SUPERSYMMETRY BREAKING WITH EXTRA DIMENSIONS

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Dedicated to Stefan Pokorski on his 60th birthday

It is a great pleasure to be here today and celebrate Stefan Pokorski. Given the breadth of his scientific activity, it is no surprise that this talk touches yet another subject to which Stefan, as well as other younger members of the Warsaw group, have given and are giving important contributions.

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1. Introduction

Coordinate-dependent compactifications of higher-dimensional theories, first proposed by Scherk and Schwarz [1], provide an elegant and efficient mechanism for mass generation and symmetry breaking. The basic idea is very simple: one twists the periodicity conditions in the compact extra dimensions by a symmetry of the action (or, more generally, of the equations of motion). From a four-dimensional (4D) point of view, this twist induces mass terms that break the symmetries with which it does not commute (for early applications, see [2]).

This talk will discuss coordinate-dependent compactifications of field theories on orbifolds, focusing on compactifications from five to four dimensions on the orbifold S^1/Z_2, and on the issue of supersymmetry breaking. The emphasis will be on some recent results obtained in [3,4]: a new formulation of the Scherk–Schwarz mechanism that involves localized mass (and possibly interaction) terms for the bulk fields at the orbifold fixed points, allowed by the fact that at the fixed points the fields and their derivatives can jump.

These results have a wide range of applications. They can be used to generate the explicit breaking of global symmetries, such as rigid supersymmetry or flavor symmetry. They can also be used to induce the spontaneous breaking of local symmetries, such as grand unified gauge symmetries or supergravity. Indeed, as will be discussed later in this talk, they encompass the most important features of dynamical supersymmetry breaking mechanisms such as gaugino condensation at the orbifold fixed points.

The plan of the talk is as follows. We first explain the general features of the 'traditional' and 'new' versions of the Scherk–Schwarz mechanism. We then illustrate our results with the simplest example, a free 5D massless fermion with a U(1) twist. We continue with the discussion of the superHiggs effect, *i.e.* the spontaneous breaking of local supersymmetry, in the simple case of pure 5D supergravity. We conclude with a short summary of the main results and with some comments on the prospects for further work.

2. The general mechanism

As a case study, we consider a generic 5D theory compactified on the orbifold S^1/Z_2, with space-time coordinates $x^M \equiv (x^m, y)$. The circle S^1 is obtained from the real axis R^1 by identifying points connected by a $2\,\pi\,R$ translation of the fifth coordinate, where R is the compactification radius:

$$T: \qquad y \longrightarrow y + 2\,\pi\,R\,. \tag{1}$$

The orbifold S^1/Z_2 is then obtained from the circle $S^1 = R^1/T$ by further identifying points connected by a reflection of the fifth coordinate about the origin:

$$Z_2: \qquad y \longrightarrow -y\,. \tag{2}$$

We could then define the theory on the interval $[0, \pi R]$, but we prefer to work on the covering space S^1 or on the full real axis R^1.

We denote by $\Psi(x^m, y)$ all the fields of the 5D theory, classifying them in representations of the 4D Lorentz group. We define the Z_2 transformation properties of the fields by

$$\Psi(-y) = Z\,\Psi(y)\,, \tag{3}$$

where Z is a matrix such that $Z^2 = 1$. It is not restrictive for us to take a basis in which Z is diagonal,

$$\Psi = \left(\begin{array}{c} \Psi^+ \\ \Psi^- \end{array} \right)\,, \quad Z = \mathrm{diag}\,(1, \ldots, 1, -1, \ldots, -1)\,. \tag{4}$$

We assume that the theory has a symmetry (for simplicity we take a global, continuous one), whose action on the fields is given by $\Psi \to \Psi' =$

$\mathrm{U}_{\vec{\beta}}\,\Psi$, where $\mathrm{U}_{\vec{\beta}}$ is a unitary matrix depending on the real parameters $\vec{\beta}$, but not on the space-time coordinates. We implement the Scherk–Schwarz mechanism by twisting the periodicity [1] conditions on S^1. Since the fields $\Psi(y)$ are multi-valued on the circle, it is convenient to define the twist on the full real axis:

$$\Psi(y) = \mathrm{U}_{\vec{\beta}}\,\Psi(y + 2\pi R)\,. \tag{5}$$

A well-known consistency condition [5,6] between the twist and the orbifold projection is that

$$\mathrm{U}_{\vec{\beta}} Z \mathrm{U}_{\vec{\beta}} = Z\,. \tag{6}$$

The reason is that the matrices $\mathrm{U}_{\vec{\beta}}$ and Z should provide a representation of the corresponding transformations T and Z_2 acting on the extra coordinate y: starting from y, the action of Z_2 leads to $-y$, which coincides with what we would obtain by acting first with T, $y \to y + 2\pi R$, then with Z_2, $y + 2\pi R \to -y - 2\pi R$, and finally with T again, $-y - 2\pi R \to -y$. Notice that, if $[\mathrm{U}_{\vec{\beta}}, Z] = 0$, then we get $\mathrm{U}^2_{\vec{\beta}} = 1$, and the twist is quantized: $\mathrm{U}^2_{\vec{\beta}} = \pm 1$. On the other hand, if $[\mathrm{U}_{\vec{\beta}}, Z] \neq 0$ there is room, in a generic 5D field theory and at the classical level, for continuous twist parameters. If we concentrate on the continuous case, and write $\mathrm{U}_{\vec{\beta}} = \exp(i\vec{\beta}\cdot\vec{T})$, where the generator $\vec{\beta}\cdot\vec{T}$ is hermitian, we see that Eq. (6) is satisfied if $\{\vec{\beta}\cdot\vec{T}, Z\} = 0$. Then we can take $\vec{\beta}\cdot\vec{T}$ to be purely off-diagonal in the basis of Eq. (4).

The consequences of the twist of Eq. (5) can be most easily studied by moving to a basis of periodic fields, and this can be achieved by performing a non-periodic, y-dependent field redefinition. A convenient choice is to take a transformation of the form [2]:

$$\Psi(x^m, y) = V(y)\,\widetilde{\Psi}(x^m, y)\,, \tag{7}$$

where

$$V(y) = \mathrm{e}^{i\vec{\beta}\cdot\vec{T} f(y)}\,, \tag{8}$$

and

$$\widetilde{\Psi}(x^m, y + 2\pi R) = \widetilde{\Psi}(x^m, y)\,. \tag{9}$$

The twist of Eq. (5) is reproduced if

$$f(y + 2\pi R) = f(y) + 1\,. \tag{10}$$

[1] Sometimes one speaks loosely of boundary conditions, even if there are no boundaries or special points on S^1. There are instead two special points on the orbifold S^1/Z_2, the fixed points $y = 0$ and $y = \pi R$, which will play an important role in what follows.

[2] This is not the most general possibility, and there is some interest in studying the formal consequences of different but physically equivalent choices [7].

Moreover, the field redefinition of Eq. (7) preserves the Z_2 parities if

$$f(-y) = -f(y)\,. \tag{11}$$

Were it not for the y-dependence of $V(y)$, the field redefinition of Eqs. (7)–(9) would leave the action invariant. Moving to the basis of periodic fields $\widetilde{\Psi}$, the only changes in the action are those produced by the terms containing derivatives with respect to y. If there are no derivative interactions, the only such terms are the kinetic terms, and in the transition to the basis of periodic fields only mass terms are generated. If instead the original theory contains derivative interactions, then additional interaction terms do appear.

It is important here to stress a point that some recent papers seem to have missed. Barring the subtleties connected with the orbifold fixed points, that will be addressed shortly, and concentrating for a moment on the case of the circle, physics is completely fixed by the five-dimensional action and by the twist condition (5). Different local field redefinitions of the form (7) may give rise to different y-dependences of the mass terms, but they just correspond to equivalent descriptions of the same physics.

Since, in the present context, mass terms arise from twists in the y direction, it is useful to write:

$$V^\dagger \partial_y \Psi = \left[\partial_y + V^\dagger \partial_y V\right] \widetilde{\Psi} = D_y \widetilde{\Psi}\,. \tag{12}$$

This allows to interpret $D_y \widetilde{\Psi}$ as a covariant derivative, with connection

$$A_y \equiv V^\dagger(-i\partial_y)V = \vec{\beta}\cdot\vec{T} f'(y)\,. \tag{13}$$

Thus, a theory with twisted fields Ψ can be written in terms of an equivalent theory [8] with periodic fields $\widetilde{\Psi}$ and a background gauge field A_y. In the simple case under consideration, the non-local order parameter is just the 'flux'

$$\int_{y_0}^{y_0+2\pi R} dy\, A_y = \vec{\beta}\cdot\vec{T}\,. \tag{14}$$

In analogy with Bloch's theorem of solid state physics (see *e.g.* [9]), there is a 'standard' parameterization in which

$$f(y) = \frac{y}{2\pi R}\,, \tag{15}$$

so that

$$A_y = \frac{\vec{\beta}\cdot\vec{T}}{2\pi R} = \text{constant}\,. \tag{16}$$

This parameterization makes the derivation of the spectrum easier, but the fact that it leads to a y-independent 5D mass term has no intrinsic physical meaning, in agreement with the non-locality of the mechanism.

We are now ready to explain the observation of [3]. On the circle S^1, the fields Ψ must be smooth functions of the extra coordinate y. On the orbifold S^1/Z_2, however, we may allow the fields to have cusps or discontinuities (collectively denoted as 'jumps' in the following) at the orbifold fixed points:

$$\Psi(y_q + \xi) = U_q \, \Psi(y_q - \xi) \,, \tag{17}$$

where $y_q = q\pi R$, $q \in Z$, $0 < \xi \ll 1$ and U_q is a global symmetry transformation. The jumps across points related by a $2\pi R$ translation must be the same, so

$$U_{2q} \equiv U_0 \,, \quad U_{2q+1} \equiv U_\pi \,. \tag{18}$$

A consistency condition identical to (6) must hold for each of the jumps:

$$U_q Z U_q = Z \,. \tag{19}$$

The reason is that an infinitesimal translation across y_q, followed by a reflection about the origin and by another infinitesimal translation across y_{-q}, must correspond to a simple reflection.

The physical spectrum is now controlled by the Scherk–Schwarz twist and by the jumps at the orbifold fixed points. This generalization leads to field bases where the Scherk–Schwarz mechanism can be represented by localized mass terms, and the latter control the field discontinuities via properly derived equations of motion.

In the next section, we shall show on a simple example that the theory with discontinuities is equivalent to a conventional Scherk–Schwarz theory with a modified twist.

3. The simplest example

To illustrate our mechanism in a simple setting, we consider, following [3], a free 5D massless Dirac fermion, written in terms of 5D fields with 4D spinor indices. In the notation of Eqs. (3) and (4), we write:

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} , \quad \overline{\Psi} = \begin{pmatrix} \overline{\psi_1} \\ \overline{\psi_2} \end{pmatrix} , \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{20}$$

The free massless 5D Dirac Lagrangian can be decomposed as:

$$\mathcal{L} = i \overline{\Psi}^T \overline{\sigma}^m \partial_m \Psi - \tfrac{1}{2} \left[\Psi^T (i\hat{\sigma}^2) \partial_y \Psi + \text{h.c.} \right] \,, \tag{21}$$

and the corresponding equations of motion read

$$i\sigma^m \partial_m \overline{\Psi} - (i\hat{\sigma}^2) \partial_y \Psi = 0 \,, \tag{22}$$

where the hat on $\hat{\sigma}^2$ reminds us that it acts on the two-dimensional space of Eq. (20). Irrespectively of the behavior of the fields at the orbifold fixed points, Eq. (22) must be valid in each region $y_q < y < y_{q+1}$ of the real axis.

The Lagrangian (21) and the equation of motion (22) are invariant under global SU(2) transformations of the form $\Psi' = \mathrm{U}_{\vec{\beta}}\Psi$, where $\mathrm{U}_{\vec{\beta}} \in \mathrm{SU}(2)$. We take for simplicity a U(1) subgroup with a single parameter β:

$$\mathrm{U}_\beta = \exp\left(i\beta\hat{\sigma}^2\right) = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}, \tag{23}$$

$$U_q = \exp\left(i\delta_q\hat{\sigma}^2\right) = \begin{pmatrix} \cos\delta_q & \sin\delta_q \\ -\sin\delta_q & \cos\delta_q \end{pmatrix}, \tag{24}$$

where $\delta_{2q} = \delta_0$ and $\delta_{2q+1} = \delta_\pi$ for any $q \in Z$. The consistency conditions of Eqs. (6) and (19) are obviously satisfied. In contrast with the 'traditional' case, the generalized boundary conditions are now specified by three real parameters, the twist β and the jumps $\delta_{0,\pi}$.

To determine the four-dimensional spectrum, we seek solutions $\Psi(y)$ to Eq. (22), with the boundary conditions of Eqs. (23) and (24). Exploiting the fact that $i\sigma^m\partial_m\overline{\Psi} = m\Psi$, we find

$$\Psi(y) = \chi\begin{pmatrix} \cos[my - \alpha(y)] \\ \sin[my - \alpha(y)] \end{pmatrix}, \tag{25}$$

where χ is a y-independent 4D spinor,

$$m = \frac{n}{R} - \frac{(\beta - \delta_0 - \delta_\pi)}{2\pi R}, \quad (n \in Z), \tag{26}$$

and

$$\alpha(y) = \frac{\delta_0 - \delta_\pi}{4}\varepsilon(y) + \frac{\delta_0 + \delta_\pi}{4}\eta(y). \tag{27}$$

Here $\varepsilon(y)$ is the 'sign' function defined on S^1, and

$$\eta(y) = 2q + 1, \quad y_q < y < y_{q+1}, \quad (q \in Z), \tag{28}$$

is the 'staircase' function that steps by two units every πR along y. The function $\alpha(y)$ satisfies

$$\alpha(y + 2\pi R) = \alpha(y) + \delta_0 + \delta_\pi. \tag{29}$$

so the solution (25) has the correct Scherk–Schwarz twist.

The spectrum (26) is characterized by a uniform shift with respect to a traditional Kaluza–Klein compactification. In contrast to the usual Scherk–Schwarz mechanism, however, the shift depends on the jumps δ_0 and δ_π, as well as on the twist β. In particular, it is possible to have a vanishing shift for nonvanishing β. In the limit $\delta_q \to 0$, our results reduce to the conventional Scherk–Schwarz spectrum. Note that the eigenfunction of Eq. (25) is discontinuous: the even part has cusps and the odd part has jumps at $y = y_q$, as required by the boundary conditions. In the limit $\delta_q \to 0$ the eigenfunction becomes regular everywhere.

For any δ_q, the system is equivalent to a conventional Scherk–Schwarz compactification with twist $\beta^c = \beta - \delta_0 - \delta_\pi$. The new field variable, Ψ_c, is related to the discontinuous variable, Ψ, via the generalized function $\alpha(y)$,

$$\begin{pmatrix} \psi_{1c} \\ \psi_{2c} \end{pmatrix} = \begin{pmatrix} \cos\alpha(y) & \sin\alpha(y) \\ -\sin\alpha(y) & \cos\alpha(y) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} . \tag{30}$$

This is reminiscent of strong CP violation, where the physical order parameter is not θ, but the combination $\theta - \arg\det m_q$, where m_q is the quark mass matrix. Similarly, the mass shift of our system is controlled not by β alone, but by the twist β^c, which includes contributions from jumps in the fermion fields. As in QCD, where we can eliminate the phase in $\det m_q$ by a chiral transformation, here we can remove the jumps by a redefinition of the fermion fields. In the new basis, there are no jumps, but the twist acquires an additional contribution.

Discontinuous field variables arise from mass terms localized at the fixed points. This can be seen by starting from a Lagrangian $\mathcal{L}$ of the form (21) for the continuous fields $\psi_c^{1,2}(y)$, characterized by a twist $\beta^c = \beta - \delta_0 - \delta_\pi$ but no jumps:

$$\begin{aligned} \mathcal{L}(\psi_c) \;&=\; i\overline{\psi_c^1}\overline{\sigma}^m\partial_m\psi_c^1 + i\overline{\psi_c^2}\overline{\sigma}^m\partial_m\psi_c^2 \\ &+\; \left[\frac{1}{2}\left(\psi_c^2\partial_y\psi_c^1 - \psi_c^1\partial_y\psi_c^2\right) + \text{h.c.}\right] . \end{aligned} \tag{31}$$

If we perform the field redefinition of Eq. (30), the 5D Lagrangian becomes:

$$\mathcal{L}(\psi_c) = \mathcal{L}(\psi) + \mathcal{L}_{\text{brane}}(\psi) , \tag{32}$$

where

$$\mathcal{L}_{\text{brane}}(\psi) \;=\; -\frac{1}{2}\alpha'(y)\left(\psi_1\psi_1 + \psi_2\psi_2\right) + \text{h.c.} , \tag{33}$$

and

$$\alpha'(y) = \sum_{q=-\infty}^{+\infty} \left[\delta_0\,\delta(y - y_{2q}) + \delta_\pi\,\delta(y - y_{2q+1})\right] . \tag{34}$$

The fields $\psi_{1,2}$ have now jumps $\delta_{0,\pi}$ and a twist $\beta = \beta^c + \delta_0 + \delta_\pi$. We see that the jumps δ_q arise from mass terms localized at the fixed points.

The discontinuities of the fields can be recovered by integrating the equations of motion. The trick is to find the correct equations. To understand the type of subtleties that may arise, when trying to apply the naive variational principle to localized actions, imagine taking the variation of a localized Lagrangian $\mathcal{L}$ bilinear in a discontinuous field ψ_2. In the variation, products of the form $(\partial\mathcal{L}/\partial\psi_2)\,\delta\psi_2$ will appear. Since $(\partial\mathcal{L}/\partial\psi_2)$ contains a δ-function, and $\delta\psi_2$ may behave as a step function, we cannot use the naive equations of motion to infer that $(\partial\mathcal{L}/\partial\psi_2)$ must vanish at the fixed point. We can avoid all subtleties associated with discontinuous field variables by *defining* the term that appears in the brane action to be *continuous* across the orbifold fixed points. For the case at hand, this means that we must choose the field variables so that the combination $\psi_1\psi_1 + \psi_2\psi_2$ is continuous. Alternatively, we can obtain the equations of motion by first regularizing the delta functions, so that ψ_1 and ψ_2 are both continuous, and then taking the singular limit.

It is interesting to note that the same physical system can be obtained from another brane Lagrangian, one in which we treat the even field $\psi_1(y)$ as continuous. The discontinuity of the odd field $\psi_2(y)$ is then

$$\psi_2(y_q+\xi) - \psi_2(y_q-\xi) = -2\tan\frac{\delta_q}{2}\,\psi_1(y_q)\,. \tag{35}$$

This jump is reproduced by the brane Lagrangian

$$\mathcal{L}'_{\rm brane}(\psi) = -\frac{1}{2}f(y)\,\psi_1\psi_1 + {\rm h.c.}\,, \tag{36}$$

where

$$f(y) = 2\sum_{q\in Z}\left[\tan\frac{\delta_0}{2}\,\delta(y-y_{2q}) + \tan\frac{\delta_\pi}{2}\,\delta(y-y_{2q+1})\right]. \tag{37}$$

In this case, we vary with respect to $\psi_1(y)$ and $\psi_2(y)$; the discontinuous field $\psi_2(y)$ does not appear in the brane Lagrangian.

In summary, the brane Lagrangians (33) and (36) give rise to equivalent theories in the absence of brane interactions, provided we use an appropriate procedure to derive the equations of motion.

4. The superHiggs effect

We now discuss, along the lines of [4], the application of our mechanism to the spontaneous breaking of (the residual) supersymmetry, in 5D supergravity compactified on the orbifold S^1/Z_2. As anticipated at the beginning, the long list of references on the subject of supersymmetry breaking in M-theory and in 5D field-theory orbifolds [10] includes many papers by Stefan and by other members of the Warsaw group.

In the case under consideration, the 'new' formulation of the Scherk–Schwarz mechanism discussed above leads to what can be called [4] 'brane-induced supersymmetry breaking', and reproduces the main features of gaugino condensation in M-theory, *i.e.*:

- there are localized gravitino mass terms at the orbifold fixed points, characterized by two independent constants P_0 and P_π;
- the classical four-dimensional vacuum energy vanishes identically;
- the compactification radius R is a classical flat direction;
- the order parameter is the non-local quantity $P_0 + P_\pi$, thus we can have one unbroken supersymmetry with $P_0 = -P_\pi \neq 0$;
- the goldstinos, absorbed by the massive gravitinos in the superHiggs effect, are associated with the fifth components of the gravitinos.

Since all these features, apart from the first one, are shared by the 'traditional' Scherk–Schwarz mechanism, it is natural to expect that a suitable generalization of such mechanism may indeed encompass also the distinctive feature of localized gravitino mass terms.

The simplest starting point for the present discussion is pure 5D Poincaré supergravity [11] in its on-shell formulation. The supergravity multiplet contains the fünfbein $e_{\mathrm{M}}{}^{A}$, the gravitino Ψ_{M} and the graviphoton B_{M}. For the present purposes, we just need to recall the terms of the 5D bulk Lagrangian and supersymmetry transformation laws that contain derivatives of the gravitino field and of the supersymmetry transformation parameter. In the notation of [4]:

$$\kappa \mathcal{L}_{\mathrm{bulk}} = i\varepsilon^{MNOPQ}\overline{\Psi}_{\mathrm{M}}\Sigma_{NO}D_P\Psi_Q + \dots , \tag{38}$$

$$\delta\Psi_{\mathrm{M}} = \frac{2}{\kappa}D_{\mathrm{M}}\eta + \dots , \tag{39}$$

where the gravitino Ψ_{M} and the supersymmetry parameter η are described by five-dimensional Dirac spinors:

$$\Psi_{\mathrm{M}} \equiv \left(\begin{array}{c} \psi^1 \\ \psi^2 \end{array} \right)_{\mathrm{M}} , \qquad \eta \equiv \left(\begin{array}{c} \eta^1 \\ \eta^2 \end{array} \right) . \tag{40}$$

As for the orbifold projection, we assign even Z_2-parity to

$$e_m{}^a , \quad e_{5\hat{5}} , \quad B_5 , \quad \psi^1_m , \quad \psi^2_5 , \quad \eta^1 , \tag{41}$$

and odd Z_2-parity to

$$e_5{}^a , \quad e_{m\hat{5}} , \quad B_m , \quad \psi^2_m , \quad \psi^1_5 , \quad \eta^2 . \tag{42}$$

We start by recalling the essential features of the conventional Scherk–Schwarz mechanism. The Lagrangian has a global $\mathrm{SU}(2)_{\mathrm{R}}$ invariance, under which the field

$$\Phi_{\mathrm{M}} \equiv \left(\begin{array}{c} \psi^1_{\mathrm{M}} \\ \psi^2_{\mathrm{M}} \end{array} \right) , \tag{43}$$

which should not be confused with Ψ_{M}, transforms as a doublet. In analogy with the previous example, the gravitino boundary conditions can be twisted by a $\mathrm{U}(1)_{\mathrm{R}} \subset \mathrm{SU}(2)_{\mathrm{R}}$ transformation,

$$\Phi^c_{\mathrm{M}}(y + 2\pi R) = \mathrm{e}^{i\beta^c \hat{\sigma}^2}\, \Phi^c_{\mathrm{M}}(y) \, . \tag{44}$$

The label 'c' indicates that the fields are continuous across the two orbifold fixed points, *i.e.* $\delta^c_0 = \delta^c_\pi = 0$. With standard technology, we can derive the gravitino spectrum, characterized by the non-local order parameter β^c:

$$\mathcal{M}^{(\rho)}_{3/2} = \frac{\rho}{R} - \frac{\beta^c}{2\pi R} , \qquad (\rho = 0, \pm 1, \pm 2, \ldots) \, . \tag{45}$$

We are now ready to show that our generalized Scherk–Schwarz mechanism can lead to the bulk-plus-brane action of brane-induced supersymmetry breaking. We can exploit the fact that, on the orbifold S^1/Z_2, the generalized gravitino boundary conditions are characterized by an overall twist *and* by jumps at the orbifold fixed points. It is then sufficient to perform the following field redefinition:

$$\Phi^c_{\mathrm{M}}(y) = \mathrm{e}^{i\alpha(y)\hat{\sigma}^2}\, \Phi_{\mathrm{M}}(y) \, , \tag{46}$$

where the function $\alpha(y)$ is the same as in the previous section. From these expressions, it is not hard to check that the fields $\Phi_{\mathrm{M}}(y)$ have jumps δ_0 and δ_π at the orbifold fixed points, and twist $\beta^c + \delta_0 + \delta_\pi$. Indeed, if we choose

$$\beta^c = -\left(\delta_0 + \delta_\pi\right) \, , \tag{47}$$

the fields $\Phi_{\mathrm{M}}(y)$ are periodic.

The bulk action is not invariant under this field redefinition. As before, the y derivatives give rise to a singular connection, which generates a brane action localized at the orbifold fixed points:

$$\mathcal{L}_{\mathrm{brane}} = \frac{1}{2\kappa} e_4 \left[\delta(x^5)\delta_0 + \delta(x^5 - \pi\kappa)\delta_\pi\right] \left(\psi^1_a \sigma^{ab} \psi^1_b + \psi^2_a \sigma^{ab} \psi^2_b\right) + \text{h.c.} \tag{48}$$

Supersymmetry invariance of the total action $S = S_{\mathrm{bulk}} + S_{\mathrm{brane}}$ is guaranteed by the fact that we have redefined the fields of an invariant bulk action, provided that we redefine the supersymmetry parameter η accordingly.

We can now proceed with a discussion that exactly parallels the one given in the previous section. The brane action (48) must be handled with care if we want to derive the correct equations of motion. The fields $\psi_m^{1,2}$ are too singular to apply the naive variational principle without regularization. Indeed, the even fields are not piecewise smooth: for example, $\psi_m^1(0) \neq \lim_{\xi\to 0}[\psi_m^1(+\xi) + \psi_m^1(-\xi)]/2$, so we cannot apply the standard Fourier decomposition. As in the example of the previous section, we can either regularize the Lagrangian (48) or move to an equivalent brane Lagrangian,

$$\mathcal{L}_{\rm brane} = \frac{1}{\kappa}\, e_4 \left[\delta(x^5)\tan\frac{\delta_0}{2} + \delta(x^5 - \pi\kappa)\tan\frac{\delta_\pi}{2}\right] \psi_a^1\sigma^{ab}\psi_b^1 + \text{ h.c.}\,, \quad (49)$$

to which we can safely apply the naive variational principle to derive the equations of motion, since the even fields ψ_m^1 are continuous. With either method, we can compute the gravitino mass spectrum, and find

$$\mathcal{M}_{3/2}^{(\rho)} = \frac{\rho}{R} + \frac{\delta_0 + \delta_\pi}{2\pi R}\,, \qquad (\rho = 0, \pm 1, \pm 2, \ldots)\,. \quad (50)$$

This result can be matched with the one of brane-induced supersymmetry breaking. Taking for simplicity P_0 and P_π to be real, we find:

$$\delta_{0\,(\pi)} = 2\ \arctan\frac{\kappa^3 P_{0\,(\pi)}}{2}\,. \quad (51)$$

5. Conclusions and outlook

In this talk we have explained how coordinate-dependent compactifications on field-theory orbifolds can be generalized, to include localized mass terms for bulk fields at the orbifold fixed points. We have stressed the fact that, in a basis where fields are only piecewise smooth, physics depends not only on the overall twist of the fields, but also on their jumps at the orbifold fixed points. As an important application, we have discussed the phenomenon of brane-induced breaking of local supersymmetry, but several other applications are conceivable.

There are several aspects that would deserve further investigations.

Here and in [3, 4] the discussion was kept at the purely classical level, but the quantum consistency of the different models should be examined, especially in connection with localized anomalies and Fayet–Iliopoulos terms [12]. The study of the quantum corrections to the effective potential, in the presence of the MSSM fields, could also lead to a dynamical determination of the radius R, along the lines of [13].

The examples considered in this talk have focused on twists and jumps affecting fermions. The bosonic case, relevant for the discussion of gauge symmetry breaking, can be discussed along similar lines [14].

It would be interesting to give an interpretation 'a la Hosotani' of spontaneous supersymmetry breaking via the Scherk–Schwarz mechanism, going beyond the attempts performed so far [15].

Finally, an interesting open problem is the extension of the traditional Scherk–Schwarz mechanism and its generalization to the case of warped compactifications.

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