HARD DIFFRACTION IN LEPTON–HADRON AND HADRON–HADRON COLLISIONS

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It is argued that the breakdown of factorization observed recently in the diffractive dijet production in deep inelastic lepton induced and hadron induced processes is naturally explained in the Good–Walker picture of diffraction dissociation. An explicit formula for the hadronic cross-section is given and successfully compared with the existing data.

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1. Diffractive production of hard jets has been recently measured by the CDF collaboration [1]. When compared with the hard diffraction observed earlier at HERA [2, 3], these measurements revealed a dramatic violation of Regge factorization. The measured diffractive structure function is about one order of magnitude smaller than that predicted from factorization [1,4,5].

Two mechanisms were invoked to explain this discrepancy between the data from (virtual)photon-induced and hadron-induced diffraction.

The first one [6, 7] explains the reduction of the diffractive cross-section in hadron-induced processes by multiple exchanges of “soft” gluons carrying color and thus destroying the rapidity gap (which defines — experimentally — the diffractive dissociation). Consequently, the original result must be multiplied by a “gap survival probability” which measures the probability that no soft gluon was exchanged between the colliding particles.

In the second mechanism the "Pomeron flux" (which cannot be uniquely defined in Regge theory) is renormalized when the incident photon is replaced by the proton (to prevent violation of the unitarity condition) [8].

In the present note I would like to suggest that:

(a) The observed effect can be understood in terms of the Good–Walker picture [9] in which the diffractive dissociation is treated as a consequence of absorption of the particle waves[1].

(b) The magnitude of the factorization breaking can be *quantitatively* estimated from the data on proton–proton elastic scattering.

2. In the Good–Walker formulation of diffraction dissociation the incident particle state $|\psi\rangle$ is expanded into a complete orthonormal set of "diffractive eigenstates" $|\psi_n\rangle$ which are eigenstates of the scattering operator T:

$$T|\psi_n\rangle = t_n|\psi_n\rangle \,, \tag{1}$$

where the eigenvalues t_n are positive numbers[2], not greater than 1.

To calculate the amplitude for the transition from the incident state $|\psi\rangle$ to a final state $|\psi'\rangle$ (orthogonal to $|\psi\rangle$) one expands also $|\psi'\rangle$ into the set $|\psi_n\rangle$. Then the amplitude for the transition from $|\psi\rangle$ to $|\psi'\rangle$ can be expressed in terms of the expansion coefficients and the eigenvalues t_n.

This relation takes a particularly simple form [10, 11] if the expansion of the observed states into the diffractive states is quasi-diagonal, *i.e.* if we consider only small quantum fluctuations:

$$|\psi\rangle = |\psi_1\rangle + \varepsilon|\psi_2\rangle + \ldots \qquad |\psi'\rangle = -\varepsilon^*|\psi_1\rangle + |\psi_2\rangle + \ldots \,, \tag{2}$$

where ε, the probability amplitude for the fluctuation, is a small number (we shall neglect ε^2)[3]. The relation between the expansion coefficients of $|\psi\rangle$ and $|\psi'\rangle$ follows from the orthogonality condition.

Using (2) we obtain (keeping only the terms linear in ε)

$$\begin{aligned}\langle\psi'|T|\psi\rangle = \varepsilon\,(t_2 - t_1) \;&=\; \varepsilon\,(\langle\psi_2|T|\psi_2\rangle - \langle\psi_1|T|\psi_1\rangle) \\ &=\; \varepsilon\,\big(\langle\psi'|T|\psi'\rangle - \langle\psi|T|\psi\rangle\big)\;. \end{aligned} \tag{3}$$

This formula, discussed in a similar context already some time ago [10, 11], is the starting point of our further discussion.

[1] Another version of this idea (rather different from the one presented here) was recently discussed in [4].

[2] We use the convention in which the high-energy elastic amplitudes (in impact parameter representation) are real.

[3] ... denote other possible small terms of the order ε. They do not affect our argument.

To give a definite physical meaning to the Good–Walker picture we have to define the diffractive eigenstates. Following [12] (see also [4, 13]) we assume that the diffractive eigenstates are states with a fixed parton number and configuration in the transverse (impact parameter) space. This is a natural choice since the partons, being elementary, cannot be excited and, at high energy, their transverse configuration is expected to remain unchanged during the collision.

3. Consider first the photon-induced reaction: $|\gamma^*\rangle \to |\text{jets}\rangle$. We write

$$|\gamma^*\rangle = |0\rangle + \varepsilon|\text{hard}\rangle \qquad |\text{jets}\rangle = -\varepsilon^*|0\rangle + |\text{hard}\rangle\,, \tag{4}$$

where $|0\rangle$ denotes the state with no partons and $|\text{hard}\rangle$ a state containing some hard partons (decaying into the large transverse momentum jets in the final state).

Substituting (4) into (3) we obtain

$$\langle\text{jets}|T|\gamma^*\rangle = \varepsilon\left(\langle\text{hard}|T|\text{hard}\rangle - \langle 0|T|0\rangle\right) = \varepsilon\langle\text{hard}|T|\text{hard}\rangle \tag{5}$$

because $\langle 0|T|0\rangle = 0$. Eq. (5) is well known since the early discussion of vector dominance model [14]. One sees that it can be interpreted in the Regge language: the elastic amplitude $\langle\text{hard}|T|\text{hard}\rangle$ represents the "Pomeron exchange" and ε is the corresponding coupling[4].

4. Consider now the production of jets in diffractive proton–proton collisions[5], *i.e.* the transition $|P\rangle \to |P' + \text{jets}\rangle$, where $|P\rangle$ denotes the incident proton and $|P' + \text{jets}\rangle$ contains the soft proton remnants (P') and hard jets observed in the final state.

We thus write

$$\begin{aligned} |P\rangle &= |\text{soft}\rangle + \varepsilon|\text{soft}' + \text{hard}\rangle\,, \\ |P' + \text{jets}\rangle &= -\varepsilon^*|\text{soft}\rangle + |\text{soft}' + \text{hard}\rangle\,. \end{aligned} \tag{6}$$

When introduced into (3) this gives

$$\langle P' + \text{jets}|T|P\rangle = \varepsilon\Big(\langle\text{soft}' + \text{hard}|T|\text{soft}' + \text{hard}\rangle - \langle\text{soft}|T|\text{soft}\rangle\Big)\,. \tag{7}$$

To exploit this formula we have to estimate the elastic amplitudes in the r.h.s. To this end we first find that up to first order in ε

$$\langle\text{soft}|T|\text{soft}\rangle = \langle P|T|P\rangle\,. \tag{8}$$

[4] Note, however, that (5) is more general: $\langle\text{hard}|T|\text{hard}\rangle$ represents the full elastic amplitude, so it may contain the exchange of any number of Pomerons. Note also that, unlike the standard Regge formula, (5) is written in the impact parameter space.

[5] The same argument applies for any hadron–hadron collision.

To estimate $\langle\text{soft}'+\text{hard}|T||\text{soft}'+\text{hard}\rangle$ we observe that it can be treated as amplitude for scattering of a system composed of two objects: the *soft* partons from the incident proton and the *hard* partons which decay into the observed final jets. One can thus apply the Glauber prescription [15] and write[6]

$$\begin{aligned}\langle\text{soft}'+\text{hard}|T|\text{soft}'+\text{hard}\rangle &= \langle\text{soft}'|T|\text{soft}'\rangle \\ +\langle\text{hard}|T|\text{hard}\rangle &- \langle\text{hard}|T|\text{hard}\rangle\langle\text{soft}'|T|\text{soft}'\rangle\,. \end{aligned} \tag{9}$$

Assuming, furthermore, that

$$\langle\text{soft}'|T|\text{soft}'\rangle \approx \langle\text{soft}|T|\text{soft}\rangle \tag{10}$$

we see that the soft amplitudes in (7) cancel and we obtain

$$\langle P'+\text{jets}|T|P\rangle = \varepsilon\langle\text{hard}|T|\text{hard}\rangle\left(1-\langle P|T|P\rangle\right)\,, \tag{11}$$

where we have used (8).

When compared to (5), this formula explains the breakdown of the factorization between the (virtual)photon-induced and hadron-induced processes. The factor $(1-\langle P|T|P\rangle)$ is usually interpreted as "absorption" of the initial state particles. One sees, however, from its derivation that it is actually a result of rather subtle cancellations between the interactions in the initial and final states.

5. Using (3) and the formula for (2×2) scattering [16], it is also not difficult to calculate the result for the process of double diffraction dissociation. It reads

$$\begin{aligned}\langle P'_{\mathrm{L}}+J_{\mathrm{L}},P'_{\mathrm{R}}+J_{\mathrm{R}}|T|P_{\mathrm{L}},P_{\mathrm{R}}\rangle &= \varepsilon_{\mathrm{L}}\varepsilon_{\mathrm{R}}[1-\langle P|T|P\rangle] \\ &\quad [1-(1-J_{\mathrm{L}})(1-J_{\mathrm{R}})(1-J_{\mathrm{LR}})]\,,\end{aligned} \tag{12}$$

where the subscripts (L,R) denote left-moving and right-moving objects. $J_{\mathrm{L}}(J_{\mathrm{R}})$ is the elastic amplitude for scattering of the left(right)-moving *hard* jet system on the right(left)-moving proton, and J_{LR} is the elastic amplitude for scattering of the left-moving *hard* jet system on the right-moving one. This formula is fairly complicated but it can be substantially simplified by observing that the *hard* jet systems are represented by small size dipoles (because of large transverse momenta of the jets) and thus the corresponding elastic amplitudes are expected to be small. In the first approximation (*i.e.* neglecting J_{LR} and the higher powers of J_{L} and J_{R}) one obtains

$$\langle P'_{\mathrm{L}}+J_{\mathrm{L}},P'_{\mathrm{R}}+J_{\mathrm{R}}|T|P_{\mathrm{L}},P_{\mathrm{R}}\rangle \approx \varepsilon_{\mathrm{L}}\varepsilon_{\mathrm{R}}[1-\langle P|T|P\rangle](J_{\mathrm{L}}+J_{\mathrm{R}})\,. \tag{13}$$

[6] This idea was already proposed in [11].

For the *symmetric* situation (and using the notation of the previous section) we thus have

$$\langle P'_{\rm L}+J_{\rm L},P'_{\rm R}+J_{\rm R}|T|P_{\rm L},P_{\rm R}\rangle \approx 2\varepsilon^2\langle {\rm hard}|T|{\rm hard}\rangle[1-\langle P|T|P\rangle]\,. \quad (14)$$

Comparing this with (5) and (11) one sees that the breaking of factorization should be about four times less effective in the double diffraction dissociation than the single one[7]. This result seems not too far from the recent experimental findings [17].

6. To estimate the size of the discussed effect we have taken the elastic pp amplitude in the form suggested in [18]

$$\langle P|T|P\rangle \equiv F(t) = \frac{\sigma_{\rm tot}}{8\pi^2}\frac{\exp[.25t\log(s/4)]}{(1-t/.71)^4} \quad (15)$$

from which one can calculate the impact parameter representation needed in (11)[8]. The product $\varepsilon \times \langle{\rm hard}|T|{\rm hard}\rangle$ was taken as a Gaussian

$$\varepsilon\times\langle{\rm hard}|T|{\rm hard}\rangle \sim \exp(-b^2/2B)\,, \quad (16)$$

where B is the slope of the *cross-section* in the (virtual)photon-induced process (3).

The hadron-induced diffraction dissociation *cross-section* can then be expressed as

$$\sigma(P\to P'+{\rm jets}) = R\,\sigma_{\rm factorized}(P\to P'+{\rm jets})\,, \quad (17)$$

where $\sigma_{\rm factorized}$ denotes the cross-section extrapolated from the deep inelastic scattering data, and

$$\begin{aligned} R \;=\; & 1-2\pi\int dt\exp(tB/4)F(t) \\ & +\pi^2\int dtdt'\exp(tB/4)F(t)\exp(t'B/4)F(t')I_0(\sqrt{tt'}B/2)\,. \end{aligned} \quad (18)$$

This expression depends on one unknown parameter, B — the slope in the momentum transfer dependence of the diffractive jet production in deep inelastic scattering. For production of heavy vector mesons $B\approx 4$ GeV^{-2} [20]. One can speculate that this is a lower limit for B which may be approximately valid for production of jets with a small mass (large β)[9].

[7] The factor 2 in the amplitude becomes 4 in the cross-section.

[8] The exact proton form factor used in [18] was replaced here by its dipole approximation.

[9] $\beta = x/\xi$ where ξ is the fractional longitudinal momentum transfer.

As the mass increases (β decreases), one may expect that B should increase (the system becomes more complicated and its transverse size is expected to grow)[10].

In figure 1 we show, plotted *versus* B, the ratio R calculated from (18), using $\sigma_{\rm tot} = 71.7 \pm 2$ mb [19]. The recent phenomenological estimates of R, given in [5] are also shown. One sees that the result is certainly not far from the data.

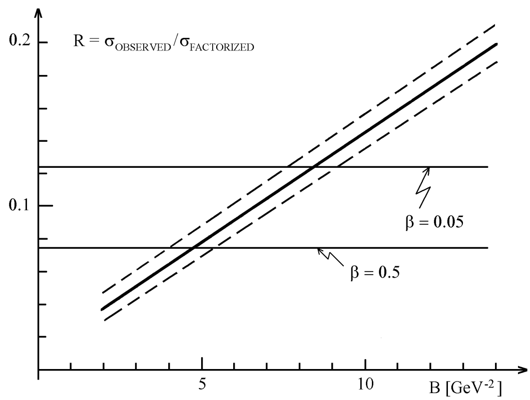

Fig. 1. Ratio R plotted versus B. The horizontal lines represent the phenomenological estimates of R given in [5].

6. Some comments are in order.

(i) One sees from the discussion in Section 3 that the *uncorrected* formula (5) is valid independently of the virtuality of the incident photon: The same formula applies to photoproduction and to deep inelastic scattering. This emphasizes the (already mentioned) point: the effect we consider cannot be simply identified with absorption in the initial state of the process.

(ii) Using the cross-sections at other energies, one can investigate the energy dependence of the correction factor R. Taking $\sigma_{\rm tot}(630) = 63$ mb, one finds that $R(630)/R(1800)$ varies from ~ 1.5 ($B = 4$ GeV^{-2}) to ~ 1.2 ($B = 10$ GeV^{-2}), in a reasonable agreement with recent data from the CDF Collaboration [22].

[10] This is confirmed by the measurements of the *inclusive* diffraction at HERA where one finds $B \approx 7$ GeV $^{-2}$ [21].

(iii) In the numerical estimate of Section 5 we have assumed that the dipole corresponding to the two jets is created at the same impact parameter as the incident proton. This assumption seems rather natural[11] but some deviations cannot be excluded. They would increase somewhat the correction factor R.

(iv) Our result given in Eq. (11) resembles, to some extent, the "renormalization" of the Pomeron flux, proposed in [8]. One should keep in mind, however, that the Eq. (11) refers to impact parameter space and thus it can be at best only approximately interpreted as the (corrected) Regge formula.

7. In conclusion, we have shown that the breakdown of Regge factorization between the diffractive production of hard jets observed at HERA and at FERMILAB is naturally explained in the Good–Walker picture of diffraction dissociation. The correction to the factorization formula is explicitely given in terms of the elastic $p\bar{p}$ amplitude at small momentum transfers. The numerical estimates seem to be consistent with the experimental findings.

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[11] As long as the diffractive system is produced in the proton vertex. If, however, a large rapidity gap develops (*i.e.* for jet production in the central rapidity region, corresponding to "double Pomeron exchange" processes) one may expect that the produced system is far from the proton remnants in the impact parameter space. In this case the ratio R would be close to 1.

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