Note that the cosmological constant does not contain the step function and is given by

$$\Lambda_5 = -\frac{16}{3}\left(g_1^2\vec{R}^2 + g_2^2\vec{S}^2\right)\,. \tag{6}$$

The supergravity action with prepotential (5) contains both symmetric and antisymmetric (multiplied by $\varepsilon(x^5)$) gravitino masses. The important thing to note is that the presence of the $Z_2$-symmetric piece $\mathcal{S}$ in the prepotential results in the supersymmetric detuning between the brane tensions and the bulk cosmological term. This detuning takes place independently of the value of the 'flip' parameter $\alpha$. If $\mathcal{S}$ is set to zero, and $\alpha = +1$, then the supersymmetric relation between brane and bulk tensions results in a warp factor that is precisely the one of the Randall–Sundrum model.

The part of the gravitino transformation law due to gauging is now:

$$\delta\psi_\alpha^A = -i\frac{\sqrt{2}}{3}\gamma_\alpha(g_1\varepsilon(x^5)\mathcal{R}_B^A + g_2\mathcal{S}_B^A)\varepsilon^B. \tag{7}$$

The presence of the step function in the above transformation law implies that the 5d action is not supersymmetric. The fifth derivative in the gravitino kinetic term acts on the step function producing an expression multiplied by a delta function. The uncanceled variation is:

$$\delta\mathcal{L} = -2i\sqrt{2}g_1(\delta(x^5) - \delta(x^5 - \pi\rho))e_4\mathcal{R}_B^A\overline{\psi_\mu}_A\gamma^\mu\gamma^5\varepsilon^B\,. \tag{8}$$

Notice that when $g_1 = 0$ the above variation vanishes implying that the Lagrangian is supersymmetric. Using the fact that the matrix $\mathcal{R}$ is proportional to $\mathcal{Q}$ we have the following relations:

$$\begin{aligned}\gamma_5\mathcal{R}_B^A\varepsilon^B(0) &= i\sqrt{\vec{R}^2}\varepsilon^A(0)\,,\\ \gamma_5\mathcal{R}_B^A\varepsilon^B(\pi\rho) &= i\alpha\sqrt{\vec{R}^2}\varepsilon^A(\pi\rho)\,.\end{aligned} \tag{9}$$

Thus:

$$\delta\mathcal{L} = 2\sqrt{2}g_1\sqrt{\vec{R}^2}e_4\overline{\psi_\mu}_A\gamma^\mu\varepsilon^A(\delta(x^5) - \alpha\delta(x^5 - \pi\rho))\,. \tag{10}$$

The variation (10) can be canceled by the variation of the determinant in the brane tension term:

$$\mathcal{L}_T = -4\sqrt{2}g_1\sqrt{\vec{R}^2}e_4(\delta(x^5) - \alpha\delta(x^5 - \pi\rho))\,. \tag{11}$$

Summarizing (and changing the normalization to that used by Randall and Sundrum), we constructed a *locally supersymmetric Lagrangian*, which has the following bosonic gravity part:

$$M^{-3}S = \int d^5x\sqrt{-g_5}(\tfrac{1}{2}R + 6k^2) - 6\int d^5x\sqrt{-g_4}kT\left(\delta(x^5) - \alpha\delta(x^5 - \pi\rho)\right)\,, \tag{12}$$

where we have defined

$$k = \sqrt{\frac{8}{9}(g_1^2R^2 + g_2^2S^2)} \tag{13}$$

and

$$T = \frac{g_1\sqrt{\vec{R}^2}}{\sqrt{(g_1^2R^2 + g_2^2S^2)}}\,. \tag{14}$$

The BPS relation between the bulk cosmological constant and the brane tensions ('the Randall–Sundrum fine-tuning') corresponds to $T = 1$, which holds only when $\langle \mathcal{S} \rangle = 0$. In such case the vacuum solution is $AdS_5$ in the bulk with flat Minkowski branes [12]. This vacuum preserves one half of the supercharges corresponding to unbroken $N = 1$ supersymmetry in four dimensions [3]. As soon as we switch on non-zero $\mathcal{S}$, we get $T < 1$, the BPS relation is destroyed and the vacuum breaks all supersymmetries.

The $N = 1$ supersymmetry is broken when $\alpha = \pm 1$ and $\langle \mathcal{S} \rangle \neq 0$. If $\alpha = -1$, then supersymmetry is always broken globally, independently of the expectation value of $\mathcal{S}$. To see this explicitly, let us take $\langle \mathcal{S} \rangle = 0$ and note that the fermions which are allowed to propagate on the left and right branes have to obey the conditions $W_0{}^A_B \psi^B = 0$ and $W_\pi{}^A_B \psi^B = 0$ respectively, where $W_{0,\,\pi}$ are given by (4). The projection operators $\Pi^A_{\pm\,B} = \frac{1}{2}(\mathbf{1}\delta^A_B \pm \gamma_5 \mathcal{Q}^A_B)$ split each spinor into two components, one of which is annihilated by $W_0$: $W_0 \varepsilon_+ = W_0 \Pi_+ \varepsilon = 0$. The second component, $\Pi_- \varepsilon$, is annihilated by $W_\pi$ if $\alpha = -1$; for $\alpha = +1$ $W_0 = W_\pi$. The BPS conditions imply (we take here $ds^2 = a^2(x^5) dx^2 + (dx^5)^2$)

$$\frac{a'}{a} \gamma_5 \varepsilon^A + \frac{2\sqrt{2}}{3} g_1 \varepsilon(x^5) \sqrt{R^2}\, \mathcal{Q}^A_B \varepsilon^B = 0 \tag{15}$$

(this holds for $AdS_4$ and Minkowski foliations). When we apply the operator $\Pi_+$ to (15), we obtain the conditions $\frac{a'}{a} + \frac{2\sqrt{2}}{3} g_1 \varepsilon(x^5) \sqrt{R^2} = 0$ or $\varepsilon_+ \equiv 0$. The first possibility leads to discontinuities of the warp factor at the fixed points: $[\frac{a'}{a}]_0 = -2kT$, $[\frac{a'}{a}]_{\pi\rho} = +2kT$. However, the matching conditions in the equations of motion give $[\frac{a'}{a}]_0 = -2kT$, $[\frac{a'}{a}]_{\pi\rho} = +2\alpha kT$, which are in contradiction with the BPS condition for $\alpha = -1$, unless $\varepsilon_+ \equiv 0$. Applying to (15) the second projector, $\Pi_-$, one finds out immediately that boundary conditions and BPS conditions agree on both branes only for $\varepsilon_- \equiv 0$. Thus there exists no globally defined Killing spinor in the setup with flipped $Z_2$ acting on fermions (bosons are acted on as in the unflipped case), and all supersymmetries are broken spontaneously.

Now we move on to the 'flipped susy' case $\alpha = -1$. A vacuum solution in the warped product form can be found

$$ds^2 = a^2(x^5) g_{\mu\nu} dx^\mu dx^\nu + R_0^2 (dx^5)^2, \tag{16}$$

where $g_{\mu\nu}$ is the $AdS_4$ metric with cosmological constant $\bar{\Lambda}$

$$g_{\mu\nu} dx^\mu dx^\nu = \mathrm{e}^{-2\sqrt{-\bar{\Lambda}} x_3} (-dt^2 + dx_1^2 + dx_2^2) + dx_3^2 \tag{17}$$

and the third coordinate $x_3$ has been singled out. As long as $T < 1$ the static vacuum solution is $AdS_5$ in the bulk and the warp factor can be parametrized

as [13,14]:

$$a(x^5) = \frac{\sqrt{-\bar{\Lambda}}}{k} \cosh(kR_0|x_5| - C)\,. \tag{18}$$

The matching conditions for $AdS_4$ branes embedded in $AdS_5$ read

$$\tanh(C) \;=\; T\,, \tag{19}$$

$$\tanh(kR_0\pi\rho - C) \;=\; -\alpha T \;=\; T\,. \tag{20}$$

The first condition sets the integration constant $C$ and the second fixes the size of the fifth dimension. The radion is stabilized at the value

$$\pi\rho k R_0 = \ln\left(\frac{1+T}{1-T}\right)\,. \tag{21}$$

Moreover, the magnitude of the brane cosmological constant is fixed by the normalization $a(0) = 1$. This leads to

$$\bar{\Lambda} = (T^2 - 1)k^2 < 0\,. \tag{22}$$

The cosmological constant on the brane depends directly on the scale of supersymmetry breaking on the brane. The same is true for the expectation value of the radion. Notice, that $\langle R_0 \rangle$ can be expressed solely in terms of $k$ and $\bar{\Lambda}$. The formula equivalent to (21) in the case $\alpha = +1$ gives $R_0 = 0$. To summarize, the nonzero expectation value of $\mathcal{S}$ gives rise to detuning between brane and bulk tensions and as a consequence to stabilization of the radion. The $\langle \mathcal{S} \rangle$ contributes also to supersymmetry breakdown, but in the case of $\alpha = -1$ supersymmetry is broken even if $\langle \mathcal{S} \rangle = 0$.

One finds that (18) is not a valid solution in the case $T = 1$. Indeed, this implies that the brane cosmological constant vanishes and the second brane is sent to infinity. In that case we expect a global mismatch due to the boundary condition on one of the branes. It turns out that a static solution with maximally symmetric foliations do not exist in this case, but one can find cosmological solutions which may be considered as a background for the physics on the brane [6].

### *2.2. Matter on the branes and in the bulk: backreaction on geometry*

After describing the supergravity background for brane world models, one needs to enhance them by putting matter, scalar and gauge fields on the branes and in the bulk. Although some work on bulk gauge theories coupled to supergravity is available, in order to perform a detailed analysis it is convenient to restrict the standard gauge sectors to the branes, and to leave in the bulk only matter supermultiplets coupled to the particular local

$\mathrm{U(1)_R}$, which is the gauged subgroup of the $R$-symmetry of the $N = 2$ supersymmetry algebra. This role is conveniently played by the bulk universal hypermultiplet (the gauge field being the graviphoton), whose introduction is also well motivated by stringy considerations. Non-Abelian gauge sectors on the branes are both models for the Standard Model physics, and simultaneously supply the seeds for the supersymmetry breakdown. The situation in the $\mathcal{S} = 0$, $\alpha = +1$ case has been discussed in great detail in [15]. The signature of supersymmetry breakdown in these models is the nonzero expectation value of the $Z_2$-odd complex scalar from the hypermultiplet, $\xi$, and of its transverse derivative $\partial_5\xi$. To excite a nontrivial vacuum configuration for this field one needs to switch on its sources on the branes. These sources can be represented by the effective superpotentials on the branes, $W_i$, $i = 1, 2$. However, once this is done, new contributions to energy densities on the branes and in the bulk are created. This leads to modifications of the vacuum configuration of the moduli and of the warp factor. In the case where the purely gravitational background had 4d flat foliations, the backreaction of the supersymmetry breaking physics leads naturally towards anti-de Sitter geometry on the branes, with negative four-dimensional cosmological constant. One should notice, that the non-zero value of $\mathcal{S}$ would not help, since the way it acts is to make the size of the bulk cosmological constant larger, but without changing its sign, so that it remains negative. The good thing that happens is that the expectation value of the radion, hence the distance between branes, becomes determined (it is a modulus as long as supersymmetry is preserved) in terms of the supersymmetry breaking sources. However, to obtain the required size of mass splitting within supermultiplets and the right hierarchy between the 4d Planck scale and the electroweak scale one needs a tuning of the sources, which signals an instability. To see this more explicitly consider the issue of making the effective four-dimensional cosmological constant zero by including positive contributions to the potential. A first obvious source for such contributions are the $F$-terms borne by the matter sector localized on the branes. They contribute the terms $\delta V_{\mathrm{boundary}} = \frac{1}{2V}\delta(x^5)|\frac{\partial W_1}{\partial \Phi_1}|^2 + \frac{1}{2V}\delta(x^5 - \pi\rho)|\frac{\partial W_2}{\partial \Phi_2}|^2$. However, this modification does not work on its own and the 4d geometry stays anti-de Sitter. To find a vacuum with the flat geometry one needs to create a potential for the second bulk field, the dilaton $S$, and in addition to put a Polonyi field on the Planck brane. The effective 4d superpotential that does the job is

$$W = W_1(\Phi_1) + (\mathrm{e}^{-a_1 S} + d\mathrm{e}^{-a_2 S})\mathrm{e}^{-3T} \tag{23}$$

with $|W_1'|^2 \approx 2W_1^2$. One obtains in this case $V_0 \approx 1/a$, $\mathrm{e}^{k\pi\rho R_0} \approx |W_2|/|W_1|$, and $F^{\Phi_1}$ and $F^S$ become dominant. Mass splittings are due to universal soft scalar masses $\sim m_{3/2}^2$, $A_3$ terms $\sim m_{3/2}$, and gaugino masses $\sim m_{3/2}$, that

are universal due to $F^S \gg F^T$. The price one has to pay for the vanishing cosmological constant is the active role of the Polonyi field. In general, this example amplifies the observation, that to obtain a phenomenologically interesting vacuum in brane models one needs certain correlations between parameters of the different brane sectors. This is somewhat unnatural in view of the fact, that the branes are spatially separated.

It is interesting to extend this discussion to the flipped case, where $\alpha = -1$. The observations are likely to be relevant for the case of the stringy brane-antibrane models, and for models similar to these of Barbieri, Hall and Nomura [16]. The gravitational background for the flipped case has been discussed earlier, let us only remind here that there is no static solution with the Minkowski foliation, but we have found solutions with anti-de Sitter foliation and stable radion due to the introduction of the detuning parameter $\mathcal{S}$. Hence we should couple bulk and brane matter to such a background with negative cosmological constant in 4d, and try to cancel the cosmological constant dynamically by positive contributions coming from the branes. It is easy to see that one can include the bulk universal hypermultiplet in the same way as in the unflipped case. The general difference is that now on the flipped brane certain terms needed to compensate delta-type variations of the bulk terms will have the opposite sign to that on the unflipped brane. The reason for that is precisely the same as the change of sign of the brane tension on the flipped brane in the purely gravitational case. The second important difference is the coupling of the bulk fermions to the flipped brane. At this brane the fermions that couple to brane operators are the $Z_2$-odd components of the bulk symplectic-Majorana fermions (gravitini and hyperini), while the components of these fermions that enter the unflipped brane are the $Z_2$-even ones. More precisely, the relevant parts of the flipped-brane-bulk coupling are $S = S_{\rm bulk} + S_{\rm YM} + S_{\rm matter}$, where

$$
\begin{aligned}
S_{\rm bulk} &= \int d^5x\, e_5 \left( \frac{R}{2} - \frac{1}{V}|\xi'|^2 g^{55} - \frac{1}{4V^2} V'^2 g^{55} \right. \\
&\quad \left. + \Lambda^2 \left( \frac{1}{6} + \frac{1}{12V}|\xi|^2 - \frac{1}{12V^2}|\xi|^4 \right) \right) , \\
S_{\rm YM} &= \int d^5x \frac{e_5}{e_5^5} \delta(x^5) \left( -\frac{V}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4}\sigma F^a_{\mu\nu} \tilde{F}^{a\mu\nu} - \frac{V}{2} \overline{\chi^a} \not{D} \chi^a \right. \\
&\quad \left. - \frac{\sqrt{V}}{2e_5^5} ( (\overline{\chi^a}_{\rm L} \chi^a_{\rm R}) \partial_5 \overline{\xi} + (\overline{\chi^a}_{\rm R} \chi^a_{\rm L}) \partial_5 \xi ) + \delta(0)\, \frac{V^{3/2}}{8} ((\chi^a)^2)^2 \right) + \dots , \\
S_{i\ \rm matter} &= \int d^5x\, \frac{e_5}{e_5^5} \delta(x^5 - x_i) (-\varepsilon_i \Lambda \left( 1 - \frac{|\xi|^2}{V} \right)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{2}{V}\sqrt{g^{55}}(W\xi' + \bar{W}\bar{\xi}') - \frac{\sqrt{g^{55}}}{V}\delta(0)(4W\bar{W} + V^{3/2}\bar{W}(\overline{\chi^a_{\rm R}}\chi^a_{\rm L})) \\
&-D_\mu C_i D^\mu \bar{C}^i - \frac{4}{V}\frac{\partial W_i}{\partial C_i}\frac{\partial \bar{W}_i}{\partial \bar{C}^i} \\
&+\left(\frac{W}{\sqrt{V}}(\overline{\psi}_{{\rm L}\mu}\gamma^{\mu\nu}\psi_{{\rm R}\nu}) - \frac{1}{V^{2/3}}W(\overline{\psi}_{{\rm L}\mu}\gamma^\mu\lambda_{\rm L})\right. \\
&\left.-\frac{i}{V^{3/2}}\sqrt{g^{55}}W(\overline{\psi}_{\rm R5}\lambda_{\rm L}) + {\rm h.c.})\right)\,,
\end{aligned}
$$

where $i = 1, 2$ labels branes and $\varepsilon_{1,2} = +1, -1$. The $Z_2$-odd 4d Majorana supersymmetry generator on the flipped brane is given by $\tilde{\varepsilon} = \begin{pmatrix} i\varepsilon^1_{\rm L} \\ i\varepsilon^2_{\rm R} \end{pmatrix}$ and analogous replacements hold for gravitini and hyperini. This implies $\psi^A_\mu(2\pi\rho + x^5) = -\psi^A_\mu(x^5)$ (and the same for hyperini) while bosonic fields remain periodic. Obviously, there are no fermionic zero modes in the bulk, so supersymmetry is broken in that sector of the model. This has been explored in [17] and more recently in [2, 18]. Bulk moduli still couple to both walls and participate in the transmission of information between branes already at the classical level. The best example is again the odd scalar from the hypermultiplet, $\xi$. If the expectation values of sources to which it couples on the walls do not vanish, the $\xi$ and $\partial_5\xi$ also assume nontrivial $x^5$ dependence, which creates operators breaking softly global supersymmetries on the walls, similarly to what happens in the unflipped $(+,+)$ case. From that case we know that on the unflipped wall the even components of $\delta_{\rm susy}\psi_5$ and $\delta_{\rm susy}\lambda$ receive nonhomogeneous contributions in their supersymmetry transformations. What happens on the $\alpha = -1$ wall? Let us inspect the transformation law of the odd part of the hyperini

$$
\delta_-\lambda^1_{\rm R} = ... - \frac{i}{2\sqrt{2}V}\partial_5(V + i\sigma)\varepsilon^1_{\rm R} - \frac{i}{\sqrt{2V}}\partial_5\xi\,\varepsilon^2_{\rm R}\,. \tag{24}
$$

The generator $\varepsilon^1_{\rm R}$ is even, so it does not enter the wall. The coefficient of the odd generator which generates supersymmetry on the wall is multiplied by the parameter $\partial_5\xi$, the same which induces susy-breaking terms on the even wall. Hence, indeed, the walls talk to each other already at the level of the classical vacuum through the messenger $\xi$. This communication obviously includes creation of supersymmetry breaking terms on both walls, and these terms are sensitive to mass scales from the opposite wall, as in the $(+,+)$ models. The detailed analysis of this case will be given elsewhere, here we just point out the full analogy to the unflipped case in the necessity of arranging correlations between terms located at different walls in designing a phenomenologically relevant vacuum.

The above examples have illustrated the intimate interplay between the gravitational background and the gauge sector physics in the case of flipped and unflipped locally supersymmetric brane world models. The backreaction of the gauge sector on the geometry is explicit, and shows that both sectors need to be tuned against each other to create a phenomenogically relevant vacuum. In fact, generic vacua for generic values of the parameters in the Lagrangian are likely to be cosmological ones, with time dependent geometry of the orbifold and physics on the brane. The example of such a solution in models discussed here has been given in [6].

## 3. Quiver models and deconstructed dimensions

So far we were discussing the issues pertaining to a nondecoupling of gravity in brane world models. However, there exist models where the decoupling of gravity from extra dimensions is natural, and can be arranged by standard methods known from four-dimensional field theory. Such are the models of deconstructed dimensions, [19, 20], where gravity is always four-dimensional, and extra dimensions are fictitious and fully contained within the 4d gauge sectors. The example of such a situation is provided by quivers with the custodial supersymmetry [21]. The UV properties of these models are better than these of a generic non-supersymmetric model, and a separation between the vevs on the gauge sector and the cut-off scale, which may be taken to be the 4d Planck scale can be arranged.

### *3.1. Orbifolding and supersymmetry breaking*

Consider the type IIB string theory with a stack of $n$ coinciding D3 branes. It is well known that the gauge bosons and fermions living on the worldvolume of the D branes form a 4d $N = 4$ supersymmetric Yang–Mills model with gauge group $\mathrm{U}(K)$. The six transverse dimensions form, from the point of view of the 4d theory living on the worldvolume, six extra nongravitational dimensions. One can obtain a theory with fewer supersymmetries than $N = 4$ $\mathrm{U}(K)$ by dividing the extra dimensions by a discrete group $Z_\Gamma$ and embedding this orbifold group into the gauge group $\mathrm{U}(n\Gamma)$. The resulting theory is called a quiver theory. We will focus on non-supersymmetric quiver theories. They are obtained by retaining in the spectrum only the fields which are invariant under the combined geometric and gauge actions of $Z_\Gamma$. Their interactions are consistently truncated to yield a smaller daughter gauge theory. The truncation process breaks the gauge group and some (or all) supersymmetries. The gauge symmetry breaking is dictated by the embedding of the generator of $Z_\Gamma$ into $\mathrm{U}(n\Gamma)$. The matrix $\gamma$ that represents the gauge action of $Z_\Gamma$ is chosen to be of the form of a direct sum of $\Gamma$ unit matrices of dimensions $n \times n$, each multiplied respectively by $\omega^i$

with $\omega = \mathrm{e}^{\frac{2\pi}{\Gamma}i}$. Then the invariant components of the gauge fields fulfill the condition

$$A = \gamma A \gamma^{-1}\,, \tag{25}$$

where $A$ is a matrix in the adjoint representation of $\mathrm{U}(n\Gamma)$. This leaves invariant the subgroup $\mathrm{U}(n)^{\Gamma}$. There are four generations of Weyl fermions, each in the adjoint of $\mathrm{U}(n\Gamma)$, whose invariant components must obey the condition

$$\psi^i = \omega^{a_i}\gamma\psi^i\gamma^{-1}\,, \tag{26}$$

where $i = 1, .., 4$ and

$$a_1 + a_2 + a_3 + a_4 = 0\,. \tag{27}$$

The invariant fermions transform in the bifundamental representations of the broken gauge group $(\boldsymbol{n}_l, \bar{\boldsymbol{n}}_{l+a_i})$ where $l$ numbers blocks of the original $n\Gamma \times n\Gamma$ matrices. Furthermore, one obtains three generations of complex bosons $\phi^i$, $i = 1, 2, 3$, in the adjoint of $\mathrm{U}(K)$, whose invariant components fullfil the condition

$$\phi^i = \omega^{\tilde{a}_i}\gamma\phi^i\gamma^{-1}\,. \tag{28}$$

The invariant scalars transform as $(\boldsymbol{n}_l, \bar{\boldsymbol{n}}_{l+\tilde{a}_i})$ under the broken gauge group. The truncated fields have a block structure in the $\mathrm{U}(n\Gamma)$ mother gauge group

$$\phi^i_{lp} = \phi^i_l \delta_{p,l+\tilde{a}_i}\,,\ \ \psi^i_{lp} = \psi^i_l \delta_{p,l+a_i}\,. \tag{29}$$

Supersymmetry is preserved when the group $Z_\Gamma$ is embedded in SU(3)

$$\tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 = 0\,. \tag{30}$$

In that case $a_4 = 0$ and at least one of the fermions can be paired with the gauge bosons, *i.e.* becoming a gaugino of $N = 1$ supersymmetry. We focus on the non-supersymmetric case $a_4 \neq 0$.

Let us move a stack of $n$ D3 branes from the origin. From the field theory point of view, moving the stacks of $n$ D3 branes from the origin is equivalent to going to the Higgs branch of the theory where all the off-diagonal scalars with $\tilde{a}_i \neq 0$ take a vev

$$\phi^i_l = v^i 1_{n\times n}\,. \tag{31}$$

Due to the $Z_\Gamma$ action the stacks have $\Gamma$ copies around the fixed point. The gauge group is broken to the diagonal subgroup $\mathrm{U}(n)_{\mathrm{D}}$. This is the deconstructed phase.

We will be interested in the one-loop divergences of the non-supersymmetric quivers. The vanishing of the quadratic divergences in models based upon the low energy dynamics of branes in string theory is a general phenomenon. It turns out to be an extension to the case of type II string

theory with D branes of misaligned supersymmetry. Let us consider a model where the low energy fields live on a $p$-brane in a configuration where many possible branes coincide. Now assume that the background geometry is a solution of the string equations with no closed string tachyons. Let us first consider that all the low energy fields vanish so that all the branes coincide. The open string mass spectrum comes from the oscillators $M_{\rm O}^2$. When considering the case where the vev of some of the low energy fields living on the brane does not vanish, *i.e.* some of the brane have been moved, the mass spectrum of open strings is shifted corresponding to the minimal length of open strings between the branes. Consider now the string amplitudes between any two of the displaced branes

$$\mathrm{STr}\int_0^\infty \frac{dt}{t^{1+\frac{p+1}{2}}}\mathrm{e}^{-2\pi\alpha' tM^2}\mathrm{e}^{-2\pi\alpha' tM_{\rm O}^2}\,, \tag{32}$$

where $M^2$ is the mass matrix of the low energy fields. Open-closed duality relates this amplitude to

$$\int_0^\infty dl\; l^{\frac{p-9}{2}}\langle Dp'|\mathrm{e}^{-\pi\alpha' M^2/l-\alpha' lM_C^2}|Dp\rangle\,, \tag{33}$$

where $l=1/2t$. As $l\to\infty$ we find that $\mathrm{STr}(M_{\rm T}^{2k})=0,\ k=0\ldots 3$, where $M_{\rm T}$ is the total mass matrix of all the open string states. Now we consider the decoupling limit $l_s\to 0$ sending all the stringy modes to infinity while preserving the vev $v$ of the brane fields. The low energy field theory on the stack of D3 branes is obtained after decoupling gravity. In the decoupling limit the mass matrix splits in two blocks $M_{\rm T}=M\oplus M_{\rm O}$ acting on decoupled states. We conclude that

$$\mathrm{STr}M^{2k}=0,\ k=0\ldots 3\,. \tag{34}$$

This is the vanishing of the supertraces corresponding to the breaking of the low energy field theory by small (compared to the string scale) vevs. Of course, this result is only valid when no closed string tachyon propagate between the branes. As soon as $a_4\neq 0$ there are closed string tachyons for non-supersymmetric quiver theories. The twisted tachyons do not intervene in the string amplitude when the branes are off the center of the orbifold. Indeed the boundary states of the branes are coherent closed string states satisfying $X^i|Dp\rangle = x^i|Dp\rangle$, where $x^i$ is the location of the brane. If the twisted states couple to the brane we must have $\theta x^i\equiv x^i$ *i.e.* the brane is at a fixed point. So we obtain the vanishing of the quadratic divergences in the deconstructed phase. More can be said here in the deconstructed phase.

The low energy degrees of freedom come from the open strings corresponding to the invariant states in the string spectrum The action of $Z_\Gamma$ leads to a truncation of the spectrum as only invariant states are kept. The orbifold $Z_\Gamma$ acts on the Chan–Paton indices by permutation implying that the action of $Z_\Gamma$ on states is

$$\theta|lp\rangle = \omega^a|l+1, p+1\rangle\,, \tag{35}$$

where $\theta$ is the generator of $Z_\Gamma$, $|lp\rangle$ is a state (either boson or fermion) with shift $a$ and the Chan–Paton indices $(lp)$ label the branes on which the open strings end. Here each label $l$ corresponds to a stack of $n$ D3 branes and the strings are connected to the mirror images of this stack. The construction of invariant states follows

$$|lp\rangle_{\rm orb} = \sum_{k=0}^{\Gamma-1} \omega^{ka}|l+k, p+k\rangle\,. \tag{36}$$

Notice that there are $\Gamma$ invariant states for each species. When the stack of $n$ D3 branes is at the origin the length of the open strings vanishes and the associated masses to the invariant state is zero. This gives rise to the low energy fields that we have discussed previously. Consider the $N=4$ mother theory written in terms of $N=1$ chiral multiplets. This is obtained by breaking the $R$-symmetry group from SU(4) to SU(3)$\otimes Z_\Gamma$ where the three complex bosons $\phi^i$ are in the **3** of SU(3) and we decompose the Weyl fermions as $\mathbf{4} = \mathbf{3} + \mathbf{1}$. The four spinors are distinguished by their $Z_\Gamma$ charges which are respectively $a_i,\ i = 1\ldots 3$ for the **3** and $a_4$ for the singlet. The $N=4$ fields can be arranged into $N=1$ supermultiplets $(\phi^i, \psi^i)$ and $(A_\mu, \psi^4)$. In the orbifold theory, this $N=1$ invariance generated by a space-time supersymmetry generator $Q$ is broken when $a_4 \neq 0$ as can be seen from the gauge numbers of the fermions and bosons. Nevertheless the spectrum contains equal number of fermions and bosons, and these are paired up in a certain way. To see this denote any scalar by

$$|\phi\rangle_{\rm orb}(lp) = \sum_{k=0}^{\Gamma-1} \omega^{k\tilde{a}}|\phi\rangle(l+k, p+k) \tag{37}$$

and its associated fermion state by

$$|\psi\rangle_{\rm orb}(lp) = \sum_{k=0}^{\Gamma-1} \omega^{ka}|\psi\rangle(l+k, p+k)\,, \tag{38}$$

where $|\psi\rangle(lp)$ and $|\phi\rangle(lp)$ are superpartners under the action of $Q$. Let us define the twisted supersymmetry operator

$$\tilde{Q} = \gamma^R Q \gamma^{-R}\,, \tag{39}$$

where $R$ is the $Z_T$ charge. The action of $\tilde{Q}$ on the orbifold states is

$$(\tilde{Q}|\phi\rangle_{\rm orb})(lp) = \omega^{-la_4}|\psi\rangle_{\rm orb}(lp)\,. \tag{40}$$

This implies that the string states, *i.e.* the physical fields in the low energy limit are classified into twisted supersymmetry multiplets. However, this is a kinematical statement, and one needs to examine masses and interactions to draw stronger conclusions.

### *3.2. Orbifolding of the field-theoretical Lagrangian and custodial supersymmetry*

To prove more about non-supersymmetric quivers it is very useful to study the effective Lagrangian of such theories. Inserting the block decomposition into the $\mathcal{N}=4$ Lagrangian we find the daughter theory Lagrangian

$$\begin{aligned}
\mathcal{L} \;=\; & {\rm Tr}\Big\{-\tfrac{1}{2}F_{\mu\nu,p}F_{\mu\nu,p} + i\overline{\lambda_p}\gamma^\mu D_\mu\lambda_p + 2D_\mu\phi^\dagger_{i,p}D_\mu\phi_{i,p} + i\overline{\psi_{i,p}}\gamma^\mu D_\mu\psi_{i,p} \\
& -g_0\left[2i\sqrt{2}\left(\overline{\psi_{i,p}}P_{\rm L}\lambda_{p+a_i}\phi^\dagger_{i,p} - \overline{\psi_i}\phi^\dagger_{i,p-a_4}P_{\rm L}\lambda_{p-a_4}\right) + {\rm h.c.}\right] \\
& -g_0\left[i\sqrt{2}\varepsilon_{ijk}\left(\overline{\psi_{i,p}}P_{\rm L}\psi_{j,p+a_i}\phi_{k,p-\tilde{a}_k} - \overline{\psi_{i,p}}\phi_{k,p+a_i}P_{\rm L}\psi_{j,p-a_j}\right) + {\rm h.c.}\right] \\
& -g_0^2\left(\phi_{i,p}\phi^\dagger_{i,p} - \phi^\dagger_{i,p-\tilde{a}_i}\phi_{i,p-\tilde{a}_i}\right)\left(\phi_{j,p}\phi^\dagger_{j,p} - \phi^\dagger_{j,p-\tilde{a}_j}\phi_{j,p-\tilde{a}_j}\right) \\
& + 4g_0^2\left(\phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{i,p+\tilde{a}_j}\phi^\dagger_{j,p} - \phi_{i,p}\phi_{j,p+\tilde{a}_i}\phi^\dagger_{j,p+\tilde{a}_i}\phi^\dagger_{i,p}\right)\Big\}\,,
\end{aligned} \tag{41}$$

where $\lambda \equiv \psi_4$. The covariant derivative acting on scalars is $D_\mu\phi_{i,p} = \partial_\mu\phi_{i,p} + ig_0A_p\phi_{i,p} - ig_0\phi_{i,p}A_{p+\tilde{a}_i}$. It is then a tedious exercise to obtain the mass matrices and compute the supertrace

$$\begin{aligned}
& {\rm STr}(\mathcal{M}^2) = 4g_0^2\sum_k\sum_p\delta_{\tilde{a}_k,0} \\
& \times\Big[\Big({\rm Tr}(\phi^\dagger_{k,p}){\rm Tr}(\phi_{k,p+a_4}) + {\rm Tr}(\phi^\dagger_{k,p+a_4}){\rm Tr}(\phi_{k,p}) - 2{\rm Tr}(\phi^\dagger_{k,p}){\rm Tr}(\phi_{k,p})\Big) \\
& + \sum_i\Big({\rm Tr}(\phi^\dagger_{k,p}){\rm Tr}(\phi_{k,p+a_i}) + {\rm Tr}(\phi^\dagger_{k,p+a_i}){\rm Tr}(\phi_{k,p}) \\
& - {\rm Tr}(\phi^\dagger_{k,p}){\rm Tr}(\phi_{k,p+\tilde{a}_i}) - {\rm Tr}(\phi^\dagger_{k,p+\tilde{a}_i}){\rm Tr}(\phi_{k,p})\Big)\Big]\,.
\end{aligned} \tag{42}$$

One can check that (42) vanishes identically if at least one of the following conditions is satisfied:

- $a_4 = 0$ or $a_i = 0$, that is when at least $\mathcal{N} = 1$ supersymmetry is preserved by the orbifolding,

- $\tilde{a}_1 \neq 0$, $\tilde{a}_2 \neq 0$ $\tilde{a}_3 \neq 0$, that is when there are no scalars in adjoint representation of $\mathrm{U}(n)$ group.

In the first case the vanishing of the supertrace is of course guaranteed by unbroken supersymmetry of the daughter theory. Surprisingly, the absence of quadratic divergences can also occur if the daughter theory is completely non-supersymmetric, the only condition being that all scalars are in bifundamental representations of the $\mathrm{U}(n)^\Gamma$ gauge group. This result is stronger than the result that we derived previously from stringy arguments. Indeed it is valid for any background value of the six scalar fields.

Let us now come back to the deconstructed case. One can explicitly diagonalize the mass matrices. For instance the gauge bosons acquire mass terms:

$$\mathcal{L} = \sum_p \sum_{k=1}^{3} g_0^2 v_k^2 (A_p^a - A_{p+\tilde{a}_k}^a)^2 \,. \tag{43}$$

(We have rewritten the gauge fields as $A = A^a T^a$ and evaluated the trace over generators. In the following we often omit the adjoint index $a$.) These mass terms are diagonalized by the following mode decomposition[1]:

$$A_p = \sqrt{\frac{2}{\Gamma}} \left( \sum_{n=0}^{(\Gamma-1)/2} \eta_n \cos\left(\frac{2n\pi}{\Gamma} p\right) A^{(n)} + \sum_{n=1}^{(\Gamma-1)/2} \sin\left(\frac{2n\pi}{\Gamma} p\right) \tilde{A}^{(n)} \right) , \tag{44}$$

where $\eta_0 = 1/\sqrt{2}$ and $\eta_n = 1$, $n \neq 0$. Plugging in this decomposition we get:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \sum_n \sum_k \left(m_k^{(n)}\right)^2 \left(A^{(n)} A^{(n)} + \tilde{A}^{(n)} \tilde{A}^{(n)}\right) , \\ m_k^{(n)} &\equiv 2\sqrt{2} g_0 v_k \sin\left(\frac{n\pi}{\Gamma} \tilde{a}_k\right) , \end{aligned} \tag{45}$$

so that the $n$-th level gauge bosons have masses $(m^{(n)})^2 = \sum_k m_k^2$. Similar calculations can be done for the other fields resulting in the fact that the spectrum is perfectly boson–fermion degenerate. We already know that this degeneracy can be traced back to a custodial supersymmetry. Let us now

[1] The decomposition is given for odd $\Gamma$. For even $\Gamma$ the first sum goes to $\Gamma/2$ and the second to $\Gamma/2 - 1$.

investigate it further. We define the vector superfields in the Wess–Zumino gauge as:

$$V^{(n)}(y,\theta) = \frac{i}{2}(\overline{\theta}\gamma_5\gamma_\mu\theta)A^{(n)} - i(\overline{\theta}\gamma_5\theta)(\overline{\theta}\lambda^{(n)}) - \frac{1}{4}(\overline{\theta}\gamma_5\theta)^2 D^{(n)}\,. \tag{46}$$

Similarly we define chiral superfields:

$$\Phi_i^{(n)}(y,\theta) = X_i^{(n)} - \sqrt{2}(\overline{\theta}P_{\rm L}\psi_i^{(n)}) + F_i^{(n)}(\overline{\theta}P_{\rm L}\theta)\,. \tag{47}$$

Analogous expressions for the tilded fields hold.

First, we note that the self-couplings in the zero-mode sector are those of the $\mathcal{N}=4$ supersymmetric theory. Indeed, the interactions of the zero-modes can be found by making in (41) the replacement $\phi_{i,p} \to \frac{1}{\sqrt{\Gamma}}\phi_i^{(0)}$ (and similarly for fermion and gauge fields). Since all memory of the block indices is lost, as a result we obtain the $\mathcal{N}=4$ Lagrangian with gauge coupling $g = \frac{g_0}{\sqrt{\Gamma}}$. Second, we have already shown that the mass pattern in the deconstruction phase is supersymmetric. It turns out that the custodial supersymmetry has a much wider extent and all the terms quadratic in the heavy modes (including triple and quartic interactions with the zero-modes) match the structure of a globally supersymmetric theory! As an example we present a superfield Lagrangian which reproduces the Yukawa terms and the scalar potential of the daughter theory:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum_n \sum_k \mathrm{Tr}\left[ 4g_0 v_k \sin\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\tilde{V}^{(n)}\Phi_k^{(n)} - V^{(n)}\tilde{\Phi}_k^{(n)}\right)\right. \\
& +2g\cos\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\left[\Phi_k^{(0)\dagger},\Phi_k^{(n)}\right]V^{(n)} + \left[\Phi_k^{(0)\dagger},\tilde{\Phi}_k^{(n)}\right]\tilde{V}^{(n)}\right) \\
& \left. +\,2g\sin\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\left\{\Phi_k^{(0)\dagger},\Phi_k^{(n)}\right\}\tilde{V}^{(n)} - \left\{\Phi_k^{(0)\dagger},\tilde{\Phi}_k^{(n)}\right\}V^{(n)}\right) + \mathrm{h.c.}\right]_{\rm D} \\
& +[W]_{\rm F} + [W^*]_{\rm F}\,,
\end{aligned} \tag{48}$$

where the superpotential is:

$$\begin{aligned}
W \;=\; & -i\sqrt{2}\sum_n \sum_{ijk} \varepsilon_{ijk}\mathrm{Tr}\left[ 4g_0 v_k \sin\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\Phi_i^{(n)}\tilde{\Phi}_j^{(n)}\right. \\
& -g\cos\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\left[\Phi_k^{(0)},\Phi_i^{(n)}\right]\Phi_j^{(n)} + \left[\Phi_k^{(0)},\tilde{\Phi}_i^{(n)}\right]\tilde{\Phi}_j^{(n)}\right) \\
& \left. +g\sin\left(\frac{n\pi\tilde{a}_k}{\Gamma}\right)\left(\left\{\Phi_k^{(0)},\Phi_i^{(n)}\right\}\tilde{\Phi}_j^{(n)} - \left\{\Phi_k^{(0)},\tilde{\Phi}_i^{(n)}\right\}\Phi_j^{(n)}\right)\right]\,.
\end{aligned} \tag{49}$$

Supersymmetry is explicitly violated by triple and quartic self-interactions of the heavy modes. Nevertheless, the presence of the custodial supersymmetry in the Lagrangian is sufficient to ensure the vanishing of one-loop corrections to the zero-mode masses. A mass-splitting of the zero-mode multiplets can appear only at the two-loop level and we expect the supersymmetry breaking scale to be suppressed $M_{\rm SUSY} \ll v \ll \Lambda$.

### *3.3. Theory space dimensions*

From the previous discussion we know that the daughter theory is the low-energy field theory of branes located at the fixed point of an orbifold. The low energy degrees of freedom on a brane are those combinations of the open string states that are invariant under the action of $Z_\Gamma$. When one moves a stack of $n$ D3 branes at a distance $d$ away from the fixed point, due to the $Z_\Gamma$ symmetry there appear $\Gamma$ copies of the stack, spaced symmetrically in the transverse directions around the fixed point, see [22]. The custodial supersymmetry implies an extension of the results of Arkani-Hamed *et al.* to nonsupersymmetric orbifoldings. For instance, it was shown that in the large $\Gamma$ limit, when the distances between images of the stack are much smaller than $d$, one can redefine the orbifold metric in such a way, that consecutive boson–fermion degenerate mass levels correspond to open strings winding around a circular direction of the transverse geometry. This geometric picture allows for the straightforward computation of the massive string spectrum:

$$m_n^2 = 4\frac{d^2}{l_s^4}\sum_{i=1}^{3}\sin^2\left(\frac{n\pi\tilde{a}_i}{\Gamma}\right),$$

where $l_s$ is the string scale and the shifts $\tilde{a}_i$ represent the action of $Z_\Gamma$ on the three complex coordinates. When all vevs are equal, this is precisely the field theoretical spectrum in the deconstruction phase of the nonsupersymmetric model. In fact one can forget about the underlying stringy picture, and view the additional dimensions as fictitious, theory space, dimensions[2]. The ladder of scales which appears in a deconstructed field theoretical quiver model is as follows. The first scale one encounters, taking the bottom-up direction in available energy, is the fictitious compactification scale $1/R_5 = agv/\Gamma$, where $a^2 = \sum_i \tilde{a}_i^2$. At this scale a seeming fifth dimension opens up and one sees the tower of Kaluza–Klein states with masses of order $1/R_5$. Hence above this scale the theory looks five-dimensional. Moreover, the spectrum of massive states is determined by the custodial supersymmetry. This picture holds up to the deconstruction scale $v$ where non-diagonal gauge bosons become massless again. Above the deconstruction scale the theory is explicitly

[2] Actually, each allowed set of shifts defines a closed subset of links in quiver diagrams, which can be interpreted as an internal dimension

four-dimensional, nonsupersymmetric and renormalizable. Quadratic divergences are absent at the one-loop level. Also at one-loop the deconstruction scale is a flat direction of this four dimensional theory, hence it stays decoupled from any UV cut-off scale, including the 4d Planck scale. Moreover, the compactification scale $1/R_5$ can be arbitrarily smaller than the deconstruction scale and it is determined by the discrete parameter which is the order $\Gamma$ of the orbifold group $Z_\Gamma$.

## 4. Summary

In this paper we have discussed the interplay between gravity/moduli and gauge/matter sectors in creating physically relevant vacua in higher-dimensional brane worlds. We started with five-dimensional brane-bulk supergravities, with flipped and unflipped boundary conditions, constructed and discussed in [3, 4, 6, 15]. We have demonstrated that in both, flipped and unflipped cases, adding nontrivial gauge/matter sectors on the branes changes geometry of the four-dimensional sections and affects stabilization of the orbifold. To achieve vacua with hierarchically broken supersymmetry, static orbifold and nearly vanishing 4d cosmological constant one needs a tuning involving all sectors of the brane-bulk Lagrangian. Thus the physics of visible and gravity/moduli sectors cannot be treated separately in such models, and the need for a tuning translates into the issue of stability of mass scales. A possibility for a natural separation between the gauge and gravity sectors, achieved by field-theoretical methods known from four-dimensional theories, appears in models with deconstructed dimensions. As an example of deconstruction we have discussed quiver theories which result from a non-supersymmetric orbifolding of the $\mathcal{N} = 4$ U($K$) gauge theories. In a generic situation these non-supersymmetric models exhibit an improved UV behavior — the quadratically divergent contributions to the effective potential vanish at the one-loop level. If the gauge group resulting from orbifolding becomes broken down to the diagonal subgroup by universal vevs, then the resulting low-energy theory exhibits custodial supersymmetry and theory space extra dimensions. The hierarchy $v \ll M_{\mathrm{cut-off}}$ is protected at the one-loop level, and at one-loop universal vevs remain a flat direction and zero-mode multiplets do not suffer from a mass splitting. Of course, the situation becomes even better in $N = 1$ supersymmetric orbifoldings. The deconstructed extra dimensions are fictitious, and belong to a renormalizable 4d gauge model. On the other hand gravity is four-dimensional at all scales. Hence, while retaining at low energies signatures of extra dimensions, these models simplify the physics of the gauge sector/gravity interface.

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