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MIRROR SYMMETRY IN CALABI–YAU COMPACTIFICATIONS WITH BACKGROUND FLUX

Jan Louis

Fachbereich Physik, Martin-Luther-Universität Halle-Wittenberg
Friedemann-Bach-Platz 6, D-06099 Halle, Germany

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Dedicated to Stefan Pokorski on his 60th birthday

We discuss Calabi–Yau compactifications with background fluxes at the level of the low energy effective action. For the fluxes in the RR-sector mirror symmetry is manifest at the level of the effective action while for the fluxes in the NS-sector the compactification manifold has to be deformed.

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1. Introduction

The standard compactifications of string theory consider a space-time background of the form

$$\mathcal{M}_{10} = \mathcal{M}_4 \otimes Y_6 \,, \tag{1}$$

where $\mathcal{M}_4$ is a non-compact manifold of Minkowskian signature while Y_6 is a compact manifold with Euclidean signature. The structure and properties of Y_6 determine the amount of unbroken supersymmetry of the theory.

The interactions of the light modes (L) of the string excitations are captured by a low energy effective Lagrangian $\mathcal{L}_{\rm eff}(L)$. This Lagrangian can be computed in various ways, for example via a Kaluza–Klein (KK) reduction of the ten-dimensional Lagrangian. This procedure produces a Lagrangian in $d = 4$ space-time dimensions which generically is a supergravity coupled to vector and matter multiplets.

Recently there has been some interest in studying "generalized" compactifications (see, for example, [1–3]) where a non-trivial background flux on Y_6 is turned on

$$\int_{\gamma_p^I} F_p = e^I = \text{const.} \tag{2}$$

F_p is the field strength of a $(p-1)$ form A_{p-1} but one allows for the possibility that F_p also can depend on a harmonic p-form on Y_6

$$F_p = dA_{p-1} + e^I \omega^I \,, \tag{3}$$

where ω^I is some harmonic p-form. Note that the Bianchi identity and the equation of motions

$$dF_p = 0 = d^\dagger F_p \tag{4}$$

still hold as long as the e^I are constant and the ω^I are harmonic. Integrating over an appropriate p-cycle γ_p^I on Y_6 then produces (2).

The flux parameters e^I are quantized in string theory but can be considered as continuous parameters in supergravity. Thus one can treat the e^I as small perturbations such that the light spectrum of the compactification does not change. It has been argued [2, 3] that this procedure is a consistent KK-reduction and that supersymmetry is spontaneously broken by the presence of the flux parameters e^I. The resulting low energy effective Lagrangian thus is a gauged or massive supergravity with the e^I appearing as gauge or mass parameters.

The motivation to study these generalized compactifications is derived from the fact that the flux parameters induce a non-trivial potential for the string theoretic moduli fields and hence lift the vacuum degeneracy. Furthermore, supersymmetry is generically broken which is another phenomenological attraction. In the talk we focus, however, on two different aspects of these compactifications. First of all, we uncover new aspects of $N = 2$ supergravity in $d = 4$ which were not fully appreciated previously. Secondly, we discuss the fate of the perturbatively established mirror symmetry in the presence of non-vanishing fluxes. The talk is based on [4, 5] where also a more complete list of references can be found.

2. RR-flux

Let us start by recalling the light spectrum of type II string theories. Both type IIA and type IIB feature in the NS-sector the metric g_{MN}, a two-form B_2 and the dilaton ϕ. In the RR-sector type IIA has a one-form C_1 and a three-form C_3 while type IIB has a second scalar l, a second two-form C_2 and a four-form C_4 with a self-dual field strength $F_5^* = dC_4$. For the other field strengths we use the notation

$$H_3 = dB_2 \,, \quad F_2 = dC_1 \,, \quad F_4 = dC_3 \,. \tag{5}$$

Furthermore, the type IIA theory has a one-parameter family of deformations, called massive type IIA theories where the two-form B_2 becomes massive with the mass parameter denoted by m_0 [6].

In order to study, Calabi–Yau compactifications of these theories let us briefly recall some facts about Calabi–Yau threefolds Y. The Einstein equations require the compact manifold Y to be Ricci-flat

$$R_{MN} = 0\,, \tag{6}$$

while the amount of unbroken supersymmetry constrains the holonomy group of Y. The minimal amount of supersymmetry is obtained for holonomy SU(3) and indeed Calabi–Yau threefolds are Ricci-flat Kähler manifolds of SU(3) holonomy. This in turn implies the existence of a single covariantly constant spinor and a unique holomorphic three-form Ω.

The light spectrum after compactification is determined by the zero modes which are elements of the de-Rahm cohomology $H^{(p,q)}(Y)$ of Y. The dimensions of the cohomology group $H^{(p,q)}(Y)$ are the Hodge numbers $h^{(p,q)}$ and for Calabi–Yau threefolds the only non-trivial ones are

$$\begin{aligned} h^{(0,0)} &= h^{(3,0)} = h^{(0,3)} = h^{(3,3)} = 1\,, \\ h^{(1,1)} &= h^{(2,2)}\,, \quad h^{(1,2)} = h^{(2,1)}\,, \end{aligned} \tag{7}$$

with all others vanishing.

Mirror symmetry states that type IIA compactified on Y is quantum equivalent to type IIB compactified on the mirror manifold $\tilde{Y}$. $\tilde{Y}$ has interchanged Hodge numbers, *i.e.*

$$h^{(1,1)}(\tilde{Y}) = h^{(1,2)}(Y)\,, \qquad h^{(1,2)}(\tilde{Y}) = h^{(1,1)}(Y)\,. \tag{8}$$

After these preliminaries let us first display the possible fluxes. For type IIA theories one has in the RR-sector

$$F_2 = \sum_{i=1}^{h^{(1,1)}} m^i \omega_2^i\,, \qquad F_4 = \sum_{i=1}^{h^{(1,1)}} e^i \omega_4^i\,, \tag{9}$$

where ω_2^i is a basis of $H^{(1,1)}$ while ω_4^i is a basis of $H^{(2,2)}$. In addition there are the two extra parameters m^0 and e^0, where e^0 is the dual of $F_{4\,\mu\nu\rho\sigma}$. So altogether there are $2(h^{(1,1)}+1)$ flux parameters (e^0, e^i, m^0, m^i) in the RR-sector of massive type IIA. In the NS-sector one has

$$H_3 = \sum_{I=0}^{h^{(1,2)}} q^I \alpha^I + p^I \beta^I\,, \tag{10}$$

where (α^I, β^I) form a basis of $H^3(Y)$. Thus we have $2(h^{(1,2)}+1)$ flux parameters (q^I, p^I) in the NS-sector.

For type IIB one finds

$$F_3 = \sum_{I=0}^{h^{(1,2)}} \tilde{e}^I \alpha^I + \tilde{m}^I \beta^I \,, \tag{11}$$

in the RR-sector and

$$H_3 = \sum_{I=0}^{h^{(1,2)}} \tilde{q}^I \alpha^I + \tilde{p}^I \beta^I \,, \tag{12}$$

in the NS-sector. Thus there are $2(h^{(1,2)}+1)$ flux parameters $(\tilde{e}^I, \tilde{m}^I)$ in the RR-sector and $2(h^{(1,2)}+1)$ flux parameters $(\tilde{q}^I, \tilde{p}^I)$ in the NS-sector.

Just from counting parameters it is already obvious that mirror symmetry can hold in the RR-sector but not obviously in the NS-sector. This is confirmed by an explicit Kaluza–Klein reduction of the respective effective actions [4]. In both cases one finds an $N = 2$ low energy supergravity in $d = 4$ featuring a gravitational multiplet coupled to $h^{(1,1)}$ vector multiplets, $h^{(1,2)}$ hypermultiplets and one tensor multiplet. For only RR-fluxes mirror symmetry is manifestly established at the level of the effective action in that

$$\mathcal{L}_{\text{eff}}^{\text{IIA}}(Y, e^I, m^I) \equiv \mathcal{L}_{\text{eff}}^{\text{IIB}}(\tilde{Y}, \tilde{e}^I, \tilde{m}^I) \tag{13}$$

with the fluxes identified, *i.e.* $e^I = \tilde{e}^I, m^I = \tilde{m}^I$.

For $m^I = 0$ one finds a standard $N = 2$ gauged supergravity with a potential for the moduli scalars of the vector multiplets. For $m^I \neq 0$ a non-standard supergravity is found where the two-form B_2 becomes massive. For a more detailed discussion and a derivation of the effective action we refer the reader to [4].

3. NS-flux

Let us instead now turn to a discussion of the 'missing' $2(h^{(1,1)} + 1)$ NS-fluxes. Since they have to arise in the NS-sector they can only come from the metric/dilaton fields. Thus one has to consider generalized compactifications where apart from non-trivial background fluxes also a different compactification manifold $\hat{Y}_6$ is chosen, *i.e.*

$$\mathcal{M}_{10} = \mathcal{M}_4 \otimes \hat{Y}_6 \,. \tag{14}$$

$\hat{Y}_6$ can be viewed as a deformation of the original Calabi–Yau manifold Y [7]. The deformation has to be such that it does not change the light spectrum and the amount of supersymmetry. The KK reduction of type IIA string theory on such manifolds is currently in progress [5] but it seems that on $\hat{Y}_6$ the holomorphic three-form Ω is no longer closed but instead satisfies

$$d\Omega \sim e^i \omega_4^i \,. \tag{15}$$

This in turn implies that the complex structure J is no longer integrable in that its Nijenhuis tensor N is non-zero. Similarly the Kähler form K ceases to be closed and instead obeys

$$dK \sim \Omega \,. \tag{16}$$

The details of such compactification are still under investigation but our preliminary results do indicate that by choosing a different compactification manifold $\hat{Y}_6$ it is possible to obtain a type IIA low energy effective Lagrangian which is mirror symmetric to type IIB compactified on the mirror of the original Calabi–Yau manifold Y with non-trivial NS-fluxes turned on [5].

It is a great honor to contribute to this special volume dedicated to Stefan Pokorski on the occasion of his 60th birthday. I wish him all the best for the coming years. Stefan has always worked at the forefront of the current research but I also admire him for successfully educating many young students and guiding them to the relevant questions of High Energy Physics. I do hope that Stefan will continue to play an active role in both respects for many more years.

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