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BOSE–EINSTEIN CORRELATIONS AS REFLECTION OF CORRELATIONS OF FLUCTUATIONS

O.V. Utyuzh, G. Wilk

Andrzej Sołtan Institute for Nuclear Studies
Hoża 69, 00-681 Warsaw, Poland

and

Z. Włodarczyk

Institute of Physics, Świętokrzyska Academy
Świętokrzyska 15, 25-405 Kielce, Poland

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Dedicated to Stefan Pokorski on his 60th birthday

Bose–Einstein correlations (BEC) observed between identical bosons produced in high energy multiparticle collisions are regarded as very important tool in investigations of multiparticle production processes. We present here their stochastic feature stressing the fact that they can be regarded as a reflection of correlations of fluctuations present in hadronizing system. We show in particular that such approach allows for simple modeling of BEC in numerical event generators used to describe the multiparticle production processes at high energy collisions.

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1. Introduction

Bose–Einstein Correlations (BEC) between identical bosons are since long time recognized as very important tool in searching for dynamics of multiparticle production processes because of their ability to provide the space–time information about them [1]. This is particularly important for heavy ion collisions which are expected to provide us with the new state of matter, the Quark Gluon Plasma (QGP) [2]. However, because of their complexity, all these processes can be investigated only by numerical modeling methods using different sorts of Monte Carlo (MC) event generators [3]. Their *a priori* probabilistic structure prevents occurring of genuine BEC which are of purely quantum statistical origin. The best one can do is to *model* BEC by changing the outputs of these generators in such a way as to reproduce the characteristic signals of BEC obtained experimentally. In the most widely investigated case of 2-particle BEC it is the fact that two-particle correlation function

$$C_2(Q=|p_i-p_j|) \;=\; \frac{N_2(p_i,p_j)}{N_1(p_i)\,N_1(p_j)} \tag{1}$$

defined as ratio of the two-particle distributions to the product of single-particle distributions increases towards $C_2=2$ when Q approaches zero.

2. More about BEC

2.1. BEC — space–time approach

There are two possible approaches towards BEC. The first stresses their space–time features and is based on the symmetrization of the respective multiparticle wave function [1] expressed by plain waves[1], $e^{\pm ikx}$, representing the produced particles. After symmetrization (and squaring) one gets the respective many-particle production rates depending on combination of variables of the type: $(k_i-k_j)(x_i-x_j)$. To get C_2 as given by Eq. (1), one has to integrate them, with some assumed weight function $\rho(x_1,x_2,\dots)$, over unmeasured space–time positions $\{x_i\}$ of the production points. The distribution $\rho(x_1,x_2,\dots,)$ is customarily assumed to be separable in terms of single particle distributions $\rho_i(x_i)=\rho(x)$ and in this way the information on the space–time distribution of points of production of finally observed particles enters here. It can be then show that, under some assumptions [1],

$$C_2(Q)=1+\left|\int dx\rho(x)e^{iQx}\right|^2=1+\mid\tilde{\rho}(Q)\mid^2, \tag{2}$$

i.e., $C_2(Q)$ can be regarded as a (kind of) Fourier transform of the space–time dimensions of the emitting source[2]. So far this approach is dominating in what concerns description of BEC.

[1] This is idealization neglecting both the possible final state and Coulomb interactions inclusion of which is possible by a suitable modifications of these plane waves. We shall not discuss it here.

[2] Actually, after closer inspection [4] it turns out that one rather gets in this way a Fourier transform of the distributions of two-particle separations (or *correlation lengths* [1]).

2.2. BEC — quantum-statistical approach

The second approach is based on observation that one encounters similar correlations in quantum optics [5] where they are known as the so called HBT effect. They are described there as arising because of correlations of some specific fluctuations present in physical systems considered (known as *photon bunching* effect [5]). Following [4,6] one can apply such possibility to description of hadronizing sources as well. Because

$$\langle n_1 n_2\rangle = \langle n_1\rangle\langle n_2\rangle + \langle (n_1 - \langle n_1\rangle)(n_2 - \langle n_2\rangle)\rangle = \langle n_1\rangle\langle n_2\rangle + \rho\sigma(n_1)\sigma(n_2) \quad (3)$$

(where $\sigma(n)$ is dispersion of the multiplicity distribution $P(n)$ and ρ is the correlation coefficient depending on the type of particles produced: $\rho = +1, -1, 0$ for bosons, fermions and Boltzmann statistics, respectively) one can write two-particle correlation function (1) in terms of the above covariances (3) stressing therefore its stochastic character:

$$C_2(Q = |p_i - p_j|) = \frac{\langle n_i\,(p_i)\, n_j\,(p_j)\rangle}{\langle n_i\,(p_i)\rangle\langle n_j\,(p_j)\rangle} = 1 + \rho\frac{\sigma\,(n_i)}{\langle n_i\,(p_i)\rangle}\frac{\sigma\,(n_j)}{\langle n_j\,(p_j)\rangle}. \quad (4)$$

It means therefore that $C_2(Q)$ can be regarded as being a measure of correlation of fluctuations. This fact has been used for numerical modeling of BEC in [7] where a special MC generator, based on application of information theory, was constructed for this purpose. In it the identical pions produced in a given event were bunched on a maximal possible way (restricted only by conservation laws constraints) in a limited number of *elementary emitting cells* of phase space according to Bose–Einstein distribution, $P(E_i) \sim \exp\left[n_i\,(\mu - E_i)\,/T\right]$ (n_i is their multiplicity and E_i are their energies)[3], with size (in rapidity, as only one dimensional phase space was considered) given by parameter δy. It turns out that in this approach one gets at the same time both the correct BEC pattern (*i.e.*, correlations) and fluctuations (as characterized by the observed intermittency pattern) [7]. This is very strong advantage of this model, which is so far the only example of hadronization model, in which Bose–Einstein statistics is not only included from the very beginning on a single event level, but it is also properly used in getting the final secondaries. In all other approaches [9–12] at least one of the above elements is missing. The shortcoming of method [7] are numerical difficulties to keep the energy-momentum conservation as exact as possible and its limitation to the specific event generator only.

[3] Values of two Lagrange multipliers, T and μ, were fixed by the energy-momentum and charge conservation constraints, respectively. Such distribution represents typical example of nonstatistical fluctuations present in the hadronizing source. Similar concept of elementary emitting cells has been also proposed in [8].

2.3. Existing methods of numerical modeling of BEC

In all other approaches the effect of BEC is obtained by a suitable changing the original output of MC generators used and introducing this way (more or less artificially) desired bunching in the phase-space of the finally produced identical particles [9, 10][4]. This is achieved either by (a) shifting (in each event) momenta of adjacent like-charged particles in such a way as to get desired $C_2(Q)$ [9] (one has to correct afterwards for the energy-momentum imbalance introduced this way), or by (b) screening all events obtained from a particular MC generator against the possible amount of bunching they are already showing and counting them as many times as necessary to get desirable $C_2(Q)$ [10][5]. The original energy-momentum balance remains in this case intact whereas the original single particle distributions are changed (this fact can be corrected by running again generator with suitably modified input parameters). In both cases one uses specific weights constructed from the assumed shape of $\rho(x)$ functions. However, the size parameters occurring there bear no direct resemblance to the size parameter R obtained by directly fitting data on $C_2(Q)$ in Eq. (1) using simple Gaussian or exponential forms. They rather represent instead the corresponding correlation lengths between the like particles [1].

3. Numerical modeling of BEC understood as correlations of fluctuations

Recently we have proposed [13] a new method of numerical modeling of BEC understood as manifestation of correlations of fluctuations, which applies already on a single event level, does not violate any conservation laws and can be applied to data provided by essentially any event generator modeling multiparticle production. Here we would like to present physical ideas underlying our approach in more detail.

Let us start with very simple example of what we are aiming at. Suppose that our MC event generator provides us with a number $N(+)$, $N(-)$ and $N(0)$ of positively and negatively charged particles and neutral ones located in phase space, *cf.* Fig. 1, left panel. They are all uniformly distributed and show no BEC pattern. Suppose now that *the same particles* (*i.e.*, located at the same space–time points and possessing the same momenta as before, with the same $N(+)$, $N(-)$ and $N(0)$) have now *different allocation of charges*, namely the one shown in the right panel of Fig. 1. The like charges are in visible (albeit strongly exaggerated) way bunched (correlated)

[4] The specific approaches proposed for LUND model [11] and the afterburner method discussed in [12], which we shall not discussed here, also belong here.

[5] Technically this is realized by multiplying each event by a special weight calculated using the output provided by event generator used.

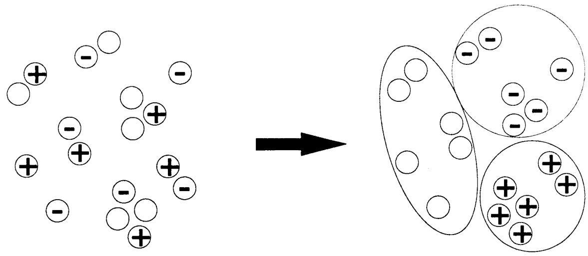

Fig. 1.

together leading to signal of BEC. What we have done in this example is the following: (a) we have resigned from the (not directly measurable) part of the information provided by event generator concerning the charge allocation to produced particles, and (b) we have allocated charges anew in such a way as to keep the like charges as near in phase space as possible (keeping also the total charge of any kind the same as the original one). It is interesting to note that this can be regarded as introduction of quantum mechanical element of uncertainty to the otherwise classical scheme of MC generator used (however, it differs completely from the usual attempts to introduce quantum mechanical effects discussed in [14]).

That such simple scheme really works can be seen in Fig. 2, which shows the $C_2(Q)$ for one dimensional lattice of N pions (positive, negative and neutral) with momenta $p_i = -p_{\rm max} + (i-1)\cdot\Delta p$ where spacing $\Delta p = 2p_{\rm max}/N$). When their charges are assigned in a purely random way (what corresponds to the situation shown at the left part of Fig. 1) it can be shown that the corresponding $C_2(Q) = 1$. However, assigning charges in a specific way (following prescription used in [13]) one gets strong enhancement of $C_2(Q)$ which normally is attributed to BEC. The procedure used is very simple. First one of the particles (from N_π) is selected and some charge (out of $(+,-,0)$) is randomly allocated to it. After that the same charge is allocated to as many particles located nearby in phase space as possible in some prescribe way forming a cell in phase-space occupied by particles of the same charge only (*cf.* right part of Fig. 1). This process is then repeated until all particles are used. The important point is to ensure that the above selection is done in such way as to get geometrical (Bose–Einstein) distribution of particles in a given cell. This can be achieved by selecting each next particle with some fixed probability P till the first failure, after which the new cell is formed. In this case $\sigma = \langle n\rangle = P/(1-P)$ and second term in the Eq. (4) is now maximal. We refer to [13] for details of the algorithm used. The characteristic pattern emerging here is that the so called "radius parameter" R (in the usual fitting formula for $C_2(Q) = \gamma[1+\exp(-R\cdot Q)]$) increases with number of particles allocated to our lattice (*i.e.*, with decreasing of their momentum separation Δp, *cf.* Fig. 2(b) (in terms of the number of particles considered it is the same, see Fig. 2(a)). On the other hand it decreases with the number of particles one can allocate to a given cell (*cf.* Fig. 2(c)).

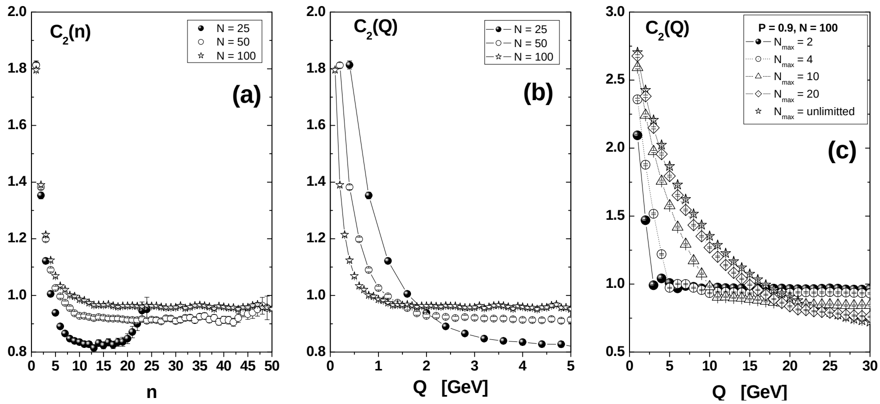

Fig. 2. Example of C_2 occurring for a pionic lattice in (one-dimensional) momentum space, $Q = |p_i - p_j| = 2p_{\max}|i-j|/N = 2p_{\mathrm{m}}n/N$: (a) as a function of n; (b) as a function of Q; (c) as a function of Q but for limitation of cell occupancy to $i < N_{\max}$. In all cases $p_{\max} = 10$ GeV. In (a) and (b) $P = 0.5$ whereas in (c) $P = 0.9$ (to allow for large cells).

We shall illustrate now action of our algorithm on simple cascade model of hadronization (CAS) (in its one-dimensional versions and assuming, for simplicity, that only direct pions are produced) [15] and on equally simple model based on application of information theory [16] (*cf.* Fig. 3). In CAS the initial mass M hadronizes by series of well defined (albeit random) branchings ($M \to M_1 + M_2$, with $M_{1,2} = r_{1,2}M$ such that $r_1 + r_2 < 1$) and is endowed with a simple spatio-temporal pattern. It shows no traces of Bose–Einstein statistics whatsoever. In MaxEnt particles occur instantaneously in all phase space with distribution given by the thermal-like formula obtained by maximalization of the accordingly defined information entropy. In both cases the masses and multiplicities were kept the same. There is no BEC here either. However, as can be seen in Fig. 3, when endowed with charge selection provided by our algorithm, a clear BEC pattern emerges in $C_2(Q)$ (and is very similar in both cases considered here). Two kind of choices of probabilities are shown in Fig. 3. First is constant $P = 0.75$ and $P = 0.5$. The other is what we call the "minimal" weight constructed from the output information provided by CAS (P_{M}) or MaxEnt (P_{ME}) event generators:

$$P_{\mathrm{C}}(ij) = \exp\left[-\frac{1}{2}\delta_{ij}^2(x)\cdot\delta_{ij}^2(p)\right] \quad \text{or} \quad P_{\mathrm{ME}}(ij) = \exp\left[-\frac{\delta_{ij}^2(p)}{2\mu_i T_l}\right], \qquad (5)$$

where $\delta_{ij}(x) = |x_i - x_j|$, $\delta_{ij}(p) = |p_i - p_j|$ and T_l is the corresponding "temperature" (with μ being mass of the produced particles). In this way one connects P with details of hadronization process by introducing to it a kind of overlap between particles as a measure of probability of their bunching in a given emitting cell.

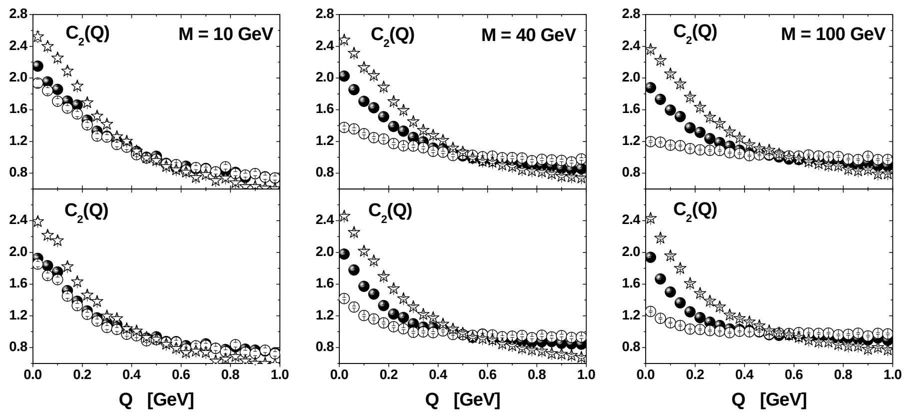

Fig. 3. Examples of BEC patterns obtained for $M = 10$, 40 and 100 GeV for constant weights $P = 0.75$ (stars) and $P = 0.5$ (full symbols) and for the weight given by Eq. (5) (open symbols). Upper panels are for CAS, lower for MaxEnt (see text for details).

It turns out that BEC effect shown in Fig. 3 depends only on the (mean) number of particles of the same charge in phase-space cell and on the (mean) numbers of such cells. This depends on P, the bigger P the more particles and bigger $C_2(Q = 0)$; smaller P leads to the increasing number of cells, which, in turn, results in decreasing $C_2(Q = 0)$, as already noticed in [8]. For small energies the number of cells decreases in natural way while their occupation remains the same (because P is the same), therefore the corresponding $C_2(0)$ is bigger, as seen in Fig. 3. The fact that there is tendency to have $C_2(0) > 2$ for larger P means that one has in this case more cells with more than 2 particles allocated to them, *i.e.*, it is caused by the influence of higher order BEC. Therefore, the "sizes" R obtained from the exponential fits to results in Fig. 3 (like $C_2(Q) \sim 1 + \lambda \cdot \exp(-Q \cdot R)$ where λ being usually called chaoticity parameter [1]) correspond to the sizes of the respective elementary cells rather than to sizes of the whole hadronizing sources itself. For $P = 0.5$ the "size" R varies weakly between 0.66 to 0.87 fm from $M = 10$ to 100 GeV whereas for the "minimal" weight (5) it varies from 0.64 to 0.44 fm.

So far we were considering only single sources. Suppose now that source of mass M consists of a number ($n_l = 2^k$) of subsources hadronizing independently. It turns out that the resulting C_2's are very sensitive to whether in this case one applies our algorithm of assigning charges to all particles from subsources taken together ("Split" type of sources) or to each of the subsource independently ("Indep" type of sources), *cf.* Fig. 4. Whereas the later

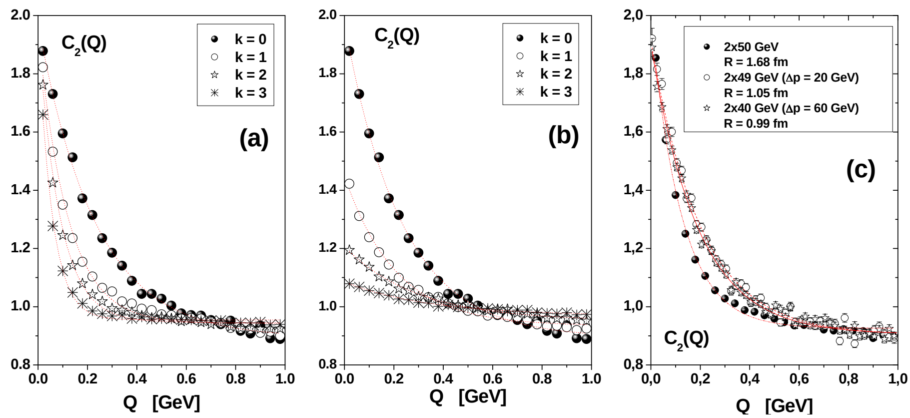

Fig. 4. Examples of BEC for CAS model with $P = 0.5$ calculated for different number of subsources ($n_l = 2^k$, $k = 1, 2, 3$ existing in the source $M = 100$ GeV for (a) "Split" and (b) "Indep" types of sources, as discussed in text. In (c) we show examples of BEC pattern for 2 "Split" type of sources moving apart with constant momentum difference $\delta p = 0$, 10 and 60 GeV/c (achieved by assuming in CAS first rank cascade parameters $r_1 = r_2 = 0.5$, 0.4975 and 0.4, respectively).

case (in which particles remember from which source they have originated) results in the similar "sizes" R (defined as before) with $C_2(Q = 0) - 1 = \lambda$ falling dramatically with increasing k (roughly like $1/2^k$, *i.e.*, inversely with the number of subsources, n_l, as expected from [8]), the former case (in which particle loose memory of which subsource they are coming from) leads to roughly the same $C_2(Q = 0)$ but the "size" R is now increasing substantially[6]. This is again entirely due to the fact that in the "Split" type of source one has higher concentration of particles in the elementary emitting cells rather then bigger number of such cells. This results in smaller average Q, and this in turn leads to bigger R. A special type of "Split" source is shown in Fig. 3(c). In it two initial sources (of equal masses) have from the beginning a well defined difference in momenta, δp (corresponding to branching parameter $r_1 = r_2 = \frac{1}{2}\sqrt{1 - (\delta p/M)^2}$), modeling in this way a possible influence of some collective flow existing in the system (the total energy remains always the same and equal to M, here $M = 100$ GeV).

[6] It is equal to, respectively, 0.87 fm, 1.29 fm, 1.99 fm and 3.35 fm for $P = 0.5$ and 0.57 fm, 3.26 fm, 4.01 and 5.59 fm for the "minimal" weight $P_{\rm C}$ given by (5).

Notice that, contrary to the normal expectations, the bigger is the "flow" the smaller is "radius" parameter R obtained from the typical exponential fit mentioned above. This is because "flow" results in our case in smaller number of particles in the elementary emitting cells.

Actually dependence on the expected BEC pattern on the number and type of subsources formed in the process of hadronization is very important and interesting feature of our model. It allows to understand the increase of the extracted "size" parameter R with the nuclear number A in nuclear collisions. That is because with increasing A the number of collided nucleons, which somehow must correspond to the number of sources in our case, also increases. If they turn out to be of the "Split" type, the increase of R follows then naturally. On the contrary, for the independently treated sources the density of particles subjected to our algorithm does not change, hence the average Q and R remain essentially the same. However, because in this case the influence of pairs of particles from different subsources increases, the effective $\lambda = C_2(0) - 1$ now decreases substantially (as was already observed in [8]). Our "Indep" type sources can therefore be used as a possible explanation of the so called inter-W BEC problem, *i.e.*, the fact that essentially no BEC is being observed between pions originating from a different W in fully W^+W^- final states [17]. This phenomenon can be understood in our model by assuming that produced W's should be treated as "Indep" type sources for which λ falls dramatically.

It also allows to attempt to fit (even using such unsophisticated hadronization model as CAS) some experimental data. As example we present in Fig. 5 our "best fit" to the e^+e^- annihilation data on BEC by DELPHI Collaboration [18] for $M = 91.3$ GeV. It turns out that such fit can be obtained only for two or more subsources [13]. A the same figure we show also intermittency pattern (with moments F_q and $M_{\rm bin}$ defined as in [19]) obtained together with the BEC after application of our algorithm and the examples of the expected charge fluctuations in different rapidity windows (defined as $D = 4\frac{\langle \delta Q^2\rangle}{\langle N_{\rm ch}\rangle}$ [20]). This is done for two sets of parameters (P and number of sources): one leading to the best possible (which turns out very good) fit to $C_2(Q)$ (Fig. 5(a)) and one leading to the best possible intermittency patter (actually only 2-nd moment F_2 can be fitted, all other moments remain still below data indicating that intermittency connected with BEC and provided by our algorithm as a kind of by-product, is still not the whole effect seen in data [19])[7].

[7] The charge fluctuations D are actually important for heavy ion collisions [20] and are shown here just for illustration of predictive power of our algorithm.

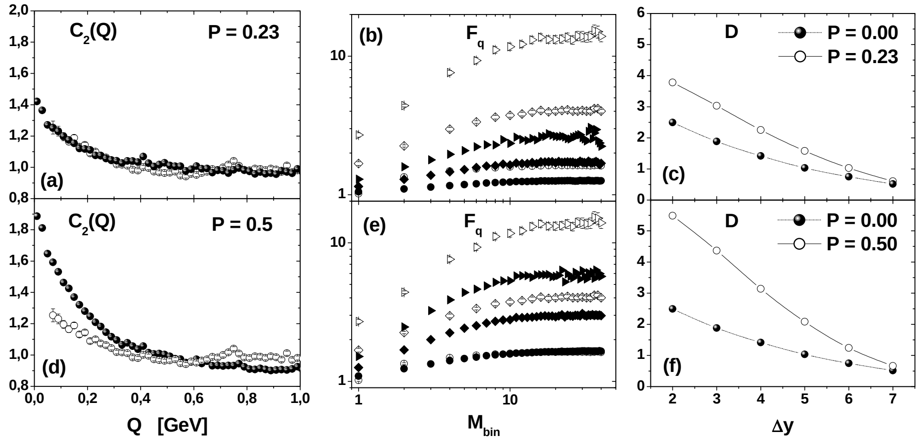

Fig. 5. Examples of results for $e^{+}e^{-}$ annihilation at $M = 91.3$ GeV obtained using CAS with two subsources: (a) best fit (full symbols) to data BEC [18] by DELPHI (open symbols) obtained with $P = 0.23$; (b) and (c) the resulting intermittency and charge fluctuations patterns, respectively. Lower panels contain in (e) the best possible fit (full symbols) to the data on intermittency [19] (open symbols, only second moment can be reproduced, both here and in (b) only F_2 to F_4 are displayed) obtained with $P = 0.5$ and resulting BEC (d) and charge fluctuations patterns (f) (here and in (c) $P = 0$ corresponds in this case to results of CAS without BEC).

4. Summary and conclusions

To summarize: we propose a new way of looking on the BEC phenomenon observed in high energy multiparticle production processes of all kind. Instead of cumbersome and practically very difficult (if not outright impossible) symmetrization of the corresponding multiparticle wave function we propose, following ideas developed in [4,5,7], to look at this phenomenon as originating due to correlations of some specific fluctuations present in such stochastic systems as blob of hadronizing matter. As result we get new and simple method of numerical modeling of BEC. It is based on reassigning charges of produced particles in such a way as to make them look like particles satisfying Bose statistics, conserves the energy-momenta and does not alter the spatio-temporal pattern of events or any single particle inclusive distribution (but it can change the distributions of, separately, charged and neutral particles leaving, however, the total distribution intact). It is intended to generalize algorithm presented in [7] in such a way as to make it applicable to essentially any event generator in which such reassignment of charges is possible. It amounts, however, to some specific changes taking place in physical picture of the original generator. The example of CAS is very illustrative in this respect. In it, at each branching vertex one has, in addition to the energy-momentum conservation, imposed strict charge conservation and one assumes that only $(0) \to (+-)$, $(+) \to (+0)$ and $(-) \to (-0)$ transitions are possible. It means that there are no multi-charged vertices (*i.e.*, vertices with multiple charges of the same sign) in the model. However, after applying to the finally produced particles our charge reassignment algorithm one finds, when working the branching tree "backwards", that precisely such vertices occur now (with charges "$(++)$", or "$(- \ -)$", for example). The total charge is, however, still conserved as are the charges in decaying vertices (*i.e.*, no spurious charge is being produced because of action of our algorithm). It is plausible therefore that to numerically get BEC pattern in an event generator it is enough to allow in it for accumulation of charges of the same sign at some points of hadronization procedure modeled by this generator. This would lead, however, to extremely difficult numerical problem with ending such algorithms without producing spurious multicharged particles not observed in nature[8]. So far only direct pions were considered but short living resonances can easily be included as well. The same (at least in principle) is true in what concerns any kind of final state interactions, not mentioned here.

GW wants to mention that his interest in multiparticle production which resulted in this presentation dates back to the seminal paper by Pokorski and Van Hove [22], which spurred formulation of the so called Interacting Gluon Model [23], a simple but powerful description of high energy processes in terms of gluonic component of hadrons. It is used (albeit in an appropriately modified form) even at present [24]. He is grateful to Professor Pokorski for his constant interest and encouragement in this kind of research. The partial support of the Polish State Committee for Scientific Research (KBN) grants 2P03B 011 18 and 621/ E-78/ SPUB/CERN/P-03/DZ4/99 is acknowledged.

[8] It should be noted that possibility of using multi(like)charged resonances or clusters as possible source of BEC has been recently mentioned in [21]. There remains problem of their modeling, which although clearly visible in CAS model, as discussed here, is not so straightforward in other approaches. However, at least in the string-type models of hadronization, one can imagine that it could proceed through the formation of charged (instead of neutral) color dipoles, *i.e.*, by allowing formation of multi(like)charged systems of opposite signs out of vacuum when breaking the string. Because only a tiny fraction of such processes seems to be enough in getting BEC in the case of CAS model, it would probably be quite acceptable modification in the string model approach [11]. We are indebted to late B. Andersson for very inspiring discussion at this point at the last ISMD2001 meeting at Datong, China.

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