

PRECISION MUON $g - 2$ RESULTS AND LIGHT HIGGS BOSONS IN THE 2HDM (II)*

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Dedicated to Stefan Pokorski on his 60th birthday

We discuss the implication of recent evaluation of the SM contribution to $(g - 2)_\mu$ in light of the latest E821 measurement, for the light Higgs-boson scenarios in a Two-Higgs-Doublet-Model ("Model II"). If the constraints from the new $(g - 2)_\mu$ results are combined with the other existing constraints, one can exclude a light-scalar scenario at 95% CL while a light-pseudoscalar scenario can be realized, for a pseudoscalar mass between 25 and 70 GeV with $\tan \beta$ in the range $25 \leq \tan \beta \leq 115$.

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1. Introduction

A precision measurement of the $g - 2$ for the muon at BNL is expected to test the electroweak (EW) sector of the Standard Model (SM) and at the same time to shed light on possible effects due to "new physics". After a release of the new E821 result [1], based on the μ^+ data collected in the year 2000, a current mean of experimental results for $(g - 2)_\mu$ is [1]:

$$a_\mu^{\text{exp}} \equiv \frac{(g - 2)_\mu^{\text{exp}}}{2} = 11\,659\,203\,(8) \times 10^{-10}, \quad (1)$$

with the uncertainty (in parentheses) which is almost two times smaller than in the previous measurement [2], and only two times larger than the ultimate goal of the E821 experiment.

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The Standard Model prediction for a_μ consists of the QED, EW and hadronic contributions :

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}}.$$

The QED contribution, which constitutes the bulk of the SM contributions, is calculated up to five loops [4], with a very small uncertainty equal to 3×10^{-11} . The EW contribution, based on one and two loop diagrams, is also known to a similar accuracy [5–8]. The EW contribution is small (152×10^{-11}) and is only two times larger than the present experimental uncertainty (see (1)). The hadronic contribution, $\sim 7000 \times 10^{-11}$, is the second largest (after the QED one) contribution to a_μ^{SM} . Its uncertainty, presently about $\sim 70 \times 10^{-11}$, is the main source of the uncertainty in the SM prediction. Various predictions for the hadronic contribution [7–15] differ among themselves (see recent discussions in [3,9–12,24]). However, these differences are no longer so significant, see below. The dominant contribution to a_μ^{had} , as well the dominant error in its value, come from the leading vacuum polarization (vp1) term. Some preliminary results of the improved calculations of this part have been presented recently [13,14]. These are data driven analyses using the most recent data on hadron production in e^+e^- collisions from BES, CMD-2, SND [15]. The corresponding uncertainties for the vp1 are now even smaller than those obtained previously by using the data on τ decays in addition to the then available e^+e^- data, *e.g.* see [11,12]. Another important issue has been the hadronic contribution to the light-by-light (lbl) scattering, and its contribution to $(g-2)_\mu$. After a sign error, first pointed out in [19], was found in the earlier calculations of this contribution, few re-evaluations [20] of this part have appeared during the last few months. All of them confirm the finding of [19], *i.e.* a positive sign of the lbl contribution. The central value for the lbl varies from 80 to 110×10^{-11} , depending on the analysis. Since this contribution can be estimated only on a purely theoretical ground, it has a sizable uncertainty of the order of 40×10^{-11} [21] (or maybe even larger, as discussed in [22]).

The latest SM predictions and the present world average of the experimental result (1) differ by $\sim 3 \sigma$ (a combined in quadrature theoretical and experimental error), if preliminary results of the evaluation of the leading vacuum polarization contribution from [13,14] are used together with the estimation for the lbl given in [21]. The significant progress in the reduction of the error on the experimental side and the stabilization of the SM prediction of a_μ^{had} , makes a reanalysis of the possible implications of the $(g-2)_\mu$ results, essential. The issue whether this is a signal of supersymmetry is already being addressed [23]. Here we focus on the implication of the latest $(g-2)_\mu$ results for the light-Higgs boson scenarios in the non-supersymmetric, CP

conserving Two-Higgs-Doublet-Model (2HDM) for its version called “Model II”; this is a continuation of our earlier studies [24, 25].

In Sec. 2 we collect some results based on the recent calculations of a_μ^{SM} . As a reference for the vp1 contribution we take the e^+e^- data-driven analysis done by Jegerlehner [13], FJ02, where the new CMD-2 results were used. The difference between the experimental data and the SM prediction, $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$, can be used to derive stringent constraints on the parameters of models, which give additional contribution(s) to a_μ . We calculate interval δa_μ , which can be then used to constrain any such a contribution, at 95% CL. In Sec. 3 we introduce the 2HDM(II), and discuss briefly the existing constraints. In Sec. 4 we use the range of δa_μ , obtained in Sec. 2, to derive the 95% CL limits on the parameters of this model. Sec. 5 contains the combined constraints, while in Sec. 6 the conclusions and outlook are given in Sec. 6.

2. The $g - 2$ for muon — the new experimental and theoretical results

Here we collect the SM contributions (and their uncertainties), which we take into account in our analysis. First we discuss the hadronic contributions (see Table I). We use the higher order contribution from [17], and for the light-on-light scattering contribution we take an estimate from [21]. The leading vacuum polarization contribution is taken from a preliminary result of an analysis by a Jegerlehner, FJ02 [13], where the experimental input is based only on the e^+e^- data, including the latest ones from CMD-2 [15]. We sum all the hadronic contributions, adding in quadrature the corresponding errors. This leads us to the result for a_μ^{had} given in the last row of the Table I.

TABLE I

hadronic contribution	[in 10^{-11}]	
ho [17]	−100	(6)
lbl [21]	80	(40)
vp1 [13]	6889	(58)
had [FJ02]	6869	(71)

We take the QED and EW terms from [4] and [7, 8], respectively (see Table II). We then calculate the total SM prediction presented in this table, by adding the QED, EW to the full hadronic contributions, and by adding in quadrature the corresponding errors. This leads us to the SM prediction

(we label it by the author of the analysis of the vp1 contribution):

$$[\text{FJ02}] \quad a_\mu^{\text{SM}} = 116\,591\,726.7 \quad (70.9) \times 10^{-11}. \quad (2)$$

Taking the new world mean we calculate the quantity Δa_μ , defined as the difference between the central values of the experimental and theoretical predictions for a_μ , Eq. (1) and (2), respectively. The error for this quantity we estimate by adding in quadrature the corresponding experimental and theoretical errors, $\sigma = \sqrt{\sigma_{\text{exp}}^2 + \sigma_{\text{SM}}^2}$. Next we calculate the regions of δa_μ , allowed at 95% CL, assuming Gaussian errors. This leads to an interval symmetric around Δa_μ , quoted in the last row of Table II.

TABLE II

SM contribution	[in 10^{-11}]	
QED	116 584 705.7	(2.9)
had[FJ02]	6 869.0	(70.7)
EW	152.0	(4.0)
tot	116 591 726.7	(70.9)
$\Delta a_\mu(\sigma)$	303.3	(106.9)
lim(95%)	$93.8 \leq \delta a_\mu \leq 512.8$	

The 95% CL interval δa_μ so obtained is positive, and hence it leads to an *allowed positive* contribution (*an allowed band*). At the same time it also leads to the *exclusion* of any negative contribution to the a_μ . Note, that the additional positive contribution to a_μ can be even few times larger than the EW contribution.

Use of results for the vp1 contribution given by HMNT group [14], from their “exclusive” analysis gives results for a_μ^{SM} , $\Delta a_\mu(\sigma)$ and δa_μ , which are numerically very close to those obtained above using the FJ02 analysis. Keeping all the other contributions as before, but with the HMNT(ex) results for the vp1 term [14], we get in units of 10^{-11} ,

$$[\text{HMNT(ex)}] : \quad \Delta a_\mu(\sigma) = 297.0(107.2) \quad 87.2 \leq \delta a_\mu \leq 507.4, \quad (3)$$

while the results of their “inclusive” analysis leads to a more stringent constraint, namely

$$[\text{HMNT(in)}] : \quad \Delta a_\mu(\sigma) = 357.2(106.4) \quad 148.7 \leq \delta a_\mu \leq 565.7. \quad (4)$$

Note that the above 95% CL intervals, are positive for both the HMNT results, just as in the FJ02 case. Further notice that the relative difference

of the upper bounds in all the three analyses are small (up to 10%). However the use of HMNT “inclusive” analysis leads to a lower bound relatively much higher (up to 70%) than in the FJ02 and HMNT(ex) cases.

The other, recently published SM predictions [11, 12], to which the BNL paper [1] refers to, were obtained from analyses of the vp1 contribution based on the older, often not very precise e^+e^- data. To improve the accuracy of the estimation of the vp1 part, in those analyses τ decay data were included in addition. However, predictions obtained in these analyses for a_μ^{SM} and for σ_{SM} are not very different from these new preliminary results used by us. It is worth mentioning that the following trend is observed (see discussion *e.g.* in [12, 13]): if one uses the τ decay data in the calculation of vp1 then its estimation for the value of vp1 increases while for uncertainty decreases. On other hand, the preliminary FJ02 and HMNT analyses of the vp1 contribution, the ones used here, rely solely on the low-energy data for the e^+e^- collisions. The accuracy of these analyses increases significantly compared to the earlier analyses of this kind due to an inclusion of new, high precision measurements [15], and by use of more refined theoretical methods. Moreover, the central value for the vp1 contribution did not increase in these new analyses. Thus, the preliminary analyses by FJ02 and HMNT lead to a larger deviation from the experimental value for the muon ($g - 2$), than the published results mentioned above, which differ “only” by 1.6 to 2.6 σ , as pointed out in [1].

Let us conclude this general part by the following comment. It becomes clear, that already at present there is a need of a coherent and a comprehensive error analysis for all components contributing to the calculation of σ_μ^{SM} . It will be even more desired in a near future, when the BNL experiment will reach a final accuracy. As the estimate for a_μ^{SM} , and for Δa_μ , which are needed for search of new physics, are not available at the moment, “a poor man” error analysis, as this presented in this section, is unavoidable. This is enough just for a hint, for a rough estimation of new effects, however to reach a final conclusion, better error analyses are necessary.

3. 2HDM and existing constraints

3.1. The model

The non-supersymmetric, CP conserving 2HDM (“Model II”) [26] based on two doublets of complex, scalar-fields ϕ_1, ϕ_2 . This is a simple extension of the SM, in which only the Higgs sector is enlarged. To avoid possible large effects due to the Flavour Changing Neutral Currents (FCNC), the 2HDM potential can be chosen in a Z_2 -symmetric form, *i.e.* without (ϕ_1, ϕ_2) mixing. In a general case, the potential can have terms, characterized by a mass parameter μ , which break the Z_2 -symmetry softly, *e.g.* [28, 29].

The 2HDM has five Higgs particles: two neutral Higgs scalars h and H , one neutral pseudoscalar A , and a pair of charged Higgses $H^\pm$. Their masses are free parameters of the model. Other parameters are: angle α , which describes the mixing in the neutral Higgs-scalar sector, $\tan\beta$ — the ratio of two vacuum expectation values of scalar doublets, $\tan\beta = v_2/v_1$, and the parameter μ . Small values of μ parameter seem to be more natural from a point of view of the FCNC effects [28]. It is worth noticing that for such a case a non-decoupling of heavy Higgs sector can be realized [28,29].

In the 2HDM one can choose the Yukawa couplings in few different ways. Here we consider the Model (II) implementation, where one doublet of fundamental scalar fields couples to the u -type quarks, and the other to the d -type quarks and charged leptons. This way FCNC processes are avoided at the tree level [26,27]. This Higgs sector is identical to the one in MSSM, however in the 2HDM (II) considered by us, there are no tree level relations between parameters as in the MSSM case. Therefore even for very heavy supersymmetric particles, the 2HDM (II) and MSSM have very different phenomenology.

To be more specific, let us consider the ratios, of the direct coupling constants of the Higgs boson h or H to the massive gauge bosons $V = W$ or Z , as well as to the fermions (*i.e.* *Yukawa couplings*) for the u -type quarks and d -type quarks and the charged leptons, to the corresponding couplings for the SM. They are determined in terms of angles α and β [26,28]. For $\chi_i^h \equiv g_i^h/(g_i^h)_{\text{SM}}$ (and similarly for H) we have, in form suitable for a simultaneous discussion of h and H ,

$$\chi_V^h = \sin(\beta - \alpha), \quad \chi_V^H = \cos(\beta - \alpha), \quad \chi_V^A = 0, \quad (5)$$

$$\chi_u^h = \chi_V^h + \cot\beta\chi_V^H, \quad \chi_u^H = \chi_V^H - \cot\beta\chi_V^h, \quad \chi_u^A = -i\gamma_5 \cot\beta, \quad (6)$$

$$\chi_d^h = \chi_V^h - \tan\beta\chi_V^H, \quad \chi_d^H = \chi_V^H + \tan\beta\chi_V^h, \quad \chi_d^A = -i\gamma_5 \tan\beta. \quad (7)$$

Here we have $(\chi_V^h)^2 + (\chi_V^H)^2 = 1$. Observe a *pattern relation* among these couplings (for h or H): $(\chi_u - \chi_V)(\chi_V - \chi_d) = 1 - \chi_V^2$, or $(\chi_u + \chi_d)\chi_V = 1 + \chi_u\chi_d$, found in [28].

For $\chi_V^h = 1$ all couplings of h have the SM values, the couplings of H to gauge bosons are equal to zero, while one of the couplings of H to fermions may differ considerably from the corresponding SM one for a small or large $\tan\beta$. If $\chi_V^H = 1$ then the H -boson has SM couplings, while h has very different properties: $\chi_V^h = 0$ and the Yukawa coupling χ_d^h can be large, for large values of $\tan\beta$. This is a case which may correspond to a light-scalar scenario discussed below.

The Yukawa coupling χ_d , relevant for a Higgs boson coupling to a muon, plays a basic role in calculation of the 2HDM contribution to a_μ . It is equal to $\tan\beta$ for a pseudoscalar and $H^\pm$. If in addition $\chi_V^h = \sin(\beta - \alpha) = 0$,

then the same holds for a scalar h ; more precisely then $|\chi_d^h| = \tan \beta$. In the calculation of the two-loop contribution to a_μ , coupling of H^+ to a scalar h is involved as well, and it is given by

$$\chi_{H^+}^h = \left(1 - \frac{M_h^2}{2M_{H^+}^2}\right) \chi_V^h + \frac{M_h^2 - \mu^2}{2M_{H^+}^2} (\chi_d^h + \chi_u^h) \quad (8)$$

with the normalization as that for an elementary charged scalar particle in the SM. For $\chi_V^h = 0$ one gets $\chi_{H^+}^h = (\chi_d^h - 1/\chi_d^h)(M_h^2 - \mu^2)/(2M_{H^+}^2)$. We see that this coupling depends on the parameter μ . In this discussion below we consider only the case with $\mu = 0$. A more general case will be studied elsewhere.

3.2. Existing constraints

Many searches for a light Higgs particle in the 2HDM (II) were performed at various energies and machines; the most systematic studies were performed at LEP. All existing LEP data, see *e.g.* [25,30–33], allows for an existence of *one* light neutral Higgs boson, h or A , with mass even below 20 GeV. According to the results presented in Fig. 1(Left), the other Higgs particle (A or h , respectively) should be heavy enough to avoid the exclusion region in the (M_h, M_A) plane, given roughly by $M_h + M_A \geq 90$ GeV.

This is in contrast to the SM Higgs boson which should be heavier than 114.4 GeV (95% CL), also the MSSM Higgs particles should be heavier than ~ 90 GeV [30]. An analysis of the Bjorken process leads to an upper limit on the coupling of h to the gauge boson, χ_V^h . This limit obtained at 95% CL is presented at Fig. 1(Right). We see, that this coupling is much smaller than 1 for $M_h \lesssim 50$ GeV. The Yukawa couplings χ_d of a very light scalar or of a very light pseudoscalar, with mass below 10 GeV, are constrained in form of upper limits by the low energy data [34,35], whereas LEP experiments [33] do that for masses $\gtrsim 4$ GeV (see Figs. 4, 5). It is only the analysis of the decay $Z \rightarrow h/A\gamma$ at LEP [25], that gives both the upper and lower limits for $|\chi_d|$, equal to $\tan \beta$ for A and, if $\chi_V^h = 0$, also for h .

The constraints from the $\mathcal{T} \rightarrow h(A)\gamma$ process, mentioned above, have been measured by few groups [34]. We present their results in Figs. 4 (lines denoted by K, N and L). Unfortunately the corresponding predictions have large experimental and theoretical uncertainties the latter due to the QCD and relativistic corrections. Nevertheless, as we will see below, the constraints coming from this process, even with large uncertainties, play an important role in closing a low mass window for the scalar h .

Finally, note that in the 2HDM there is an important lower limit on the mass of H^+ , coming from the NLO analysis of the $b \rightarrow s\gamma$ data, given by $M_{H^+} \geq 500$ GeV at 95% CL [37].

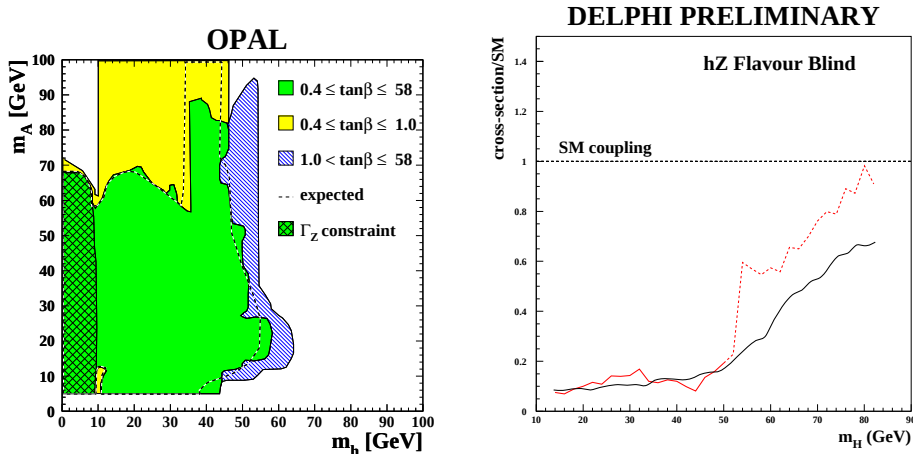


Fig. 1. Left: The (M_h, M_A) exclusion plot from OPAL [31]. Right: The upper limit on the $(\chi_V^h)^2$ (DELPHI preliminary results [32]).

4. Constraining 2HDM(II) by $g - 2$ for the muon data

We apply the δa_μ , obtained in Sec. 2, to constrain parameters of the 2HDM (II) (see also earlier papers [24,39,40]). We assume that the lightest Higgs boson, h or A , dominates the full 2HDM (II) contribution, *i.e.* we have $a_\mu^{2\text{HDM}} \approx a_\mu^h$, or a_μ^A (*a simple approach* in [24]). This approach should hold for masses below 50 GeV (see Fig. 1(Left)). For a higher masses, which are also considered here, this is essentially equivalent to an assumption of a large mass gap between the lightest one, h or A , and the remaining Higgs-bosons, which lead to a light- h or a light- A scenario. The relevant one- and two-loop diagrams, studied in [24,38,39] and [24,40], respectively, are shown in Figs. 2 for the h and A contributions.

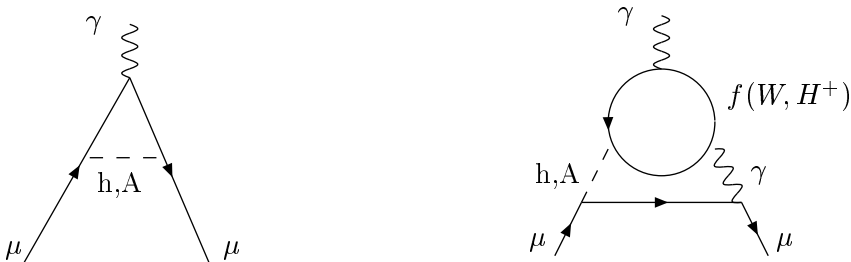


Fig. 2. One- and two-loop (W^+ and H^+ loops are only for a h -exchange) diagrams.

According to the LEP limits, discussed in Sec. 3.2, we assume that h does not couple to W/Z , and therefore we neglect the W -loop in the light- h scenario. We include, however, a H^+ -loop with M_{H^+} equal to 400, 800 GeV (and for $\mu = 0$). In Fig. 3 we present the contributions to a_μ obtained for

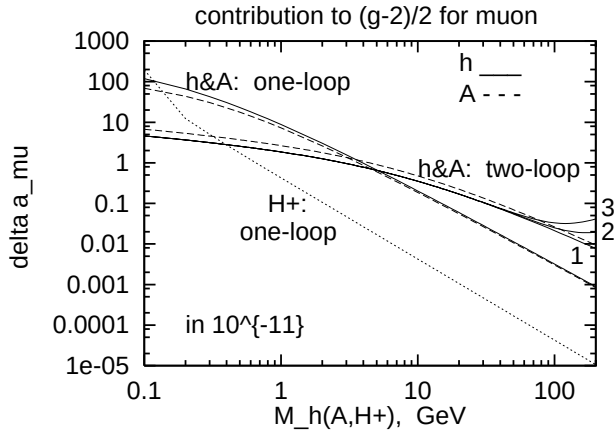


Fig. 3. The (absolute value of) individual contributions to a_μ from a scalar h (solid line), a pseudoscalar A (dashed line) and a charged Higgs boson H^+ (dotted line). The one-loop contribution for A and H^+ , and the two-loop one for h , are negative. Two-loop diagram contributions for A and h (denoted “1”) are based on the down-type fermion loops. For h also results with the H^+ -loop are shown: line “2”(“3”) corresponds to $M_{H^+} = 800(400)$ GeV.

a h (solid lines), and for a A (dashed lines), assuming Yukawa couplings χ_d equal to 1. For both h and A , the one-loop [24, 38, 39] and two-loop [24, 40] results are shown separately. For the purpose of comparison a one-loop H^+ contribution is presented. The one-loop diagram gives positive contribution to a_μ for a scalar, whereas it is negative for a pseudoscalar, independently of the value of the Higgs-boson mass. The signs of the two-loop contributions are reversed, these diagrams contribute negatively (positively) for a h (A) case [40]. These two-loop diagrams can give large contributions, since they allow to avoid one small Yukawa coupling with muon in favor of the coupling with the other, potentially heavy, particles circulating in the loop [40, 41]. Indeed, the contributions of two-loop diagrams dominate over the corresponding one-loop ones when the mass of h or A is above few GeV, see Fig. 3. As a result, in the two-loop analysis, based on a sum of the one- and two-loop (fermionic and bosonic) contributions, a positive (negative) contribution can be ascribed to a scalar h with mass below (above) 5 GeV or a pseudoscalar A with mass above (below) 3 GeV.

In our calculation, a_μ^h and a_μ^A contain contributions either proportional to χ_d^2 , equivalently to $\tan^2 \beta$ for A and for h (if $\chi_V^h = 0$), or to $\chi_d \chi_u = -1$. Assuming $a_\mu^h = \delta a_\mu$ for a light- h scenario, and $a_\mu^A = \delta a_\mu$ for a light- A one, and using the estimate of the interval δa_μ from Table II, we can derive

constraints on $\tan\beta$, for h and A . They are, as expected, in the form of allowed regions (the area between thick lines in Figs. 4) for mass below 5 GeV for h (Fig. 4(Left)) and for mass above 3 GeV for A (Fig. 4(Right)), (see also [40]).

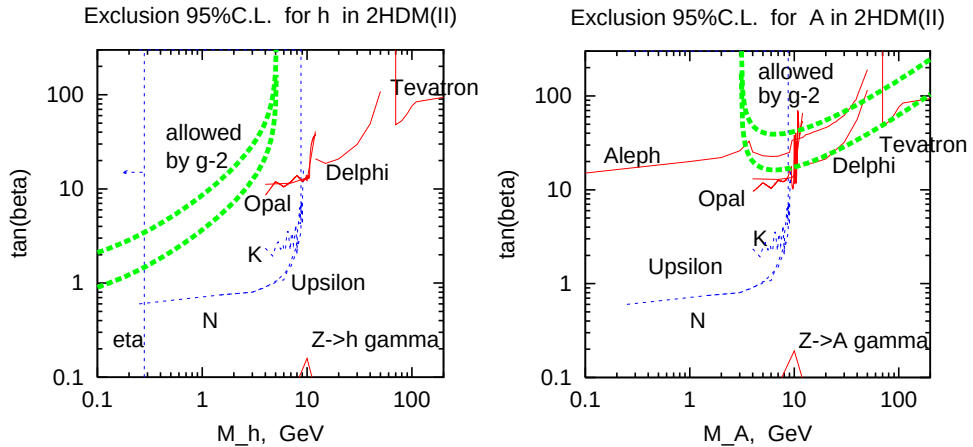


Fig. 4. Current 95% CL constraints for h (Left) and for A (Right). The $(g-2)_\mu$ data (this analysis) give allowed regions laying between thick dashed lines. Also the upper limits from LEP experiments (ALEPH, DELPHI, OPAL), Tevatron, and from the Υ decay into $h(A)\gamma$, and η decay are presented. The lower limits from LEP measurements on $Z \rightarrow h(A)\gamma$ are shown as well. See text for details.

5. Combined 95% CL constraints

The 95% constraints from the $(g-2)_\mu$ are presented in Figs. 4 (area between thick lines) together with current upper limits from LEP, from the Yukawa processes [33] (see ALEPH, DELPHI and OPAL results). Also the lower limits from the $Z \rightarrow h(A)\gamma$ [25] can be seen in Figs. 4. In addition, the upper 90% CL limits from the Υ decay (lines denoted K, N and L, with the K results rescaled by a factor 2, as discussed in [24]), and from the Tevatron [36], are presented as well.

We see, that our two-loop analysis based on the latest $(g-2)_\mu$ data and on the FJ02 estimation of a_μ^{had} (vp1), if combined with constraints from other experiments, allows in the 2HDM (II) for an existence of a pseudoscalar with mass between ~ 25 GeV and 70 GeV, and $\tan\beta$ above 25. The allowed by $(g-2)_\mu$ data mass region for A , between ~ 3 and 25 GeV, are excluded by the constraints from LEP, based on OPAL and DELPHI data. On the other hand the constraints from the Tevatron close the pseudoscalar-mass window above 70 GeV. For a light scalar the combined constraints are even

more severe; if the old constraints from the Υ decay data are taken into account, the allowed by the latest $(g - 2)_\mu$ data area disappears. Note, that the exclusion of a light h is in agreement with a conclusion of a theoretical analysis [42].

6. The conclusions and outlook

The latest precise measurement of the $(g - 2)_\mu$ if compared with the improved, theoretical estimations of the SM contribution, allows one to constrain strongly the additional contribution, which arises in a CP conserving, non-supersymmetric 2HDM (II) for a small parameter μ . The additional contribution, allowed at 95% CL, has to have a positive sign, and can lead to a clear prediction for a light-scalar and a light-pseudoscalar scenarios in the model considered here. These two scenarios correspond to the case when one of the Higgs boson, h or A , is very light, much lighter than the other Higgs particles of the model. It should be further noted that both of these scenarios are in agreement with existing data from other experiments. An exchange of such light particle dominates in the one-, and two-loop contributions to the a_μ . Constraints from $(g - 2)_\mu$ are such that they exclude a light h with a mass above 5 GeV, and a light A if its mass is below 3 GeV, as the corresponding contributions are negative in these regions.

Our two-loop analysis presented in this paper is based on the newest $(g - 2)_\mu$ data and on the (preliminary) FJ02 estimation of a_μ^{had} . Combining the constraints from the $(g - 2)_\mu$ data with those from other experiments, a pseudoscalar with mass between ~ 25 GeV and 70 GeV, and $25 \leq \tan \beta \leq 115$ is allowed. However a light scalar is excluded.

The main results will hold also if the $g - 2$ constraints will be based on the HMNT results for the vp1. For the HMNT(in) case the window for a pseudoscalar will be even smaller: $35 \leq M_A \leq 70$ GeV and $\tan \beta$ between 40 and 120. The results will not change significantly, if the uncertainty for the lbl contribution will be even two times larger than in the estimation we used in the analysis.

Finally, we stress a need for a coherent and a comprehensive error analysis for the SM contributions to the a_μ .

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