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PARAMETERS IN WEIGHT CALCULATIONS FOR THE BE EFFECT

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The weight method of implementing the BE effect into Monte Carlo generators is discussed and presented in some detail. We show how the choice of free parameters and the definition of "direct" pions influence the results for the hadronic Z^0 decays.

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1. Introductory remarks

The effect of Bose–Einstein symmetrization (BE effect) in the two-particle correlation spectra depends on the shape and size of the source. This allowed to estimate the source parameters of astronomical sources via the so-called Hanbury–Brown and Twiss effect [1]. The analogous estimates in particle physics are much more involved [2–4]. In fact, the applicability of the standard analysis assuming incoherent production in particle collisions was questioned recently and an alternative approach was presented [5]. The implementation of BE effect into Monte Carlo generators modeling multiple production is particularly difficult, as the symmetrization should be done at the level of amplitudes and generators deal usually with probabilities. As far as we know there is only one implementation based on the specific assumptions concerning amplitudes, and it applies only for a single string fragmentation processes [6]. The most widely used procedure modeling the BE effect in the popular PYTHIA generator [7] is based on the prescription for shifting the final state momenta to produce an enhancement at small momentum differences in the distributions of pairs of identical hadrons [8]. In this procedure one fits the parameters of the "input BE function"

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$$F(Q) = 1 + \lambda \exp(-Q^2 R^2)\,, \tag{1}$$

(assumed to have the same form as the standard parametrization of BE effect) to reproduce the experimentally observed effect.

There is no simple relation between the values of input parameters λ and R and the analogous parameters describing the experimental distribution. There is also no theoretical justification for this procedure and it should be regarded as a convenient parametrization, rather than the physical description of the BE effect.

The alternative approach is based on the formalism of Wigner functions [9]. One approximates the corrected distribution as a product of distribution without the BE effect and the weight function for which a definite prescription is given [10]. This allows us to produce the distributions with the BE effect by generating the events without this effect and attaching to them the weights. To calculate these weights one must adopt several simplifying assumptions [10] (and hope they do not destroy the validity of the formulae). Finally one must assume the form of "two particle weight factor" and fit its parameters to describe correctly the data.

Superficially, there is a marked similarity between these two approaches. In both cases the form of an "input function" is assumed and its parameters should be fitted to describe the data. However, there are also clear differences. Whereas the "momentum shifting method" has no theoretical justification, it has two free parameters (plus a few hidden parameters defining the choice of momenta to be shifted, which affect the results rather mildly) and is quite easy to apply. Since all generated events are used, neither the multiplicity distributions nor, *e.g.*, the decay channel probabilities in Z^0 decay are affected by shifting. There seems to be no need to change the values of the generator parameters fitted to the data before taking the BE effect into account.

On the other hand, the weight method is quite well justified (granting that simplifying assumptions are not too rough), but there are many technical problems with its use. Some of them have been solved: prohibitive increase of computational time with multiplicity may be avoided by a proper clustering procedure for final state momenta [11] and the distortion of the multiplicity distribution may be removed by simple rescaling of weights depending on the event multiplicity [12]. Obviously, the weights may in principle affect other distributions which were fitted to data without taking the BE effect into account. Thus the proper procedure would be to refit all the generator parameters comparing the weighted results with data. However, if the rescaling guarantees that average weight is equal to one for each well defined class of events (*e.g.* in each Z^0 decay channel), the changes in distributions should be minor.

Another notorious technical problem for weight methods is the instability of results due to the long tail of very high weight values. Usually it requires some arbitrary cut, but for sufficiently high number of generated events the effects of this cut are not very significant. Finally, there is a problem of selecting the particles, whose momenta are used to calculate weights and a problem of proper choice of "two particle weight factor" and its parametrization (reflecting somehow the shape and the size of the production source).

In this paper we discuss the solutions to the last two problems presenting the MC results for the BE effect in the hadronic decay of Z^0 and comparing them with some data. We consider only the distributions in the invariant four momentum difference

$$Q^2 = -(p_1 - p_2)^2. \tag{2}$$

The effect of anisotropy in various components of Q [13] was discussed elsewhere [14].

The following chapter contains the discussion of possible particle selections and the influence of various MC parameters. The effects of different forms of two particle weight functions (considered already in the earlier paper [15]) are presented in the third chapter. The last chapter presents the comparison with some data and conclusions. It should be stressed that we discuss only the standard prescription for weights justified by the Wigner function formalism [9,10]. Other proposals [12,16,17] should be regarded as viable versions of the weight method only if it is shown that they reproduce approximately the results obtained for this prescription.

2. Particle selection and MC parameters

Before discussing the details of the MC procedure implementing the BE effect we should decide which distribution will be used to present this effect. The standard quantity called "the BE ratio" is defined as

$$R_{\mathrm{BE}}(Q) = \frac{\rho_2(Q)}{\rho_2^0(Q)}\,, \tag{3}$$

where Q^2 was defined above (2) and the numerator and denominator represent the identical two-particle distribution with and without the BE effect, respectively. Obviously, this definition requires a more precise formulation of how we shall define the denominator.

In the experimental definition of the BE ratio one uses often the distribution of unlike sign pion pairs but this requires cutting off the resonance effects. Thus recently it is preferred to use the pairs of identical pions from different events

$$R_{\mathrm{BE}}(Q) = C_2^{\mathrm{BE}}(Q) \equiv \frac{\rho_2(Q)}{\rho_1 \otimes \rho_1(Q)}\,, \tag{4}$$

where the denominator is a convolution of single distributions

$$\rho_1 \otimes \rho_1(Q) = \int \rho_1(p_1)\rho_1(p_2)\delta\left(Q^2 + (p_1 - p_2)^2\right) d^3p_1 d^3p_2\,. \tag{5}$$

This choice of the denominator has other flaws (*i.e.* it removes all correlations, and not only the BE effect). Therefore, one uses often double ratios, dividing the experimental ratio by an analogous ratio of distributions from MC generator (without the BE effect)

$$R'_{\mathrm{BE}}(Q) = \frac{C_2^{\mathrm{BE}}(Q)}{C_2^{\mathrm{MC}}(Q)}\,. \tag{6}$$

For the MC generated events the simplest choice is just to run MC without the procedure implementing the BE effect

$$R_{\mathrm{BE}}^{\mathrm{MC}}(Q) = \frac{\rho_2^{\mathrm{MC,BE}}(Q)}{\rho_2^{\mathrm{MC}}(Q)}\,. \tag{7}$$

Obviously if in the experimental investigation a double ratio is used, it seems more proper to calculate for comparison an analogous double ratio from MC events

$$R_{\mathrm{BE}}^{\prime\mathrm{MC}}(Q) = \frac{C_2^{\mathrm{MC,BE}}(Q)}{C_2^{\mathrm{MC}}(Q)}\,. \tag{8}$$

Fortunately the difference between $R_{\mathrm{BE}}^{\prime\mathrm{MC}}(Q)$ and $R_{\mathrm{BE}}^{\mathrm{MC}}(Q)$ is often insignificant. This is illustrated in Fig. 1, where we show both ratios calculated for pion pairs from Z^0 decays using the weight method with the Gaussian two-particle weight factor

$$w_2(p_1, p_2) = \exp\left(-\frac{(p_1 - p_2)^2}{2\sigma}\right), \tag{9}$$

with $\sigma = 0.05\,\mathrm{GeV}^2$.

Here, as in all the later figures:

- one million of events was generated by the PYTHIA 6.2 generator [7],
- the background distributions were constructed using pairs from different events; four million pairs of events were used for this purpose,
- the BE ratios were normalized to approach smoothly the value of one at Q exceeding 1 GeV.

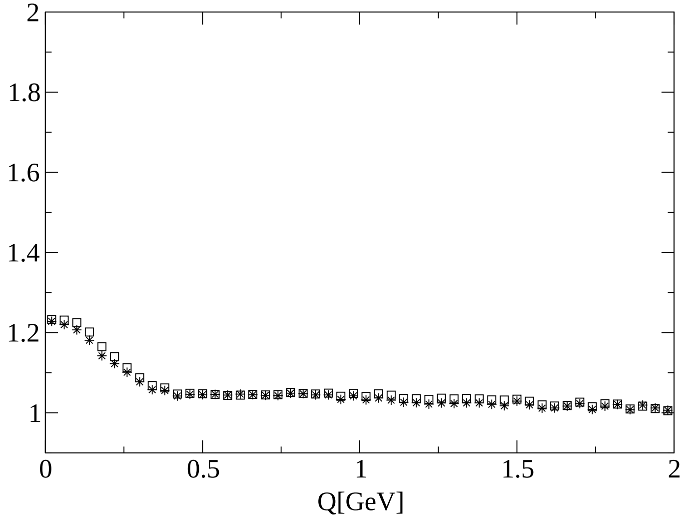

Fig. 1. Comparison of the ratio of Q distributions (7) with and without weights (squares) with double ratio of spectrum-to-background ratios (8) with and without weights (stars).

For completeness, let us remind here that the two-particle weight factor is related to the full weight calculated for each event by a formula [10]

$$W(p_1, ..., p_n) = \sum \prod_{i=1}^{n} w_2(p_i, p_{P(i)}) \,, \tag{10}$$

where the sum extends over all permutations of n elements. More precisely, the event weight is a product of such sums calculated for all kinds of identical bosons registered by the detectors (in practice it is enough to count only positive and negative pions).

This formula suggests that the σ parameter in formula (9) is the only free parameter in the method. This would be, however, an oversimplification. The calculation of the full sum over permutations is practically impossible for the number of identical pions exceeding twenty [18]. Thus we define the clusters of pions "close to each other" in the phase space and sum over permutations within clusters only. To make the results independent on the cluster definition we have to choose the value of the relevant parameter ϵ much bigger than σ. The details of this procedure have been described elsewhere [11].

Moreover, if the two particle weight factor is interpreted as a Fourier transform of the space-time distribution of pairs of pions, it seems justified to use a common shape of this factor only for pairs of "direct" pions.

The decay products of long living particles and resonances are born far away from the original source and the corresponding two-particle weight factor for pairs including these decay products would be close to the Dirac delta function, contributing negligibly to the final event weight. The same reasoning was presented by Sjöstrand [7] who choose 20 MeV as a limit defining "long living" resonances and performed the momentum shift only for pions produced directly and the decay products of broader resonances.

This suggests that we should calculate the event weight including in the sum only "direct" pions defined in a similar way. To avoid the changes of the original Monte Carlo procedures (which was the case for Sjöstrand PYBOEI procedure, called internally from the generator before the decay of "long living" resonances and particles), we form a table of momenta for "direct" pions defined in various ways and use this table for the weight calculation. We found that the modifications of the original width limit of 20 MeV are irrelevant as long as we do not include the ω decay products in the weight calculations. Including ω decay products enhances strongly the BE ratio, as shown in Fig. 2 for the Z^0 decay.

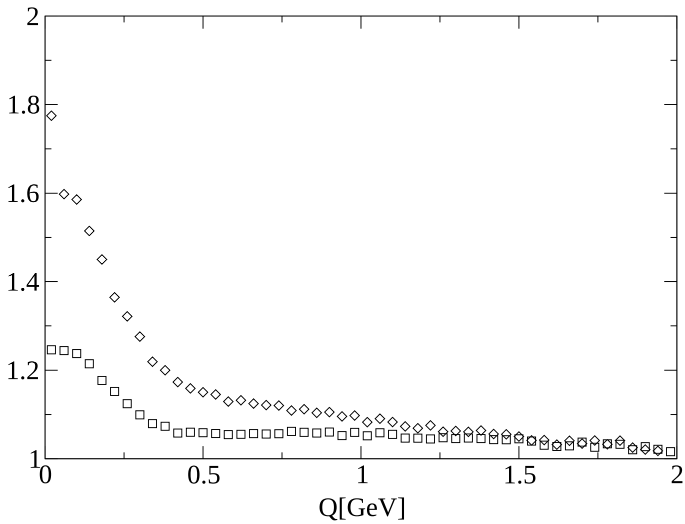

Fig. 2. Comparison of the ratio of Q distributions with and without weights (7) for weights calculated excluding (squares) and including (diamonds) ω decay products.

To justify the choice of the width limit let us require that the "direct" pion and the pion from ω decay have approximately the same momenta. The maximal momentum of a pion in the decay of ω at rest is about 300 MeV/c, and the most likely value is of the order of 100 MeV/c. This allows to estimate that the distance between "birth points" of such pions is of the order of 10 fm and the corresponding width in momentum space should be about

20 MeV, smaller than the typical resolution. This suggests that the decay products of ω (as well as the decay products of narrower resonances and other unstable particles) should not be taken into account when calculating weights.

However, this argument has some flaws. First, the BE ratio is defined as a function of Q^2 and not of the three-momentum squared (thus it reflects the space-time and not just the space structure of source). Second, the distribution of the decay length is not Gaussian. Thus we should not expect a Gaussian shape of the weight factor. Finally, excluding the decay products of narrow resonances is a very rough procedure; a better solution would be to use different two-particle weight factors for different pairs of pions (direct–direct, direct–resonance and resonance–resonance). Let us add that all this estimate is classical and does not take into account the possible quantum effects. Thus we should not treat the choice of "direct" pions excluding the ω decay products as definite. In fact, the uncertainty of this choice seems to be the biggest uncertainty of the weight method. If necessary, it may be used to improve the description of the BE effect if the observed values of the BE ratio at small Q^2 are high.

The other free parameters of the PYTHIA generator may also influence the weights and the resulting BE ratio. An example of this effect is shown in Fig. 3 where we compare the results for default values of PYTHIA parameters and for the values fitted to the L3 data [13]. Let us stress that the

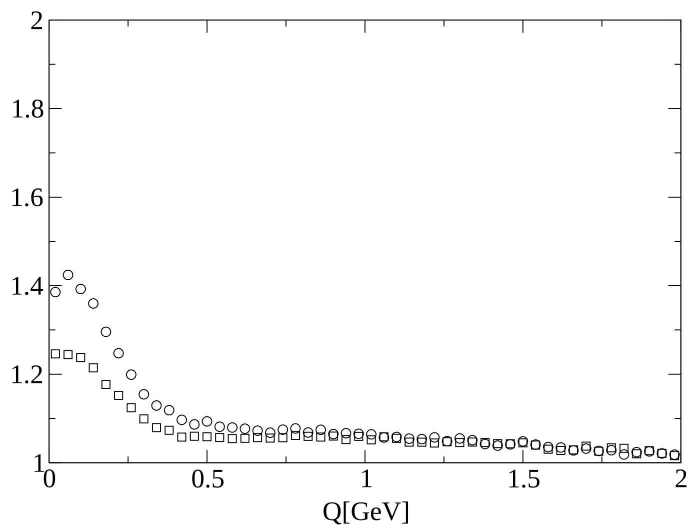

Fig. 3. Comparison of the ratio of Q distributions with and without weights (7) for weights calculated using default PYTHIA parameters (squares) and L3 parameters (circles).

choice of "direct" pions (excluding the ω decay products) and the value of σ parameter are the same in both cases, but the results are visibly different. This is probably mainly due to the suppression of η and η' mesons for the L3 parameters.

Finally, let us check the dependence of the results on the value of the σ parameter. Until now we were using the value of $0.05\,\mathrm{GeV}^2$ which corresponds to the average Gaussian source size of the order of 1 fm. In Fig. 4 we compare the results (with L3 parameters) for $\sigma = 0.05\,\mathrm{GeV}^2$ and $\sigma = 0.07\,\mathrm{GeV}^2$. We see that by increasing σ (which corresponds to a decreasing source size) we increase the width of "BE peak" and slightly increase its height.

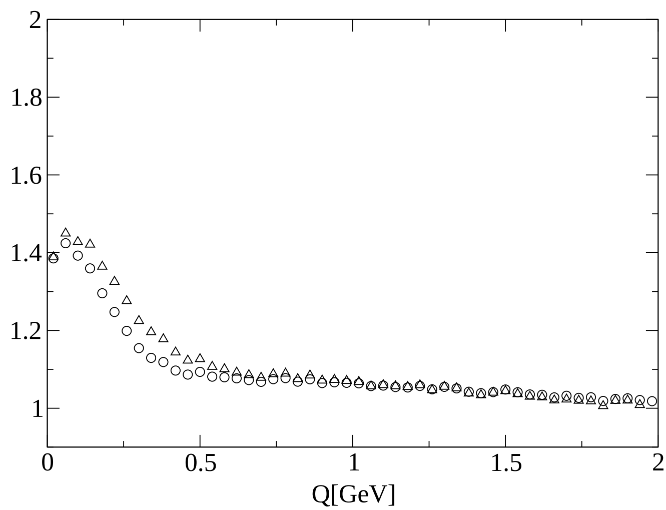

Fig. 4. Comparison of the ratio of Q distributions with and without weights (7) for L3 parameters with weights calculated using $\sigma = 0.05\,\mathrm{GeV}^2$ (circles) and $\sigma = 0.07\,\mathrm{GeV}^2$(triangles).

A notorious problem of the weight method is a possible distortion of various distributions (fitted previously to data) by the introduction of weights. First such a distortion was observed for the multiplicity distribution where the probabilities of high multiplicities were enhanced by weights. This was cured by rescaling the weights with a factor $C\Lambda^n$ [12] where n is a charge particle multiplicity (measured in experiment). The values of parameters C and Λ are fitted to restore the original values of $\overline{n}$ and the original normalization. With this method the weights cause only a moderate increase of the dispersion of the multiplicity distribution.

The weights influence also the single particle momentum distribution, reducing slightly the width, but these effects are not very significant. More important is the change in the Q^2 distribution of unlike sign pairs of pions, as shown in Fig. 5. A similar effect was observed for Sjöstrand's implementation method of the BE effect. It should be noted, however, that by including the ω decay products in the weight calculation we increase the R ratio for unlike sign pion pairs by a few percent only, whereas the ratio for like sign pairs increased by about 50%, as shown in Fig. 2. Thus it is possible to describe a big BE effect without distorting seriously the distribution for unlike sign pairs.

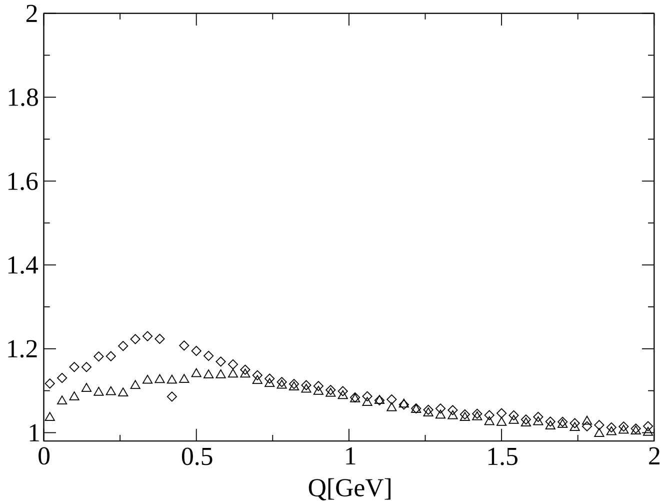

Fig. 5. Comparison of the ratio of Q distributions (7) with and without weights for unlike sign pairs of pions. Weights are calculated using L3 parameters with $\sigma = 0.07\,\mathrm{GeV}^2$ excluding ω decay products (triangles) and using default parameters with $\sigma = 0.05\,\mathrm{GeV}^2$ including ω decay products (diamonds).

3. Choice of the two particle weight factor

In the former section we used always the Gaussian two-particle weight factor (9). Obviously, there is no reason why all the space–time and momentum distributions should be described by such simple functions. However, if we restrict ourselves to the monotonically decreasing weight factors normalized to one at $Q^2 = 0$, it is easy to show that the Gaussian choice results in a curve which is the smoothest one and resembles the data best. This is demonstrated in Fig. 6 where we compare the results obtained for the default PYTHIA parameters for the Gaussian weight factor (9) with $\sigma = 0.05\,\mathrm{GeV}^2$ and for two other choices of the weight factor:

- step-like

$$w_2(p_1, p_2) = \Theta\left[(p_1 - p_2)^2 + \sigma\right] , \tag{11}$$

  with the same value of σ

- exponential

$$w_2(p_1,p_2) = \exp\left(-\frac{\sqrt{-(p_1-p_2)^2}}{2\sqrt{\sigma}}\right), \tag{12}$$

with $\sigma = 0.03\,\mathrm{GeV}^2$.

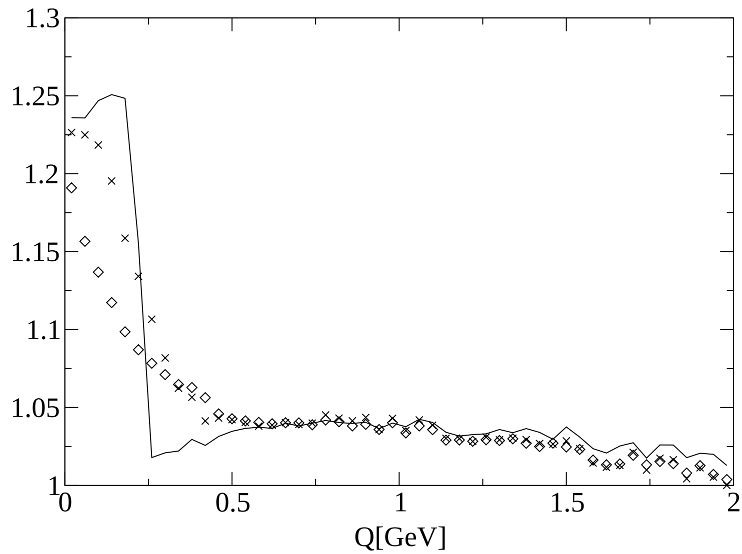

Fig. 6. Comparison of the ratio of Q distributions (7) for default PYTHIA parameters and three different choices of two-particle weight factor: Gaussian (crosses), step-like (solid line) and exponential (diamonds).

We see clearly that the shape of the obtained BE ratio reflects the shape of the two-particle weight factor. This may be written as an approximate relation

$$R_{\mathrm{BE}}(-(p_1-p_2)^2) \approx 1 + c\,w_2(p_1,p_2)\,, \tag{13}$$

where the value of c depends on the shape of the weight factor and the selection of particles used in the weight calculations.

Let us note that the Gaussian parametrization is unlikely to describe the data where the distribution of the space–time distance between the "birth points" of two pions is more complicated. This is the case for the four jet decay of W^+W^- final states if two pions originate from two W. There it is unjustified to expect monotonically decreasing and normalized two-particle weight factors. However, for the Z^0 decay the parametrization (9) seems to be the appropriate one.

4. Comparison with data, conclusions

We will not attempt here a detailed fit to any published data. There are many reasons for this reservation. First, as we have already mentioned, different experiments use different definitions of the reference sample in the denominator of the BE ratio. Thus the fit quality may depend on many factors not related to the procedure implementing the BE effect (*e.g.*, the resonance effects and other correlations). Second, the published data include usually the acceptance corrections which are difficult to reconstruct in our calculations. In fact, the Monte Carlo parameters should be also fitted to the particular set of data before implementing the BE effect. As shown in Sec. 2 there is a difference between the results obtained using default PYTHIA parameters and the parameters used by the L3 collaboration.

Therefore, we want to make only a semi-quantitative comparison between the results from our procedure and some high statistics data. To this purpose we use the recent L3 data shown as the reference sample in the paper devoted to the analysis of the WW decay [19]. We compared them with various MC results shown in previous sections, rescaled with arbitrary constants to agree with data at $Q^2 > 1\,\mathrm{GeV}^2$. We found that the modeled BE effect is too small compared with the data unless we include the ω decay products for the weight calculation. We show the comparison in Fig. 7 for two choices of the σ parameter in the weight factor (9). Normalization of both curves was

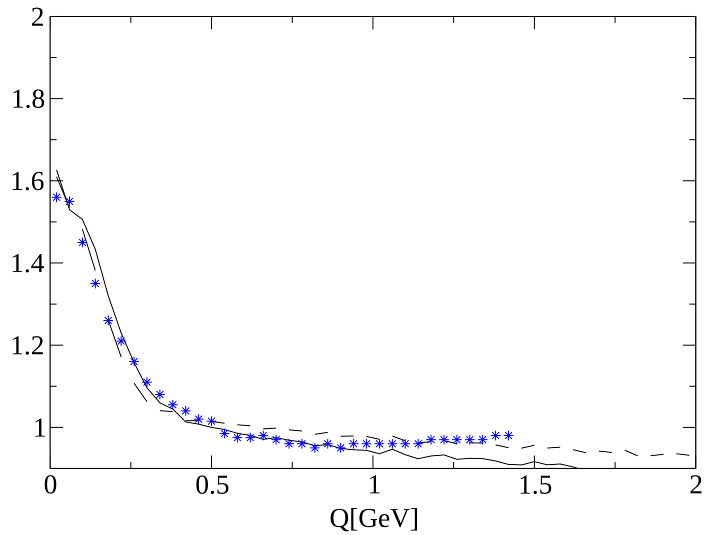

Fig. 7. Comparison of the ratio of Q distributions for L3 parameters (7) with $\sigma = 0.04$ (solid line) and $\sigma = 0.03$ (broken line) with the L3 data (stars).

adjusted to fit the data. We see that the data are qualitatively described by the PYTHIA MC with our implementation of the BE effect. One should not expect a good quantitative fit to the data for any single value of σ; as already noted, one could use at least different values of this parameter (and, even better, different shapes of w_2) for pairs of pions of different origin. Then, however, the number of free parameters would increase making the success of the fitting procedure rather trivial.

To summarize, we have discussed the freedom of the weight method for implementing the BE effect into Monte Carlo generators. We have shown that this freedom seems to be sufficient to describe the data. For pions coming from a single source which may be parametrized with a Gaussian distribution, there are three steps for choosing the weight method parameters:

1. One should decide which ratio (Eq. (4) or (6)) is used to display the BE effect and to calculate the analogous ratio from the MC with weights (Eq. (7) or (8)).

2. One should choose the selection criterion for pions used to calculate weights. The typical choice corresponds to using direct pions and decay products of broad resonances, $\Gamma > 20\,\mathrm{MeV}$ (as in Sjöstrand's method).

3. One should select a proper value of the parameter σ in (9).

Technically, our algorithm contains four Fortran procedures:

- LWBOEI, where for each event the "direct" pions are selected and their momenta are stored in the tables,

- KLASKF, where pions of one sign are assigned to clusters,

- PERCJE, where a weight factor from each cluster is calculated,

- CLUSWAGI, where the full event weight is calculated as a product of weight factors from all clusters and all pion signs.

All these procedures are available at request from us, together with a sample program calling the PYTHIA 6.2 generator and comparing the weighted and unweighted distributions for hadronic Z^0 decays. A modification of this program for other processes or other MC generators would be straightforward.

One should also remember that after the introduction of weights one should rescale them by a $C\Lambda^n$ factor to restore the original normalization and average multiplicity. This, however, does not influence significantly the shapes of the BE ratios.

To describe the process in which pions originate from two or more independent sources (as the $e^+e^- \to W^+W^-$ process with double hadronic decay of W) one needs a more elaborate procedure. Different forms of the w_2 factor should be used for pairs coming from the same and from different sources. This will be discussed in detail elsewhere.

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