$\bar{B} \to X_s\gamma$ AFTER COMPLETION OF THE NLO QCD CALCULATIONS

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Dedicated to Stefan Pokorski on his 60th birthday

Several years ago, Stefan Pokorski, Manfred Münz and us outlined a program for calculation of the NLO QCD corrections to the weak radiative $\bar{B}$ meson decay $\bar{B} \to X_s\gamma$. Very recently, just before the 60th birthday of Stefan Pokorski, this program has been formally completed. In the present paper, we summarize the existing results and discuss perspectives for further improvement of the accuracy of the Standard Model prediction for $\mathrm{BR}[\bar{B} \to X_s\gamma]$.

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1. Introduction

The radiative decay $\bar{B} \to X_s\gamma$ is known to be extremely sensitive to the structure of fundamental interactions at the electroweak scale. It is dominantly generated by the Flavor Changing Neutral Current (FCNC) decay $b \to s\gamma$ that does not arise at the tree level in the Standard Model (SM). The leading order SM diagrams are shown in Fig. 1.

Many possible non-standard contributions (*e.g.*, SUSY one-loop diagrams) are of the same order in electroweak interactions. They might remain important even for relatively heavy exotic particles. Consequently, $b \to s\gamma$ imposes severe constraints on extensions of the SM (see, for instance, [1–4]).

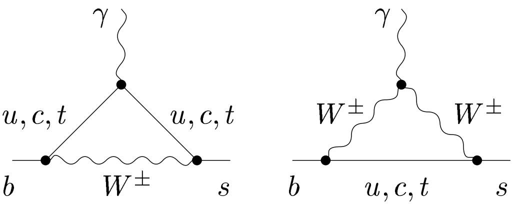

Fig. 1. Leading-order Feynman diagrams for $b \to s\gamma$ in the SM.

The inclusive branching ratio $\mathrm{BR}[\bar{B} \to X_s\gamma]$ has been measured so far by CLEO [5], BELLE [6] and ALEPH [7]. The most accurate result is the one of CLEO, where photons with energies down to 2.0 GeV are included. Extrapolation towards lower photon energy cutoffs is performed following the phenomenological models of Refs. [8, 9].

When the photon energy cutoff is chosen to be 1.6 GeV in the $\bar{B}$-meson rest frame, the experimental world average reads[1]

$$\mathrm{BR}[\bar{B} \to X_s\gamma\ (E_\gamma > 1.6\ \mathrm{GeV})]_{\mathrm{exp}} = (3.12 \pm 0.41) \times 10^{-4}\,. \tag{1.1}$$

Within 1σ, it matches the SM prediction [3, 11]

$$\mathrm{BR}[\bar{B} \to X_s\gamma\ (E_\gamma > 1.6\ \mathrm{GeV})]_{\mathrm{SM}} = (3.57 \pm 0.30) \times 10^{-4}\,. \tag{1.2}$$

One can see that the experimental and theoretical uncertainties are close in size. Without the inclusion of the Next-to-Leading Order (NLO) QCD corrections, the theoretical uncertainty in Eq. (1.2) would be around three times larger, and the constraints on new physics — much weaker.

The program of the NLO calculation was outlined by Stefan Pokorski, Manfred Münz and us in the article [1]. At that time, the only known results were the Leading Order (LO) ones that suffered from large scale uncertainties [1,10]. We analyzed these uncertainties in detail, and enumerated calculations that still had to be done in the NLO case. Very recently, the last element of this NLO program has been completed [11]. In parallel to the QCD calculations, progress was being made in evaluation of the electroweak corrections, non-perturbative effects, as well as in collecting and analyzing the experimental data.

In the present paper, we summarize all the contributions to the NLO QCD calculation of $\mathrm{BR}[\bar{B} \to X_s\gamma]$, and discuss perspectives for further improvement of the theoretical accuracy. In particular, we point out the interplay between charm-quark mass uncertainties in the perturbative calculation and non-perturbative effects.

[1] By convention, contributions to $\bar{B} \to X_s\gamma$ from intermediate real ψ and ψ' are treated as background, while all the other $c\bar{c}$ states are included.

Our article is organized as follows. The next section is devoted to a brief description of the history of perturbative calculations of QCD effects in $b \to s\gamma$. In Section 3, we summarize the electroweak corrections. Non-perturbative effects are discussed in Section 4. The main theoretical uncertainties and possibilities for their elimination are the subject of Section 5. Section 6 contains our conclusions.

2. The LO and NLO QCD calculations

In a certain range of photon energy cutoffs, the width of the hadronic decay $\bar{B} \to X_s\gamma$ is well approximated by the perturbative decay width

$$\Gamma[b \to X_s^{\rm parton}\gamma] = \Gamma[b \to s\gamma] + \Gamma[b \to s\gamma g] + \dots\,. \tag{2.1}$$

Arguments that support such a statement will be discussed in Section 4. Until then, we shall restrict our discussion to the perturbative quantity (2.1).

The framework for all the renormalization-group-improved perturbative analyzes of $b \to X_s^{\rm parton}\gamma$ is set by the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \dots\,. \tag{2.2}$$

It is obtained from the underlying theory (SM in our case) by decoupling of all the particles that are much heavier than the b-quark. The Wilson coefficients $C_i(\mu)$ play the role of coupling constants at the vertices Q_i. The generic structure of the operators Q_i is as follows:

$$Q_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_{\underline{i}} b), & i = 1,\, 2, \\ (\bar{s}\Gamma_i b)\sum_q (\bar{q}\Gamma'_{\underline{i}} q), & i = 3,\, 4,\, 5,\, 6, \qquad (q = u,d,s,c,b) \\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8\,. \end{cases} \tag{2.3}$$

Here, Γ_i and Γ'_i denote various combinations of the color and Dirac matrices (see, *e.g.*, [11]).

The dots in Eq. (2.2) stand for UV counter-terms and non-physical operators that vanish by the QCD$\times$QED equations of motion. In the present section, we neglect everything that is not important for $b \to s\gamma$ at the leading order in $\alpha_{\rm em}$, m_b/M_W, m_s/m_b and V_{ub}/V_{cb}. This includes other operators Q_i of dimension 5 and 6, higher-dimensional operators, as well as terms involving leptons.

Let us assume that the decoupling of heavy particles is performed in the $\overline{\mathrm{MS}}$ scheme, at the renormalization scale $\mu_0 \sim M_W$. The values of $C_i(\mu_0)$ are found from the so-called matching conditions, *i.e.* by imposing equality of the effective- and underlying-theory Green functions at external momenta that are much smaller than masses of the decoupled particles. Next, the Wilson coefficients are evolved from $\mu = \mu_0$ down to $\mu = \mu_b \sim m_b$, according to the Renormalization Group Equations (RGE)

$$\mu \frac{d}{d\mu} C_i(\mu) = C_j(\mu) \gamma_{ji}(\mu)\,, \tag{2.4}$$

where the anomalous dimension matrix $\hat{\gamma}$ is found from UV divergences in the effective theory. This procedure results in expressing the effective Lagrangian (2.2) in terms of

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 C_i^{(2)}(\mu_b) + \dots\,, \tag{2.5}$$

where $C_i^{(n)}(\mu_b)$ depend on α_s only via the ratio $\eta \equiv \alpha_s(\mu_0)/\alpha_s(\mu_b)$. Consequently, working at a fixed order in α_s, one truncates an expansion in powers of $\alpha_s(\mu_b)$ rather than in powers of $\alpha_s(M_W)\ln(M_W^2/m_b^2)$, as it would be the case without introduction of the effective theory. Thus, the behavior of the perturbation series improves. This is the essence of the renormalization-group improvement in the considered case.

In the LO calculations, everything but $C_i^{(0)}(\mu_b)$ is neglected in Eq. (2.5). At the NLO, one takes into account all the $\mathcal{O}(\alpha_s(\mu_b))$ contributions to $\Gamma[b \to X_s^{\rm parton}\gamma]$, including those containing $C_i^{(1)}(\mu_b)$.

The Wilson coefficients encode information on the short-distance QCD effects due to hard gluon exchanges between the quark lines of the leading one-loop electroweak diagrams (Fig. 1). Such effects enhance the branching ratio $\mathrm{BR}[\bar{B} \to X_s\gamma]$ by roughly a factor of three, as first pointed out in Refs. [12, 13].

A peculiar feature of the renormalization group analysis in $b \to s\gamma$ is that the mixing under infinite renormalization between the four-fermion operators $Q_1, \dots, Q_6$ and the "magnetic penguin" operators Q_7, Q_8, which govern this decay, vanishes at the one-loop level. Consequently, in order to calculate the coefficients $C_7(\mu_b)$ and $C_8(\mu_b)$ at LO, two-loop calculations are necessary. Such calculations were completed in Ref. [14]. Earlier analyzes [15–20] contained additional approximations or were not fully correct. The results of Ref. [14] were subsequently confirmed in Refs. [21–23].

As pointed out in Refs. [1, 10], the LO expression for $\Gamma[b \to X_s^{\rm parton}\gamma]$ suffers from large ($\sim\ \pm 25\%$) renormalization scale uncertainties. Therefore, matching the experimental accuracy of Eq. (1.1) requires performing a complete NLO QCD calculation. This goal has been achieved in a joint effort of many groups:

- Two-loop $\mathcal{O}(\alpha_s)$ corrections to the matching conditions $C_7(\mu_0)$ and $C_8(\mu_0)$ were first calculated in Ref. [24] and subsequently confirmed by several groups [25–28].

- Two-loop mixing and one-loop matching for the four-quark operators $Q_1, \ldots, Q_6$ were found in Refs. [29–32]. In Ref. [33], these results were confirmed by recalculation in a different operator basis that is more suitable for $b \to s\gamma$ analyzes.

- Two-loop mixing in the sector (Q_7, Q_8) was calculated in Ref. [34]. These results have been recently confirmed [35].

- Three-loop mixing between the sectors $(Q_1, \ldots, Q_6)$ and (Q_7, Q_8) was evaluated in Ref. [23]. It is currently being verified by another group [35].

- The leading-order matrix elements $\langle s\gamma g|Q_i|b\rangle$ and the one-loop matrix element $\langle s\gamma|Q_7|b\rangle$ were calculated in Refs. [8, 36]. Some of them were confirmed in Ref. [37] where certain BLM corrections were included, too.

- Two-loop calculation of the matrix element $\langle s\gamma|Q_{1,2}|b\rangle$ was presented in Ref. [38]. It has been recently verified and extended to the full basis of four-quark operators [11, 39]. The one-loop matrix element $\langle s\gamma|Q_8|b\rangle$ has been found in Refs. [11, 38], too.

It should be emphasized that all these ingredients enter not only the analysis of $\bar{B} \to X_s\gamma$ in the SM but are also necessary in extensions of this model. The corrections to the Wilson coefficients of the operators Q_7 and Q_8 are also relevant for $\bar{B} \to X_s l^+ l^-$.

3. Electroweak corrections

The study of electroweak corrections begins with searching for terms that might be enhanced by large logarithms. Czarnecki and Marciano [40] pointed out that large logarithms $\ln(m_b^2/m_e^2)$ are absent when $\alpha_{\rm em}^{\rm on\ shell}$ is used in the overall normalization of $\Gamma[b \to X_s^{\rm parton}\gamma]$.

Another type of large logarithm that might enhance some of the electroweak corrections is $\ln(m_W^2/m_b^2)$, *i.e.* the same logarithm that is responsible for the huge QCD enhancement of the $b \to s\gamma$ amplitude. Once $[1-\alpha_s(\mu_0)/\alpha_s(\mu_b)] \sim 0.4$ is treated as a quantity of order unity, the considered electroweak correction is formally of order $\mathcal{O}(\alpha_{\rm em}/\alpha_s)$, so it might be numerically relevant, given the accuracy in Eq. (1.1). However, as demonstrated in Refs. [9, 40, 41] through explicit calculations, it turns out to be negligible ($\sim -0.7\%$).

The articles [42] contain results for the complete electroweak corrections to the matching conditions $C_i(\mu_0)$. Some of them are proportional to $\alpha_{\rm em}(M_Z)/\sin^2\theta_W \simeq 0.034$. Their effect on $\Gamma[b \to X_s^{\rm parton}\gamma]$ amounts[2] to -1.5% for $M_{\rm Higgs} = 115$ GeV, and diminishes with increasing $M_{\rm Higgs}$. The authors of Ref. [42] resolved the numerical discrepancy between Refs. [43] and [40] in favor of the latter.

The only electroweak $\mathcal{O}(\alpha_{\rm em})$ corrections that remain unknown at present are enhanced neither by large logarithms nor by $1/\sin^2\theta_W$. Thus, we can be practically certain about their irrelevance.

4. Non-perturbative effects

The LO contribution to $\Gamma[b \to X_s^{\rm parton}\gamma]$ is given by the tree-level matrix element of the Q_7 operator[3]. Let us temporarily assume that this operator is the only one in the effective Lagrangian (2.2), and denote the corresponding contribution to the hadronic width by $\Gamma[\bar{B} \to X_s\gamma]^{(Q_7 \text{ only})}$.

In analogy to the analyzes [44, 45] of the inclusive semi-leptonic decay $\bar{B} \to X_u e\bar{\nu}$, one can apply the Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET) to show that

$$\begin{aligned}\Gamma[\bar{B} \to X_s\gamma]^{(Q_7 \text{ only})} &= \Gamma[b \to X_s^{\rm parton}\gamma]^{(Q_7 \text{ only})} \\ &\times \left[1 + a_1\frac{\lambda_1}{m_b^2} + a_2\frac{\lambda_2}{m_b^2} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^3}{m_b^3}\right)\right]. \qquad (4.1)\end{aligned}$$

Here, $\lambda_{1,2} \sim \Lambda_{QCD}^2$ are the standard HQET parameters. The value of $\lambda_2 \simeq 0.12$ GeV^2 is known from the measured B–B^* mass difference. The value of $\lambda_1 = -(0.27 \pm 0.10 \pm 0.04)$ GeV^2 has been determined in Ref. [46] from the observed semi-leptonic B-decay spectra (see Ref. [47] for more recent determinations). The coefficients a_1 and a_2 can be calculated within perturbation theory[4], which yields [48, 49]

[2] This number includes QED corrections to the matrix elements of $Q_{1,2,7}$, too.

[3] In dimensional regularization, one-loop matrix elements of $Q_3, \ldots, Q_6$ may give LO contributions, too. However, they can be absorbed into the tree level matrix element of Q_7 with a redefined Wilson coefficient [1, 18].

[4] The same refers to similar coefficients at higher orders in the $(\Lambda_{\rm QCD}/m_b)$-expansion.

$$a_1 = \frac{1}{2} + \mathcal{O}(\alpha_s(m_b)) \qquad \text{and} \qquad a_2 = -\frac{9}{2} + \mathcal{O}(\alpha_s(m_b))\,. \tag{4.2}$$

The resulting $\mathcal{O}(\Lambda^2_{\rm QCD}/m_b^2)$ non-perturbative correction on the r.h.s. of Eq. (4.1) amounts to around -3%.

The relation (4.1) still holds when a lower cutoff E_0 is imposed on the photon energy in the $\bar{B}$-meson rest frame, provided E_0 is not too close to the endpoint $E^{\rm max} = (m_B^2 - m_{K^*}^2)/(2m_B) \simeq 2.6$ GeV. Acceptable values of E_0 must correspond to much larger than $\Lambda_{\rm QCD}$ invariant masses of the recoiling hadronic state X_s. Fig. 3 in Ref. [9] suggests that $E_0 = 1.6$ GeV is sufficiently low. More than 95% of the total $\Gamma[b \to X_s^{\rm parton}\gamma]$ originates from a peak that lays above such a cutoff[5]. This peak is now clearly seen in the $\bar{B} \to X_s\gamma$ spectrum observed by CLEO (Fig. 2). Its position corresponds to the photon energy in the leading two-body decay $b \to s\gamma$.

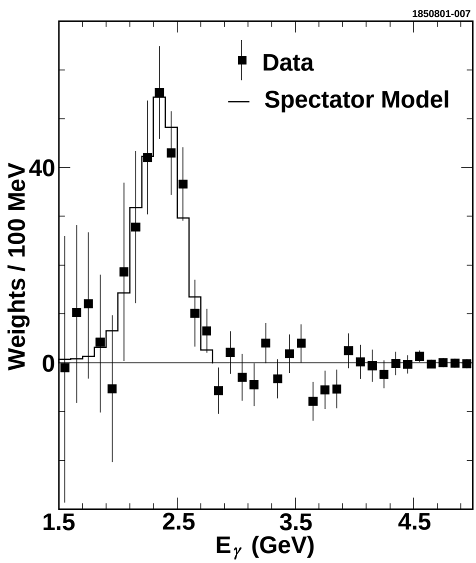

Fig. 2. The $\bar{B} \to X_s\gamma$ photon spectrum observed by CLEO [5].

There is neither experimental nor theoretical need to consider photons below 1.6 GeV. They are practically unobservable at the inclusive level, because of the overwhelming $b \to c$ background. On the theoretical side, keeping not too small E_0 facilitates the discussion of non-perturbative effects due to operators other than Q_7. Of course, we have to admit that 1.6 GeV is chosen arbitrarily. It could almost equivalently be 1.5 or 1.7 GeV.

[5] Consequently, E_0-dependence in Eq. (4.2) can be safely neglected.

However, going up to the current CLEO cutoff of 2.0 GeV would increase uncertainties on the theoretical side. Data-driven extrapolation from the experimental cutoff to the theoretically preferred one is the right choice to make at present.

The discussion of non-perturbative effects becomes much more complex when we take into account operators other than Q_7. It is no longer possible to apply OPE in analogy to $\bar{B} \to X_u e\bar{\nu}$, because the b-quark annihilation and the photon emission may now be separated in space-time by more than $\Lambda_{\rm QCD}^{-1}$.

The contribution of Q_8 to $\Gamma[\bar{B} \to X_s\gamma]$ has been analyzed in Ref. [50] with the help of fragmentation functions. Important non-perturbative effects have been found for low E_γ only, *i.e.* much below $E_0 = 1.6$ GeV. Thus, with our cutoff, a reliable approximation is given by the perturbative contribution to $\Gamma[b \to X_s^{\rm parton}\gamma]$ from the matrix elements of Q_8. The accuracy of this approximation does not need to be known precisely, because the perturbative contribution of Q_8 is smaller than 3%.

Similar conclusions can be drawn for the operators $(\bar{s}\Gamma b)(\bar{q}\Gamma' q)$, where $q = u, d, s$. They are present inside $Q_3, \ldots, Q_6$. Perturbative effects of their matrix elements are even smaller than that of Q_8. As far as non-perturbative effects are concerned, one might worry about production of virtual vector mesons that convert to a real photon. However, creation of such transverse mesons is impossible in the factorization approximation because $Q_3, \ldots, Q_6$ contain no $\bar{q}\sigma_{\mu\nu}q$ currents. Deviations from the factorization approximation are suppressed either by $\alpha_s(m_b)$ or by $\Lambda_{\rm QCD}/m_b$ [51]. This is sufficient to make them negligible here, given the smallness of $|C_{3,\ldots,6}(\mu_b)| < 0.07$, as compared to $|C_{1,2,7,8}(\mu_b)| \simeq (0.5,\ 1,\ 0.3,\ 0.15)$.

The operators $Q_3, \ldots, Q_6$ contain $(\bar{s}\Gamma b)(\bar{b}\Gamma' b)$ terms, too. The b-quark loops are localized at distances much smaller $\Lambda_{\rm QCD}^{-1}$ in space-time. Thus, they can undergo the same treatment as Q_7, as far as non-perturbative effects are concerned. Since their perturbative contributions are minor, the non-perturbative ones are totally negligible.

Charm quark loops are the most difficult to analyze. Factorization is not sufficient here because $2m_c/m_b$ is not a small number. Moreover, non-factorizable contributions may be numerically important because the Wilson coefficients C_1 and C_2 are not small at all.

Let us begin with tracing down possible contributions from intermediate real $c\bar{c}$ states. Our cutoff $E_0 = 1.6$ GeV implies that the invariant mass of the final X_s state is smaller than $m_{\eta_c} + m_K$. Consequently, real $c\bar{c}$ states might occur only *before* the photon emission, *i.e.* in a cascade decay: $\bar{B} \to Y_{c\bar{c}} X_s^{(1)}$ followed by $Y_{c\bar{c}} \to X^{(2)}\gamma$.

The importance of such processes can be tested in the case $Y_{c\bar{c}} = \psi$, because separate experimental data on both (inclusive) components of the cascade decay are available. For low E_0, the resulting branching ratio of the intermediate ψ contribution is larger than the one in Eq. (1.1). It gets reduced to (a few)$\times 10^{-5}$ for $E_0 = 1.6$ GeV, and becomes negligible for $E_0 = 2.0$ GeV [52].

The models used by CLEO [5, 53] to extrapolate from 2.0 GeV to lower cutoffs do not include the intermediate ψ contribution. They are based on perturbative calculations, in which the only diagram (Fig. 3) that might correspond to this contribution affects $\Gamma[b \to X_s^{\rm parton}\gamma]$ by 1.7% only (for $E_0 = 1.6$ GeV). Consequently, the procedure applied by CLEO is consistent with treating the intermediate ψ contribution as background.

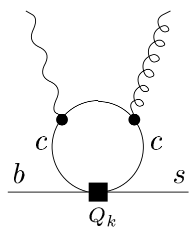

Fig. 3. Charm loop contribution to $b \to s\gamma g$.

Identical arguments work for ψ'. Higher $c\bar{c}$ states might produce higher energy photons. However, radiative charm annihilation processes in all the $c\bar{c}$ states except ψ and ψ' have negligible branching ratios. Thus, it does not really matter whether we consider their contributions as background or not. Whatever decision is made, its effect is expected to be less than the 1.7% perturbative contribution from the diagram in Fig. 3.

Having discussed the real intermediate $c\bar{c}$ states, we proceed to the virtual ones. Neither infrared nor collinear singularities occur in the perturbative contributions of $c\bar{c}$ loops to $\Gamma[b \to X_s^{\rm parton}\gamma]$ at NLO. Thus, according to the common wisdom, one expects that these perturbative results give reasonable estimates to the corresponding contributions to $\Gamma[\bar{B} \to X_s\gamma]$, up to corrections of order $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$.

The actual situation is somewhat more complicated, because the leading one-loop diagram (Fig. 4) vanishes for the on-shell photon. However, it becomes non-vanishing when a soft gluon is attached to the c-quark loop.

Such a gluon may originate from the decaying $\bar{B}$ meson. Thus, one finds a non-perturbative effect [54, 55] that is not approximated in any sense by the corresponding perturbative null. Fortunately, it can be expressed within HQET in terms of a series

$$\frac{\Delta\Gamma[\bar{B} \to X_s\gamma]}{\Gamma[\bar{B} \to X_s\gamma]} = \frac{\lambda_2}{m_c^2} \sum_{n=0}^{\infty} b_n \left(\frac{m_b \Lambda_{\rm QCD}}{m_c^2} \right)^n, \tag{4.3}$$

in which the $n \geq 1$ terms are likely to be negligible, because the coefficients b_n decrease rapidly with n [56,57]. The calculable leading $\mathcal{O}(\Lambda_{\rm QCD}^2/m_c^2)$ term enhances the decay width by around 2.5% [58].

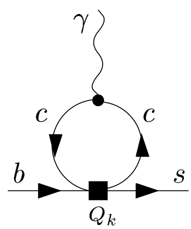

Fig. 4. One-loop matrix element that vanishes for the on-shell photon.

The perturbative $\mathcal{O}(\alpha_s)$ results described in Sec. 2 include non-vanishing two-loop diagrams with $c\bar{c}$ loops, *e.g.* the ones obtained by adding a virtual gluon to the diagram in Fig. 4. The corresponding non-perturbative effects are expected to be suppressed by both $\alpha_s(m_b)$ and $\Lambda_{\rm QCD}/m_{c,b}$. Thus, at the first glance, they might seem irrelevant. However, it remains an open question whether their suppression is numerically sufficient. No quantitative estimates of such non-perturbative effects have been performed so far. We shall discuss this issue in more detail at the end of the next section.

5. Phenomenological discussion

In the present section, we shall discuss the two main uncertainties in the present-day SM prediction for $\bar{B} \to X_s\gamma$. The analysis of Ref. [3] will be largely followed.

The prediction (1.2) is obtained from the formula

$$\begin{aligned} &{\rm BR}[\bar{B} \to X_s\gamma\ (E_\gamma > E_0)] \\ &= {\rm BR}[\bar{B} \to X_c e\bar{\nu}]_{\rm exp} \left(\frac{\Gamma[\bar{B} \to X_u e\bar{\nu}]}{\Gamma[\bar{B} \to X_c e\bar{\nu}]} \right)_{\rm th} \left(\frac{\Gamma[\bar{B} \to X_s\gamma\ (E_\gamma > E_0)]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]} \right)_{\rm th}, \end{aligned} \tag{5.1}$$

in which the following substitutions are made

$$\left(\frac{\Gamma[\bar{B} \to X_s\gamma(E_\gamma > E_0)]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}\right)_{\rm th} \simeq \left(\frac{\Gamma[b \to X_s^{\rm parton}\gamma(E_\gamma > E_0)]}{\Gamma[b \to X_u^{\rm parton} e\bar{\nu}]}\right)_{\rm NLO} + \left(\begin{array}{c}\text{non-perturbative}\\ \text{corrections (4.3)}\end{array}\right), \tag{5.2}$$

$$\left(\frac{\Gamma[\bar{B} \to X_u e\bar{\nu}]}{\Gamma[\bar{B} \to X_c e\bar{\nu}]}\right)_{\rm th} \simeq \left(\frac{\Gamma[b \to X_u^{\rm parton} e\bar{\nu}]}{\Gamma[b \to X_c^{\rm parton} e\bar{\nu}]}\right)_{\rm NNLO} + \left(\begin{array}{c}\text{known}\,\mathcal{O}\big(\lambda_2/m_b^2\big)\\ \text{corrections}\end{array}\right). \tag{5.3}$$

Such ratios are introduced in order to minimize uncertainties in Eq. (5.1) that originate from the CKM angles and the overall factors of m_b^5. The use of $b \to u$ transitions is motivated by the fact that Eq. (5.3) is known at the NNLO, while convergence of the perturbation series and non-perturbative effects are more easily controlled in Eq. (5.2) than in $\Gamma[\bar{B} \to X_s\gamma]/\Gamma[\bar{B} \to X_c e\bar{\nu}]$. The $\mathcal{O}(\Lambda^2_{\rm QCD}/m_b^2)$ terms from Eq. (4.1) have canceled in the ratio (5.2) with the analogous corrections to $\Gamma[\bar{B} \to X_u e\bar{\nu}]$. On the other hand, once the quark masses are expressed in terms of the hadronic ones, the ratio (5.3) depends on both λ_1 and λ_2.

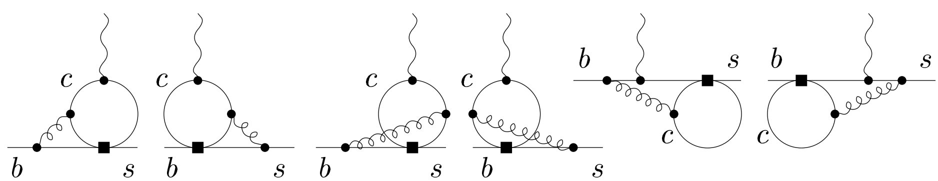

Fig. 5. Charm loop contributions to the matrix elements of four-quark operators.

The main uncertainty in the perturbative ratio on the r.h.s. of Eq. (5.2) originates from the two-loop diagrams with charm quarks presented in Fig. 5. Such diagrams are the only source of m_c-dependence of this ratio. Since the higher-order (NNLO) QCD corrections are unknown, the renormalization scheme for m_c remains arbitrary, at least within a certain class of "reasonable" schemes that do not artificially enhance the unknown corrections. As argued in Ref. [3], the uncertainty in Eq. (1.2) stemming from this scheme-dependence can be accounted for by setting $m_c/m_b = m_c(\mu)^{\overline{\rm MS}}/m_b^{1S}$ in the two-loop diagrams [6], and varying the scale μ between m_c and m_b. Such a variation is the dominant source of the error in Eq. (1.2).

[6] Here, m_b^{1S} stands for the b-quark mass in the so-called "1S-scheme" [46]. It is defined as half of the perturbative contribution to the Υ mass.

One could remove the considered uncertainty by calculating three-loop diagrams obtainable from Fig. 5 by adding one more virtual gluon. UV-divergent parts of such diagrams have been already found in the process of calculating the NLO anomalous dimensions [23]. Evaluating the finite parts would constitute an extremely tedious task, though not totally impossible, if numerical integration was applied. Finding the remaining NNLO corrections would be relatively simpler, given that fully automatized analytical methods are now available [59–61].

However, before undertaking such an ambitious task, one should make sure that all the non-perturbative effects are really under control. The main worry are the doubly-suppressed corrections mentioned in the last paragraph of Section 4. So far, they have been neither estimated nor included in the theoretical error. They are related to *precisely* the same two-loop diagrams with charm quarks (Fig. 5). Numerical importance of non-local parts of those diagrams [7] can be illustrated by the fact that the r.h.s. of Eq. (1.2) changes by 35% when m_c is shifted from the original value of $0.22\,m_b$ to the threshold for charm pair production $m_c = \frac{1}{2}m_b$. A $\Lambda_{\rm QCD}/m_b$-suppressed non-perturbative effect on the top of such a large perturbative contribution might not be negligible. Unfortunately, no systematic methods have yet been developed for calculating corrections of this type.

6. Conclusions

In the present paper, we have summarized the existing calculations of perturbative and non-perturbative contributions to the inclusive weak radiative $\bar{B}$ meson decay. We have pointed out that both the main perturbative uncertainty and the most worrisome non-perturbative effects have their origin in the fact that non-local charm quark loop contributions are particularly large. Removing the perturbative uncertainty due to m_c-dependence would be extremely tedious, but not totally impossible. However, developing a method for systematically estimating the related non-perturbative effects is desirable in advance.

The present agreement at the $\sim 10\%$ level between the experimental (1.1) and theoretical (1.2) determinations of $\mathrm{BR}[\bar{B} \to X_s\gamma]$ implies that clear signatures of new physics in this observable are not likely to be found in the foreseeable future. The importance of improving the accuracy on both the experimental and theoretical sides follows from the need for strengthening the *$b \to s\gamma$ constraints* on beyond-SM theories. Such constraints are likely to be crucial in identifying the origin of new physics effects that we expect to encounter in the LHC era.

[7] By non-local we mean those parts that cannot be removed off-shell by finite local counter-terms.

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