THE ENERGY AND COMPRESSIBILITY LIQUID DROP MODEL EXPANSION IN THE EXTENDED THOMAS–FERMI MODEL

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We used the generalized form of the Thomas–Fermi type for the density profile inside spherical nuclei to obtain a leptodermous expansion for the matter density. This expansion was used to calculate the energy coefficients of the liquid drop model formula. We obtained analytical expressions for the volume, surface, curvature and higher order energy coefficients. These analytical expressions were used to derive a liquid drop model expansion for compressibility of spherical nuclei. We studied the energy and compressibility expansion coefficients and also their convergence. Particular interest was focused on the study of surface and curvature properties.

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1. Introduction

For nuclei that are not too small the surface thickness is much smaller than the nuclear radius (leptodermous system) and we can expand the nuclear energy of a finite nucleus into volume, surface, curvature and higher order contributions. This leptodermous expansion (or the Liquid Drop Model (LDM) formula) has proved enormously useful in calculating fission barriers, ground-state masses and other nuclear properties [1–3]. In the limit of a very large nuclear systems, the density may be considered to vary along one axis and extending to infinity in the two other directions. This one-dimensional geometry is called Semi-Infinite Nuclear Matter (SINM) and is used to extract the surface and curvature energy coefficients of the liquid drop formula [3, 4].

In several problems of nuclear physics and astrophysics the surface and curvature properties of nuclei play a crucial role. In nuclear physics they are important, for example, for barrier heights and saddle-point configuration in nuclear fissions and also for fragment distribution in heavy ion collision. In astrophysics they are important in studying neutron stars and supernovae.

Being a quantal system, the nucleus is best described by the shell model or one of its self-consistent versions, such as the Hartree–Fock (HF) or random phase approximations. However, a fairly good description of the nucleus can be obtained within the semiclassical approach such as the liquid-drop model, the droplet model or the extended Thomas–Fermi model [1–3]. Comparison of Hartree–Fock and fully self-consistent semiclassical calculations has shown that the quantal oscillations in the density are confined to the interior of the nucleus and that the nuclear surface is well described by semiclassical methods [5,6]. This allows us to calculate surface properties of nuclei by means of semiclassical methods. Although the surface energy can be calculated in a full quantal way, only one part of the curvature energy can be obtained from a quantum mechanical calculation [7–9]. This is an important reason to compute the surface and curvature coefficients using semi-classical methods [9].

The nuclear curvature energy posses a problem, the so-called curvature energy anomaly [10]. The value of the curvature energy obtained from theoretical calculations [1, 3, 7–10] is about 10 MeV. On the contrary, the empirical value determined from the analysis of experimental data on ground state nuclear masses is compatible with a vanishing value [2,11,12]. Several explanations have been suggested to solve this anomaly, for instance the consideration of finite range interaction, Friedel oscillations and relativistic effects. The effects of the finite range interactions and the influence of Friedel oscillations have been analyzed in reference [7] and it has been found that they are not responsible for the curvature anomaly. Also, the relativistic effects have been found [9] not to be responsible for the anomaly. Myers and Swiatecki [13] pointed out that the reason for this puzzle is due to neglecting terms of higher order in the LDM formula. They added a higher order term to dispose of the curvature energy puzzle. The value of this term was chosen empirically so as to make the binding energy vanish at $A = 1$.

The nuclear compressibility is one important factor in the equation of state of nuclear matter which is accessible to experiment. It enters static properties of nuclei and also in dynamical properties. It also plays a role in determining the strength of the shock waves following the collapse of supernovae. The direct relation between the nuclear compressibility and the second derivative of the binding energy of the nucleus suggests a liquid drop type formula for the compressibility. One way to determine the compressibility coefficients in this LDM formula is done by fitting the compressibility liquid drop formula to the experimental data from the giant monopole resonance of nuclei [14–17]. Recently Satpathy *et al.*, [17] used an analytical form for the energy LDM expansion to calculate the compressibility LDM formula. They found that the LDM expansion of the compressibility shows anomalous behavior in contrast to the rapidly converging LDM energy expansion. They concluded that the LDM expansion of the compressibility is not quite suitable for the extraction of the compressibility coefficients of this formula.

The present paper has two aims. The first aim, is to calculate the higher order terms in the LDM energy expansion. We used a variety of Skyrme interactions together with the Extended Thomas–Fermi (ETF) functionals including fourth order correction terms to obtain analytical formulae for energy expansion coefficients. These analytical formulae were used to analyze the surface and curvature properties of spherical symmetric nuclei. The second aim, was to calculate and study the LDM expansion coefficients of compressibility. The convergence of this expansion was also studied.

We presented the calculation method in Section 2. Section 3 presents our results and discussions.

2. Calculations

The total energy of a nuclear system is usually written as an integral over all space of suitable kinetic and potential energy densities

$$E = \int H(r) d^3 r \, . \tag{1}$$

For the potential part, a most convenient interaction is the Skyrme interaction that consists of a two-body and a three-body zero range force. The three-body term simulates the density dependence, while the velocity dependent two-body term represents the finite range expansion of the force [18]. The general form of the Hamiltonian density for a symmetric spherical nucleus with no Coulomb energy is given by

$$H = H_{\mathrm{b}} + H_{\mathrm{eff}} + H_{\tau} \, , \tag{2}$$

where H_{b}, H_{eff}, and H_{τ} represent the bulk, the effective mass and the kinetic Hamiltonian energy density, respectively. They are given by

$$H_{\mathrm{b}} = \frac{3}{8}\tau_0 \rho^2 + \frac{1}{16} t_3 \rho^{\alpha+2} + C_1 (\nabla\rho)^2 \, , \tag{3}$$

$$H_{\mathrm{eff}} = C_2 \rho \tau \tag{4}$$

and

$$H_{\tau} = \frac{\hbar^2}{2m} \rho \tau \, , \tag{5}$$

where

$$C_1 = \frac{1}{16}\Big(9t_1 - t_2(5+4x_2)\Big) \tag{6}$$

and

$$C_2 = \frac{1}{16}\Big(3t_1 + t_2(5+4x_2)\Big)\,. \tag{7}$$

ρ is the local density of the system and $\tau(\rho)$ is its kinetic energy density.

The kinetic energy density in the ETF model up to the fourth order correction is given by [3]

$$\tau(\rho) = \tau_0(\rho) + \tau_2(\rho) + \tau_4(\rho)\,, \tag{8}$$

where $\tau_0(\rho)$ is the standard Thomas–Fermi kinetic energy density term and is given by

$$\tau_0(\rho) = \frac{3}{5}\left(\frac{3\pi^2}{2}\right)^{\frac{2}{3}} \rho^{\frac{5}{3}}\,, \tag{9}$$

$\tau_2(\rho)$ is the second order correction and is given by

$$\tau_2 = \frac{1}{36}\frac{(\nabla\rho)^2}{\rho} + \frac{1}{3}\Delta\rho + \frac{1}{6}\frac{\nabla\rho\cdot\nabla f}{f} + \frac{1}{6}\frac{\rho\nabla f}{f} - \frac{1}{12}\rho\left(\frac{\nabla f}{f}\right)^2 \tag{10}$$

and $\tau_4(\rho)$ is the fourth order correction and is given by

$$\tau_4(\rho) = \frac{1}{6480}\left(\frac{2}{3\pi^2}\right)^{\frac{2}{3}} \rho^{\frac{1}{3}} \left\{8\left(\frac{\nabla\rho}{\rho}\right)^4 - 27\left(\frac{\nabla\rho}{\rho}\right)^2\left(\frac{\triangle\rho}{\rho}\right) + 24\left(\frac{\triangle\rho}{\rho}\right)^2\right\}. \tag{11}$$

$f(\rho)$ is related to the effective mass by the relation

$$F(\rho) = \frac{m}{m^*} = 1 + \frac{\partial H}{\partial\tau} = 1 + c\rho\,, \tag{12}$$

where

$$c = \frac{1}{16}\frac{2m}{\hbar^2}\Big[3t_1 + t_2(5+4x_2)\Big]\,. \tag{13}$$

Whenever the energy density can be written as a function of the matter density, this is called the Energy Density Formalism (EDF), minimization of the energy leads to a variational equation known as the Euler–Lagrange (E–L) equation. Solving the E–L equation yields both the density profile and the average total energy of the nucleus [3]. The same routine was employed to calculate the surface energy of the nucleus using the SINM idea [3,6]. The curvature energy is calculated using the obtained density profile [3,6].

When solving the E–L equation numerically using the ETF limited to a second order correction, the variational densities fall off too quickly in the outer surface and lead to an overestimation of the kinetic energy [3, 18]. This problem was overcome when the full fourth order ETF was used and the results on the average were identical to that of Strutinsky-averaged HF calculations [3,4]. Introducing the fourth order correction of the ETF makes the E–L equation a highly nonlinear fourth order differential equation which seems inaccessible to numerical solutions [3]. One practical way out of this difficulty is to perform a leptodermous expansion of the energy using the SINM simplifications [4]. The second way is to parameterize the density profile and minimize the total energy with respect to the density parameters (the restricted variational method). The two-parameters Fermi distribution function (Fermi distribution to a power) for the trial density profile was proved [3, 4] to give a very good approximation to the numerical solution of the fourth order E–L equation for symmetric SINM.

In our model calculations, we choose the two-parameters Fermi distribution for the trial density

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-q}, \tag{14}$$

where ρ_0, a, and q are the nuclear matter central density, the diffuseness parameter and the skewness parameter, respectively.

For the density profile given by Eq. (11), the gradient terms ($\nabla\rho$) and the Laplacian terms ($\triangle\rho$) can be expressed in terms of the density itself. Thus, the total energy of the system contains only integrals of the form $\int \rho d^3r$. These integrals can be approximated by the form [19]

$$\int \rho^p d^3r = \frac{4\pi}{3}\rho_0^q\Big[R^3 - aR^2A_1(pq) + 6a^2RA_2(pq) - 6a^3A_3(pq)\Big], \tag{15}$$

where the coefficients $A_n(m)$ are given by [19]

$$A_n(m) = \frac{1}{(n-1)!}\int \Big\{1-[1+\exp(-x)]^{-m}+(-1)^n[1+\exp(x)]^{-m}\Big\}x^{n-1}dx\,. \tag{16}$$

The normalization condition

$$A = \int \rho(r)d^3r = \frac{4\pi}{3}\rho_0\Big\{R^3 - aR^2A_1(q) + 6a^2RA_2(q) - 6a^3A_3(q)\Big\} \tag{17}$$

is inverted to obtain the radius R in the form

$$R = r_0 A^{\frac{1}{3}} + aA_1(q) + \frac{a^2}{r_0}\Big[A_1(q) - 2A_2(q)\Big]A^{-\frac{1}{3}} . \tag{18}$$

From Eqs. (12) and (15) we can write

$$\begin{aligned}
&\int \rho^p d^3r = \rho_0^{p-1}\Big\{A - 3\left(\frac{q}{r_0}\right)[A_1(pq) - A_1(q)]A^{\frac{2}{3}} \\
&+ 6\left(\frac{a}{r_0}\right)^2 [A_2(pq) - A_2(q) - A_1(q)(A_1(pq) - A_1(q))]A^{\frac{1}{3}} \\
&+ 3\left(\frac{a}{r_0}\right)^3 [4A_2(q) - 3A_1^2(q)(A_1(pq) - A_1(q)) + 2\,A_1(q)A_2(pq) - 2A_3(pq)] \\
&+ \left(\frac{a}{r_0}\right)^4 [A_1^2(q)(7A_1(q) - 9A_1(pq)) + 12A_2(q)\,(A_1(pq) - A_1(q)) \\
&+ 6\,A_1(q)A_2(pq) - 6A_3(pq)]A^{-\frac{1}{3}}\Big\} .
\end{aligned} \tag{19}$$

This equation is the leptodermous expansion of the density. Introducing this equation in the energy Eq. (1), we obtain the leptodermous expansion of the energy in the form

$$E = E_{\mathrm{V}}A + E_{\mathrm{S}}A^{\frac{2}{3}} + E_{\mathrm{C}}A^{\frac{1}{3}} + E_0 + E_{-1}A^{-\frac{1}{3}} , \tag{20}$$

where E_{V}, E_{S}, E_{C}, E_0 and E_{-1} represent the volume, surface, curvature, zero and $-1/3$ energy coefficients, respectively. Each of these terms gets contributions from bulk potential, effective mass and kinetic energies.

Thus, the surface energy can be written as

$$E_{\mathrm{s}} = E_{\mathrm{s}}^{\mathrm{b}} + E_{\mathrm{s}}^{\mathrm{eff}} + E_{\mathrm{s}}^{k} . \tag{21}$$

The bulk surface energy is given by

$$\begin{aligned}
E_{\mathrm{s}}^{\mathrm{b}} \;=\; -3\left(\frac{a}{r_0}\right)\Bigg\{&\frac{3}{8}t_0\rho B_{\mathrm{s}}(2) + \frac{1}{16}t_3\rho^{\alpha+1}B_{\mathrm{s}}(\alpha+2) \\
&+ \frac{1}{16}\left(\frac{q}{a}\right)^2 \rho[9t_1 - t_2(5+4x_2)]\,G_1(0)\Bigg\} .
\end{aligned} \tag{22}$$

The kinetic surface energy is given by

$$E_{\mathrm{s}}^{k} = -3\,\frac{q}{r_0}\,\frac{\hbar^2}{2m}\Bigg\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_{\mathrm{s}}\,\frac{5}{3} + \frac{1}{3a}B_{d1} + \frac{q}{a}\Bigg[\frac{1}{36}G_1(-1) + \frac{c\rho}{6}G_1(0)$$
$$- \left(\frac{c\rho}{2}\right)^2 G_1(1) + \frac{1}{3}(c\rho)^3 G_1(2) - \frac{5}{12}(c\rho)^4 G_1(3) + \frac{5}{12}(c\rho)^5 G_1(4)\Bigg]$$
$$+ \frac{c\rho}{6}\left[F_1(1) - c\rho F_1(2) + (c\rho)^2 F_1(3) - (c\rho)^3 F_1(4)\right]\Bigg\}$$
$$- \frac{1}{6480}\,\frac{\hbar^2}{2m}\left(\frac{2}{3\pi^2\rho}\right)^{\frac{2}{3}}\left(\frac{3a}{r_0}\right) C_2\Big\{8\left(\frac{q}{a}\right)^4 H_1 - 27\frac{q^3}{a^4}Q_1 + 24\frac{q^2}{a^4}P_1\Big\}\,. \quad (23)$$

The effective mass surface energy is given by

$$E_{\mathrm{s}}^{\mathrm{eff}} = -3\left(\frac{q}{r_0}\right) C_2\rho\Bigg\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_{\mathrm{s}}\frac{8}{3} + \left(\frac{q}{a}\right)\Big[-\frac{11}{36}G_1(0) + \frac{c\rho}{6}G_1(1)$$
$$- \left(\frac{c\rho}{2}\right)^2 G_1(2) + \frac{1}{3}(c\rho)^3 G_1(3) - \frac{5}{12}(c\rho)^4 G_1(4) + \frac{5}{12}(c\rho)^5 G_1(5)\Big]$$
$$+ \frac{c\rho}{6}\Big[F_1(2) - c\rho F_1(3) + (c\rho)^2 F_1(4) - (c\rho)^3 F_1(5)\Big]\Bigg\}$$
$$- \frac{1}{6480}\left(\frac{2}{3\pi^2}\right)^{\frac{2}{3}}\frac{3a}{r_0}\,C_2\rho^{\frac{1}{3}}\left\{-7\left(\frac{q}{a}\right)^4 H_2 - 3\frac{q^3}{a^4}Q_2 + 30\frac{q^2}{a^4}P_2\right\}\,, \quad (24)$$

where $B_{\mathrm{s}}(n)$, B_{d1}, $G_1(n)$, $F_1(n)$, H_1, H_2, Q_1, Q_2, P_1 and P_2 are related to the coefficients $A_n(m)$ and given in the Appendix.

The curvature energy can be written as

$$E_{\mathrm{c}} = E_{\mathrm{c}}^{\mathrm{b}} + E_{\mathrm{c}}^{\mathrm{k}} + E_{\mathrm{c}}^{\mathrm{eff}}\,. \quad (25)$$

The bulk curvature energy is given by

$$E_{\mathrm{c}}^{\mathrm{b}} = 6\left(\frac{a}{r_0}\right)^2\Bigg\{\frac{3}{8}t_0\rho B_{\mathrm{c}}(2) + \frac{1}{16}t_3\rho^{\alpha+1}B_{\mathrm{c}}(\alpha+2)$$
$$+\frac{1}{16}\left(\frac{q}{a}\right)^2\rho\Big[9t_1 - t_2(5+4x_2)\Big]B(0)\Bigg\}\,. \quad (26)$$

The kinetic curvature energy is given by

$$E_{\mathrm{c}}^{k} = 6\,\frac{aq}{r_0^2}\,\frac{\hbar^2}{2m}\Biggl\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}}B_{\mathrm{c}}\frac{5}{3}+\frac{q}{a}\Biggl[\frac{1}{36}B(-1)+\frac{c\rho}{6}B(0)$$
$$-\left(\frac{c\rho}{2}\right)^2B(1)+\frac{1}{3}(c\rho)^3B(2)-\frac{5}{12}(c\rho)^4B(3)+\frac{5}{12}(c\rho)^5B(4)\Biggr]$$
$$+\frac{c\rho}{6}\Bigl[X(1)-c\rho X(2)+(c\rho)^2X(3)-(c\rho)^3X(4)\Bigr]\Biggr\}$$
$$+\frac{1}{3}\frac{\hbar^2}{2m}\Biggl\{\frac{6q}{r_0^2}B_{\mathrm{c}}-8\pi\rho q r_0^2(A_1(q+1)-A_1(q))\Biggr\}$$
$$+\frac{6}{6480}\left(\frac{2}{3\pi^2\rho}\right)^{\frac{2}{3}}\left(\frac{a}{r_0}\right)^2\frac{\hbar^2}{2m}\Biggl\{8\left(\frac{q}{a}\right)^4[H_2-H_1A_1(q)]$$
$$-27\frac{q^3}{a^4}[Q_2-Q_1A_1(q)]+24\frac{q^2}{a^4}[P_2-P_1A_1(q)]\Biggr\}$$
$$+\frac{1}{6480}\left(\frac{2}{3\pi^2}\right)^{\frac{2}{3}}\frac{\hbar^2}{2m}\,\frac{8\pi r_0q^2}{a^3}\,\rho^{\frac{1}{3}}\Biggl\{24\Biggl[(q+1)A_1\left(\frac{q+9}{3}\right)$$
$$-(3q+2)A_1\left(\frac{q+6}{3}\right)+(3q+1)A_1\left(\frac{q+3}{3}\right)-qA_1(q)\Biggr]$$
$$-27q\left[A_1\left(\frac{q+9}{3}\right)-3A_1\left(\frac{q+6}{3}\right)+3A_1\left(\frac{q+3}{3}\right)-A_1(q)\right]\Biggr\}. \quad (27)$$

The effective mass curvature energy is given by

$$E_{\mathrm{c}}^{\mathrm{eff}} = 6\rho\,\frac{aq}{r_0^2}C_2\Biggl\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}}B_{\mathrm{c}}\frac{8}{3}+\frac{q}{a}\Bigl[-\frac{11}{36}B(0)+\frac{c\rho}{6}B(1)$$
$$-\left(\frac{c\rho}{2}\right)^2B(2)+\frac{1}{3}(c\rho)^3B(3)-\frac{5}{12}(c\rho)^4B(4)+\frac{5}{12}(c\rho)^5B(5)\Bigr]$$
$$+\frac{c\rho}{6}\left[X(2)-c\rho X(3)+(c\rho)^2X(4)-(c\rho)^3X(5)\right]\Biggr\}$$
$$+\frac{6}{6480}\left(\frac{2}{3\pi^2}\right)^{\frac{2}{3}}\left(\frac{a}{r_0}\right)^2C_2\rho^{\frac{1}{3}}\Bigl\{-7q^2[R_2-R_1A_1(q)]$$
$$-3q[S_2-S_1A_1(q)]+30[T_2-T_1A_1(q)]\Bigr\}$$
$$+\frac{1}{6480}\,\frac{2}{3\pi^2}\,\frac{8\pi r_0q^2}{a^3}\,C_2\rho^{\frac{4}{3}}\Biggl\{30\Biggl[(q+1)A_1\left(\frac{4q+9}{3}\right)$$

$$- (3q+2)A_1\left(\frac{4q+6}{3}\right) + (3q+2)A_1\left(\frac{4q+3}{3}\right) - qA_1\left(\frac{4q}{3}\right)\Bigg]$$
$$- 3q\left[A_1\left(\frac{4q+9}{3}\right) - 3A_1\left(\frac{4q+6}{3}\right) + 3A_1\left(\frac{4q+3}{3}\right) - A_1\left(\frac{4q}{3}\right)\right]\Bigg\}\,, \quad (28)$$

where $B_c(n)$, B_c, $X(n)$, S_1, S_2, R_1, R_2, T_1 and T_2 are related to the coefficients $A_n(m)$ and given in the Appendix.

Also, the energy coefficient E_0 can be written as

$$E_0 = E_0^{b} + E_0^{k} + E_0^{eff}\,. \quad (29)$$

The bulk energy term E_0^b is given by

$$E_0^{b} = 3\left(\frac{a}{r_0}\right)^3 \left\{\frac{3}{8}t_0\rho B_0(2) + \frac{1}{16}t_3\rho^{\alpha+1}B_0(\alpha+2) + \frac{1}{16}\left(\frac{q}{a}\right)^2 \rho\left[9t_1 - t_2(5+4x_2)\right]C(0)\right\}. \quad (30)$$

The kinetic energy E_0^k is given by

$$E_0^{k} = 3\,\frac{a^2 q}{r_0^3}\,\frac{\hbar^2}{2m}\Bigg\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_0\,\frac{5}{3} + \frac{q}{a}\Bigg[\frac{1}{36}C(-1) + \frac{c\rho}{6}C(0) - \left(\frac{c\rho}{2}\right)^2 C(1) + \frac{1}{3}(c\rho)^3 C(2) - \frac{5}{12}(c\rho)^4 C(3) + \frac{5}{12}(c\rho)^5 C(4)\Bigg] + \frac{c\rho}{6}\left[Y(1) - c\rho Y(2) + (c\rho)^2 Y(3) - (c\rho)^3 Y(4)\right]\Bigg\} + \frac{\hbar^2}{2m}\left\{\frac{aq}{r_0^3}U_1 - \frac{8}{3}(a\pi\rho)U_2\right\}. \quad (31)$$

The effective mass energy E_0^{eff} is given by

$$E_0^{eff} = 3\,\frac{a^2 q}{r_0^3}\,C_2\rho\Bigg\{\frac{3a}{5q}\left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_0\frac{8}{3} + \frac{8\pi}{3}qaU_1 + \frac{q}{a}\Bigg[-\frac{11}{36}C(0) + \frac{c\rho}{6}C(1) - \left(\frac{c\rho}{2}\right)^2 C(2) + \frac{1}{3}(c\rho)^3 C(3) - \frac{5}{12}(c\rho)^4 C(4) + \frac{5}{12}(c\rho)^5 C(5)\Bigg] + \frac{c\rho}{6}\left[Y(2) - c\rho Y(3) + (c\rho)^2 Y(4) - (c\rho)^3 Y(5)\right]\Bigg\}, \quad (32)$$

where $B_0(n)$, U_1, U_2, $C(n)$ and $Y(n)$ are related to the coefficients $A_n(m)$ and given in the Appendix.

Also, the energy coefficient E_{-1} can be written as

$$E_{-1} = E^{\mathrm{b}}_{-1} + E^{k}_{-1} + E^{\mathrm{eff}}_{-1} \,. \tag{33}$$

The bulk energy $\mathrm{E}^{\mathrm{b}}_{-1}$ is given by

$$\begin{aligned} E^{\mathrm{b}}_{-1} &= \left(\frac{a}{r_0}\right)^4 \left\{ \frac{3}{8} t_0 \rho B_1(2) + \frac{1}{16} t_3 \rho^{\alpha+1} B_1(\alpha+2) \right. \\ &\left. + \frac{1}{16}\left(\frac{q}{a}\right)^2 \rho[9t_1 - t_2(5+4x_2)]D(0) \right\} . \end{aligned} \tag{34}$$

The kinetic energy E^{k}_{-1} is given by

$$\begin{aligned} E^{k}_{-1} &= \frac{a^3 q}{r_0^4} \frac{\hbar^2}{2m} \left\{ \frac{3a}{5q} \left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_1 \frac{5}{3} + 6\,\frac{q}{a}\left[\frac{1}{36} D(-1) + \frac{c\rho}{6} D(0) \right.\right. \\ &\left. - \left(\frac{c\rho}{2}\right)^2 D(1) + \frac{1}{3}(c\rho)^3 D(2) - \frac{5}{12}(c\rho)^4 D(3) + \frac{5}{12}(c\rho)^5 D(4) \right] \\ &\left. + \frac{c\rho}{6}[Z(1) - c\rho Z(2) + (c\rho)^2 Z(3) - (c\rho)^3 Z(4)] \right\}. \end{aligned} \tag{35}$$

The effective mass energy E^{eff}_{-1} is given by

$$\begin{aligned} E^{\mathrm{eff}}_{-1} &= \frac{a^3 q}{r_0^4} C_2 \rho \left\{ \frac{3a}{5q} \left(\frac{3\pi^2\rho}{2}\right)^{\frac{2}{3}} B_1 \frac{8}{3} + 6\,\frac{q}{a}\left[-\frac{11}{36} D(0) + \frac{c\rho}{6} D(1) \right.\right. \\ &\left. - \left(\frac{c\rho}{2}\right)^2 D(2) + \frac{1}{3}(c\rho)^3 D(3) - \frac{5}{12}(c\rho)^4 D(4) + \frac{5}{12}(c\rho)^5 D(5) \right] \\ &\left. + \frac{c\rho}{6}[Z(2) - c\rho Z(3) + (c\rho)^2 Z(4) - (c\rho)^3 Z(5)] \right\}, \end{aligned} \tag{36}$$

where $B_1(n)$, $D(n)$, and $Z(n)$ are related to the coefficients $A_n(m)$ and given in the Appendix.

The nuclear compressibility is defined by

$$K = 9\rho_0^2 \frac{d^2\ \left(\frac{E}{A}\right)}{d\rho^2}\Bigg|_{\rho_0} . \tag{37}$$

We used the analytical expressions for the energy coefficients obtained before to get the nuclear compressibility. A similar leptodermous expansion, or LDM formula, for compressibility is obtained

$$K = A_{\rm V}A + K_{\rm S}A^{\frac{2}{3}} + K_{\rm C}A^{\frac{1}{3}} + K_0 + K_{-1}A^{-\frac{1}{3}}\,. \tag{38}$$

It is easy and straightforward to calculate the compressibility coefficients $K_{\rm V}$, $K_{\rm S}$, $K_{\rm C}$, K_0 and K_{-1} using the analytical equation of the corresponding energy terms.

3. Results and discussions

One practical way to solve the fourth order E–L equations is to use the restricted variational method. The trial density profile parameters are determined by minimizing the total energy of the nucleus with respect to these parameters. The surface and curvature energies are then obtained using the SINM approximation. They are defined, until now, only for SINM systems.

In our model calculations we used the generalized form of the Thomas–Fermi distribution function for the trial density profile. We have three density parameters namely, ρ_0, a and q. The central density of the nucleus ρ_0 is determined from the saturation condition of the volume energy and it is fixed by the Skyrme force parameters. The normalization condition on the mass number of the nucleus, when using the analytical expression for the density integral, is inverted to get the LDM expansion for the matter density. This LDM expansion, when introduced in the energy equation, gives closed analytical expressions for the LDM energy coefficients. These expressions were used to calculate the surface and the curvature energies as functions of the density parameters a and q. These density parameters were determined at the minimum value of both the surface and the curvature energies. The obtained optimal values of a and q for different Skyrme forces are listed in Table I. The analytical expression for the surface energy (also for the other LDM energy coefficients) enabled us to study the interrelation between the different factors of the surface energy. We divided the energy into three parts: the bulk, the effective mass and the kinetic energy term. The sum of the bulk and the effective mass terms is the potential energy. The effective mass term (the two-body velocity dependent term in the Skyrme forces) reflects the effect of the kinetic energy formula on the potential part.

Table II shows the bulk, effective mass and the kinetic surface energies. The total surface energy is also listed and compared with previous results for some Skyrme forces. It is noted that the average value of the surface potential energy is $+37\,$MeV and the average value of the surface kinetic energy is $-19\,$MeV (to be compared with the average value of the volume potential energy $-38\,$MeV and the average value of the volume kinetic energy

TABLE I

The optimal value of the density parameters (a) and (q) for different Skyrme forces with the corresponding reference for the force parameters.

Force [Ref. number]	a	q
RATP[20]	0.64	1.84
SGII [21]	0.47	1.00
SKM* [3]	0.6	1.3
SKI [22]	0.42	1.42
SLY4 [22]	0.55	1.1
SLY7 [22]	0.47	1.2
SIII [23]	0.52	1.4
SKA [24]	0.64	1.64
SKSC4 [25]	0.53	1.44
SKSC4o [25]	0.53	1.44
SKSC14 [25]	0.53	1.44
SKSC15 [25]	0.53	1.44
SKp [26]	0.53	1.11
T6 [27]	0.56	1.65

$+22$ MeV). The negative contribution of the surface kinetic energy means that the particles entering the surfaces are slowed down to reduce the surface energy of the nucleus. The average value of the surface energy for the different Skyrme force is about $+18$ MeV which is in a good agreement with the LDM empirical value. Table II shows also that our results are in a strong agreement with that of the restricted variational method of Ref. [3].

Table III shows the bulk, effective mass and the kinetic curvature energies. The average kinetic curvature energy is about $+2$ MeV (SKp and SGII give negative values) and the average potential curvature energy is about $+8$ MeV. This gives an average value for the nuclear curvature energy of about $+10$ MeV which is the well-known value obtained using different theoretical models. The leptodermous curvature energy $E_{\rm c}$ gets contribution from the compression energy and the effective curvature energy $E^*_{\rm C}$ is given by [1,2]

$$E^*_{\rm C} = E_{\rm C} - \frac{2E_{\rm S}}{K_{\rm V}}\,, \tag{39}$$

where $K_{\rm V}$ is the volume compressibility. This reduces the leptodermous curvature energy by a factor of about 3 MeV and an average value of about 7 MeV is obtained for the effective curvature energy. Table III also shows

TABLE II

The bulk, effective mass and the kinetic surface energy together with the total surface energy for different Skyrme forces, all in MeV.

Force	$E_{\rm S}^{\rm b}$	$E_{\rm S}^{\rm eff}$	$E_{\rm S}^{k}$	$E_{\rm S}$	$E_{\rm S}$ [Ref.]
SKM*	53.27	−13.04	−22.28	17.95	17.22 [3]
SKa	68.81	−29.56	−20.85	18.39	18.52 [3]
SIII	46.66	−11.98	−16.36	18.32	18.04 [3]
RATP	64.89	−24.21	−22.26	18.43	18.48 [3]
SLY4	61.99	−20.91	−22.39	18.70	18.91 [22]
SLY7	54.28	−18.74	−17.78	17.76	17.85 [22]
SKI	35.17	−3.54	−14.01	17.62	17.31 [22]
SGII	47.26	−11.64	−19.17	16.45	—
SKSC4	37.59	0.00	−19.08	18.51	—
SKSC4o	37.60	0.00	−19.08	18.52	—
SKSC14	37.86	0.00	−19.08	18.78	—
SKSC15	37.68	0.00	−19.08	18.60	—
SKp	40.47	0.00	−21.31	19.16	—
T6	38.32	0.00	−19.55	18.77	—

TABLE III

The same as Table II but for the curvature energy.

Force	$E_{\rm C}^{\rm b}$	$E_{\rm C}^{\rm eff}$	$E_{\rm C}^{k}$	$E_{\rm C}\,(E_{\rm c}^{*})$	$E_{\rm C}$ [Ref.]
SKM*	3.69	4.82	3.98	12.49 (9.52)	12.82 [3]
SKa	−1.07	10.33	4.14	13.41 (10.84)	12.15 [3]
SIII	5.07	2.88	1.76	9.70 (7.81)	9.52 [3]
RATP	−0.23	9.21	4.77	13.75 (10.91)	12.99 [3]
SLY4	8.09	5.36	0.17	13.63 (10.58)	8.21 [22]
SLY7	4.53	4.28	1.43	10.24 (7.49)	7.47 [22]
SKI	4.94	0.71	1.58	7.23 (5.55)	5.06 [22]
SGII	8.77	2.24	−0.67	10.35 (7.82)	—
SKSC4	8.14	0.00	1.36	9.50 (6.58)	—
SKSC4o	8.13	0.00	1.36	9.49 (6.57)	—
SKSC14	8.18	0.00	1.36	9.54 (6.54)	—
SKSC15	8.15	0.00	1.36	9.50 (6 .56)	—
SKp	11.90	0.00	−1.06	10.84 (7.18)	—
T6	7.51	0.00	2.24	9.84 (6.85)	—

that our results for the curvature energy using different Skyrme forces are in a strong agreement with the corresponding values obtained by the restricted variational method of Ref. 3.

Table IV shows the coefficients of the higher order terms in the LDM energy expansion. It is noted that the coefficient of the E_0 term has a slightly greater value than the curvature energy coefficient E_{C} (but with a negative sign). This result was found in the restricted variational calculations of reference [17] and also deduced by Myers and Swiatecki [13] in their Thomas–Fermi model to overcome the curvature energy puzzle. As seen in Table IV the energy coefficient E_{-1} has a small (negative) value and can be neglected to stop the energy expansion at the E_0 term.

TABLE IV

The coefficients of the higher order terms in the LDM energy expansion.

Force	E_0	E_{-1}
SKM*	−12.969	−1.11
SKa	−12.811	−2.44
SIII	−10.217	−1.31
RATP	−13.418	−2.17
SLY4	−15.877	−1.92
SLY7	−10.472	−1.53
SLI	−7.228	−0.64
SGII	−11.125	−1.15
SKSC4	−11.386	−0.30
SKSCo	−11.383	−0.29
SKSC14	−11.60	−0.30
SKSC15	−11.451	−0.29
SKp	−14.926	−0.15
T6	−11.588	−0.21

This result is very clear in Table V, where the value of each term in the energy expansion is calculated at some representative values of the mass number (A) using different Skyrme forces. The energy coefficient E_{-1} can be neglected and the energy expansion stops at the term E_0. The energy coefficient E_0 can be neglected for mass number $A \geq 200$.

Table VI shows the energy per nucleon using the different Skyrme forces at different values of the mass number (A). We compared our results with the Thomas–Fermi model of Myers and Swiatecki $E_{\mathrm{M-S}}$[13]. We see from

TABLE V

Values of the different terms in the energy LDM expansion for the representative mass number (A) and different Skyrme forces.

Force (A)	E_{V}	$E_{\mathrm{S}}A^{-\frac{1}{3}}$	$E_{\mathrm{C}}A^{-\frac{2}{3}}$	E_0A^{-1}	$E_{-1}A^{-\frac{4}{3}}$	E
SKM* (40)	−15.776	5.25	0.81	−0.32	−0.01	−10.04
(120)	−15.776	3.64	0.39	−0.11	0.00	−11.85
(200)	−15.776	3.07	0.28	−0.06	0.00	−12.49
(280)	−15.776	2.74	0.22	−0.05	0.00	−12.85
SLY7 (40)	−15.896	5.19	0.64	−0.26	−0.01	−10.33
(120)	−15.896	3.60	0.31	−0.09	0.00	−12.07
(200)	−15.896	3.04	0.22	−0.05	0.00	−12.69
(280)	−15.896	2.71	0.18	−0.04	0.00	−13.04
SKSC4 (40)	−15.859	5.42	0.56	−0.28	0.00	−10.17
(120)	−15.859	3.76	0.27	−0.09	0.00	−11.93
(200)	−15.859	3.17	0.19	−0.06	0.00	−12.56
(280)	−15.859	2.83	0.14	−0.04	0.00	−12.92

Table VI that our calculated values for the energy per nucleon using different Skyrme forces are consistent with each other. A strong agreement is noticed for the forces SKM*, SLY7 and RATP with that of Myers and Swiatecki [13].

We used the obtained analytical formulae for the energy coefficients to calculate the values of the compressibility coefficients for the most used Skyrme forces.

Table VII shows the contribution of the bulk, effective mass and the kinetic compressibility terms to the surface compressibility. The contribution of the bulk and the effective mass compressibility is negative and gives a negative value for the potential compressibility. The contribution of the kinetic term is a small positive and the surface compressibility has a negative value. The ratio $\mid K_{\mathrm{S}}/K_{\mathrm{V}} \mid$ is also tabulated and it can be seen that the scaling model result $K_{\mathrm{S}} \simeq -K_{\mathrm{V}}$ is well satisfied for all Skyrme forces except for SIII interaction.

TABLE VI

The binding energy as a function of the mass number (A) for different Skyrme forces compared with that of Myers and Swiatecki $E_{\mathrm{M-S}}$ [13].

A	SKM*	SKa	SIII	RATP	SLY7	SGII	SKp	$E_{\mathrm{M-S}}$
20	−8.54	−8.42	−8.58	−8.48	−8.88	−9.0	−8.66	−8.57
40	−10.00	−10.0	−10.1	−10.1	−10.3	−10.4	−10.1	−10.1
60	−10.80	−10.8	−10.8	−10.9	−11.0	−11.0	−10.8	−10.9
80	−11.30	−11.3	−11.3	−11.4	−11.5	−11.4	−11.3	−11.4
100	−11.60	−11.7	−11.6	−11.7	−11.8	−11.7	−11.6	−11.8
120	−11.90	−11.9	−11.9	−12.0	−12.1	−12.0	−11.9	−12.0
140	−12.10	−12.1	−12.1	−12.2	−12.3	−12.2	−12.1	−12.2
160	−12.20	−12.3	−12.3	−12.4	−12.4	−12.3	−12.3	−12.4
180	−12.40	−12.5	−12.4	−12.5	−12.5	−12.4	−12.4	−12.5
200	−12.50	−12.6	−12.5	−12.6	−12.6	−12.5	−12.5	−12.7
220	−12.60	−12.7	−12.6	−12.7	−12.7	−12.6	−12.6	−12.8
240	−12.70	−12.8	−12.7	−12.8	−12.8	−12.7	−12.7	−12.9
260	−12.80	−12.9	−12.8	−12.9	−12.9	−12.8	−12.8	−13.0
280	−12.90	−13.0	−12.9	−13.0	−13.0	−12.9	−12.9	−13.0

TABLE VII

The contribution of the bulk, effective mass and kinetic surface compressibility terms together with the total surface compressibility. Also, the absolute value of $K_{\mathrm{S}}/K_{\mathrm{V}}$ is shown.

Force	$K_{\mathrm{S}}^{\mathrm{b}}$	$K_{\mathrm{S}}^{\mathrm{eff}}$	K_{S}^{k}	K_{S} $\mid K_{\mathrm{S}}/K_{\mathrm{V}} \mid$
SKM*	−241.6	−44.1	54.0	−231.7 (1.07)
SKa	−272.5	−81.8	72.2	−282.4 (1.07)
SIII	−476.3	−36.9	43.0	−470.2 (1.32)
RATP	−251.6	−78.0	60.0	−269.9 (1.13)
SLY4	−240.5	−68.5	57.0	−252.0 (1.10)
SLY7	−203.6	−56.2	50.0	−209.8 (0.91)
SKI	−461.9	−10.1	40.0	−431.9 (1.17)
SGII	−209.6	−37.2	48.0	−198.8 (0.93)
SKSC4	−284.1	0.0	48.0	−236.1 (1.01)
SKSC4o	−284.0	0.0	48.0	−236.0 (1.01)
SKSC14	−285.0	0.0	48.0	−237.0 (1.01)
SKSC15	−284.3	0.0	48.0	−236.3 (1.01)
SKp	−237.6	0.0	51.0	−186.6 (0.93)
T6	−291.7	0.0	49.0	−242.7 (1.03)

Table VIII shows the contribution of the bulk, effective mass and the kinetic energy terms to the curvature compressibility. The curvature compressibility has a positive value ($K_{\rm C} \approx 0.85\ K_{\rm V}$) for all Skyrme forces used in our calculations. The value of the curvature compressibility, as found in literature, is somewhat uncertain both in magnitude and in sign [28–31]. While, Treiner *et al.*, [28] found $K_{\rm C}$ to be about $+345$ MeV for SIII force using ETF model, Nayak *et al.*, [29] found within the scaling model $K_{\rm C}$ to be negative and about -150 MeV for the same force, which they assigned to the use of the fourth order correction in the ETF kinetic energy formula. Recently, Satpathy *et al.*, [17] used the ETF kinetic energy formula including only the second order correction and found that $K_{\rm C}$ to be negative for SkM* (-110 MeV), SKA (-124 MeV) and SIII (-114 MeV). In our model calculations we found that the contribution of the effective mass term to the curvature compressibility is zero for all Skyrme forces. The contribution of the bulk term is small compared to that of the kinetic term. The kinetic term has the main contribution to the curvature compressibility. For the kinetic curvature compressibility the Thomas–Fermi term is found to be zero and only the second and the fourth order corrections of the ETF formula are active. The most important terms concerned the curvature compressibility are the gradient and the Laplacian terms of the ETF kinetic formula.

TABLE VIII

The same as in Table VII but for the curvature compressibility.

Force	$K_{\rm C}^{\rm b}$	$K_{\rm C}^{\rm eff}$	$K_{\rm C}^{k}$	$K_{\rm C}\ \lvert K_{\rm C}/K_{\rm V}\rvert$
SKM*	−18.9	0.0	231	212 (0.98)
SKa	2.1	0.0	194	196.1 (0.75)
SIII	53.9	0.0	201	254.9 (0.72)
RATP	−7.9	0.0	222	214.1 (0.89)
SLY4	−20.3	0.0	201	180.7 (0.79)
SLY7	−12.6	0.0	202	189.4 (0.82)
SKI	48.8	0.0	212	260.8 (0.70)
SGII	−19.7	0.0	200	180 (0.84)
SKSC4	−16.3	0.0	223	206.7 (0.88)
SKSC4o	−16.3	0.0	223	206.7 (0.88)
SKSC14	−16.4	0.0	223	206.6 (0.88)
SKSC15	−16.3	0.0	223	206.7 (0.88)
SKp	−29.1	0.0	221	181.9 (0.91)
T6	−15.2	0.0	231	215.8 (0.92)

Table IX shows the higher order terms of the compressibility LDM expansion. The coefficient of the A^0 term (K_0) is negative for all Skyrme forces and the coefficient of the $A^{-1/3}$ term (K_{-1}) is positive. The compressibility of symmetric spherical nuclei as a function of the mass number (A) is shown in Table X where we compared our results with that of the TF model of Myers and Swiatecki $K_{\mathrm{M-S}}$[32] and a good agreement is noticed for Skm*, SLy4 and SGII forces.

TABLE IX

The higher order terms of the compressibility LDM expansion.

Force	K_0	K_{-1}
SKM*	−219.2	165.2
SKa	−385.7	204.9
SIII	−71.9	36.0
RATP	−319.2	180.1
SLY4	−345.0	148.1
SLY7	−207.2	72.2
SKI	−12.8	13.5
SGII	−187.89	10.4
SKSC4	−81.1	92.1
SKSC4o	−81.0	92.1
SKSC14	−81.3	92.4
SKSC15	−81.1	92.2
SKp	−129.5	115.5
T6	−83.8	109.4

Table XI shows the values of the compressibility coefficients in the LDM expansion for some representative mass number A and different Skyrme forces. The compressibility coefficient K_{-1} can be neglected and the LDM expansion for compressibility stops at the term K_0. The compressibility coefficient K_0 can be neglected for mass number $A \geq 200$. Table XI shows also that the LDM expansion of the nuclear compressibility is rapidly converging. Recently, Satpathy *et al.*, [17] found that this expansion shows anomalous behavior. Their result may be due to using only the second order correction ETF kinetic energy formula in their calculations. The nuclear compressibility is very sensitive to the gradient and Laplacian terms of the fourth order ETF kinetic energy formula. In summary, we used the restricted variational method to solve the energy equation of the ETF model including fourth order

TABLE X

The nuclear compressibility as function of the mass number for the different Skyrme forces compared with that of Myers and Swiatecki $K_{\mathrm{M-S}}$ [32].

A	SKM*	SKa	SIII	RATP	SLY7	SGII	SKp	$K_{\mathrm{M-S}}$
20	152	170	214	157	169	158	152	125
40	163	189	238	173	180	168	159	148
60	168	198	251	180	185	173	163	159
80	172	204	259	185	189	176	166	166
100	175	208	265	189	192	179	168	171
120	177	211	270	192	194	181	170	175
140	178	213	274	194	194	182	171	178
160	180	215	277	196	196	184	172	181
180	181	217	280	197	198	185	172	183
200	182	219	282	198	199	186	164	185
220	183	220	284	200	199	186	175	186
240	184	221	286	201	200	187	175	188
260	185	222	288	202	200	187	176	189
280	185	223	289	202	201	188	176	190

TABLE XI

The values of the different terms in the compressibility LDM expansion for the representative mass number (A) and different Skyrme forces.

Force (A)	K_{V}	$K_{\mathrm{S}}A^{-\frac{1}{3}}$	$K_{\mathrm{C}}A^{-\frac{2}{3}}$	K_0A^{-1}	$K_{-1}A^{-\frac{4}{3}}$	K
SKM* (40)	216.7	−67.79	18.13	−5.47	1.21	162.7
(120)	216.7	−46.93	8.72	−1.82	0.28	176.7
(200)	216.7	−39.58	6.20	−1.09	0.14	182.2
(280)	216.7	−35.38	4.96	−0.78	0.09	185.4
SLY4 (40)	229.9	−73.7	15.45	−8.62	1.07	164.1
(120)	229.9	−51.1	7.43	−2.87	0.25	183.6
(200)	229.9	−43.1	5.28	−1.72	0.12	190.5
(280)	229.9	−38.53	4.22	−1.23	0-0.8	194.4
SKSC4 (40)	234.6	−69.05	17.67	−2.03	0.67	181.9
(120)	234.6	−47.87	8.50	−0.68	0.16	194.7
(200)	234.6	−40.38	6.04	−0.41	0.08	199.9
(280)	234.6	−36.09	4.83	−0.29	0.05	203.1

gradient corrections. We got analytical equations for the energy coefficients in the LDM energy expansion. The LDM energy expansion stopped at the fourth term and gave good convergence behavior. The value of the higher order energy coefficient E_0 was in a good agreement with the empirical value proposed by Myers and Swiatecki in their Thomas Fermi model to overcome the curvature energy puzzle. Analytical expressions were also derived for the compressibility coefficients of the LDM compressibility expansion. The LDM compressibility expansion was found to have good converging behavior. The energy and compressibility expansion coefficients were useful in comparing the leptodermous contributions of the individual terms of a large variety of Skyrme forces on the market for the same reference density.

Appendix

In this appendix we give the explicit form of the functions of the coefficients $An(m)$ appears in the energy LDM.

$$
\begin{aligned}
B_{\mathrm{s}}(n) &= A_1(nq) - A_1(q)\,,\\
B_{\mathrm{c}}(n) &= A_2(nq) - A_2(q) - A_1(q)B_{\mathrm{s}}(n)\,,\\
B_0(n) &= \frac{1}{3}\{7A_1^3(q) - 12A_1(q)A_2(q) - 9A_1^2(q)\\
&\quad + 12A_2(q)A_1(nq) + 6A_1(q)A_2(nq) - 6A_3(nq)\}\,,\\
B_1(n) &= A_1^2(q)\{5A_1^2(q) - 14A_2(q) + 6A_2(nq) - 6A_2(nq)\\
&\quad + A_2(q)[8A_2(q) + 12A_1(q)A_1(nq) - 12A_2(nq)]\}\,,\\
B_{d0}(n) &= A_2(q+1) - A_2(q) - A_1(q)[A_1(q+1) - A_1(q)]\,,\\
B_{d1}(n) &= qA_1(q) - (2q+1)A_1(q+1) + (q+1)A_1(q+2)\,,\\
B_{d2}(n) &= qA_2(q) - (2q+1)A_2(q+1) + (q+1)A_2(q+2)\,,\\
B_{d3}(n) &= qA_3(q) - (2q+1)A_3(q+1) + (q+1)A_3(q+2)\,,\\
B_{\mathrm{c}} &= B_{d2} - B_{d1}A_1(q)\,,\\
B_0 &= B_{d1}[4A_1(q) - 3A_1^2(q)] + 2B_{d2}A_1(q) - 2B_{d3}\,,\\
B_{-1} &= B_{\mathrm{c}}[A_1^2(q) - 2A_2(q)]\,,\\
F_i(n) &= A_i(nq+2) - 2A_i(2q+1) + A_i(q)\,,\\
G_i(n) &= A_i(2q+nq) - A_i(2q+nq+1) + A_i(2q+nq+2)\,,
\end{aligned}
$$

where i runs over 1, 2 and 3.

$$
\begin{aligned}
X(n) &= F_2(n) - F_1(n)A_1(q)\,, \\
Y(n) &= F_1(n)[4A_2(q) - 3A_1^2(q)] + 2F_2(n)A_1(q) - 2F_3(n)\,, \\
Z(n) &= X(n)[A_1^2(q) - 2A_2(q)]\,, \\
B(n) &= G_2(n) - A_1(q)G_1(n)\,, \\
C(n) &= G_1(n)[4A_2(q) - 3A_1^2(q)] + 2G_2(n)A_1(q) - 2G_3(n)\,, \\
D(n) &= B(n)[A_1^2(q) - 2A_2(q)]\,,
\end{aligned}
$$

$$
\begin{aligned}
H_1 &= A_1\left(\frac{q}{3}\right) - 4A_1\left(\frac{q}{3}+1\right) + 6A_1\left(\frac{q}{3}+2\right) - 4A_1\left(\frac{q}{3}+3\right) + A_1\left(\frac{q}{3}+4\right)\,, \\
H_2 &= A_2\left(\frac{q}{3}\right) - 4A_2\left(\frac{q}{3}+1\right) + 6A_2\left(\frac{q}{3}+2\right) - 4A_2\left(\frac{q}{3}+3\right) + A_2\left(\frac{q}{3}+4\right)\,, \\
R_1 &= A_1\left(\frac{4q}{3}\right) - 4A_1\left(\frac{4q}{3}+1\right) + 6A_1\left(\frac{4q}{3}+2\right) - 4A_1\left(\frac{4q}{3}+3\right) + A_1\left(\frac{4q}{3}+4\right)\,, \\
R_2 &= A_2\left(\frac{4q}{3}\right) - 4A_2\left(\frac{4q}{3}+1\right) + 6A_2\left(\frac{4q}{3}+2\right) - 4A_2\left(\frac{4q}{3}+3\right) + A_2\left(\frac{4q}{3}+4\right)\,, \\
Q_1 &= qA_1\left(\frac{q}{3}\right) - (4q+1)A_1\left(\frac{q}{3}+1\right) + (6q+3)A_1\left(\frac{q}{3}+2\right)\,, \\
Q_2 &= -(4q+3)A_2\left(\frac{q}{3}+3\right) + (q+1)A_2\left(\frac{q}{3}+4\right) \\
&\quad + qA_2\left(\frac{q}{3}\right) - (4q+1)A_2\left(\frac{q}{3}+1\right) + (6q+3)A_2\left(\frac{q}{3}+2\right)\,, \\
S_1 &= qA_1\left(\frac{4q}{3}\right) - (4q+1)A_1\left(\frac{4q}{3}+1\right) + (6q+3)A_1\left(\frac{4q}{3}+2\right) \\
&\quad - (4q+3)A_1\left(\frac{4q}{3}+3\right) + (q+1)A_1\left(\frac{4q}{3}+4\right)\,, \\
S2 &= qA_2\left(\frac{4q}{3}\right) - (4q+1)A_2\left(\frac{4q}{3}+1\right) + (6q+3)A_2\left(\frac{4q}{3}+2\right) \\
&\quad - (4q+3)A_2\left(\frac{4q}{3}+3\right) + (q+1)A_2\left(\frac{4q}{3}+4\right)\,, \\
R_1 &= A_1\left(\frac{4q}{3}\right) - 4A_1\left(\frac{4q}{3}+1\right) + 6A_1\left(\frac{4q}{3}+2\right) - 4A_1\left(\frac{4q}{3}+3\right) + qA_1\left(\frac{4q}{3}+4\right), \\
R_2 &= A_2\left(\frac{4q}{3}\right) - 4A_2\left(\frac{4q}{3}+1\right) + 6A_2\left(\frac{4q}{3}+2\right) - 4A_2\left(\frac{4q}{3}+3\right) + qA_2\left(\frac{4q}{3}+4\right), \\
P_1 &= q^2A_1\left(\frac{q}{3}\right) - 2q(2q+1)A_1\left(\frac{q}{3}+1\right) + (6q^2+6q+1)A_1\left(\frac{q}{3}+2\right) \\
&\quad - (2q+1)(2q+1)A_1\left(\frac{q}{3}+3\right) + (q+1)^2A_1\left(\frac{q}{3}+4\right)\,, \\
P_2 &= q^2A_2\left(\frac{q}{3}\right) - 2q(2q+1)A_2\left(\frac{q}{3}+1\right) + (6q^2+6q+1)A_2\left(\frac{q}{3}+2\right) \\
&\quad - (2q+1)(2q+1)A_2\left(\frac{q}{3}+3\right) + (q+1)^2A_2\left(\frac{q}{3}+4\right)\,,
\end{aligned}
$$

$$
\begin{aligned}
T_1 &= q^2 A_1\left(\frac{4q}{3}\right) - 2q(2q+1)A_1\left(\frac{4q}{3}+1\right) + (6q^2+6q+1)A_1\left(\frac{4q}{3}+2\right) \\
&\quad - (2q+1)(2q+1)A_1\left(\frac{4q}{3}+3\right) + (q+1)^2 A_1\left(\frac{4q}{3}+4\right), \\
T_2 &= q^2 A_2\left(\frac{4q}{3}\right) - 2q(2q+1)A_2\left(\frac{4q}{3}+1\right) + (6q^2+6q+1)A_2\left(\frac{4q}{3}+2\right) \\
&\quad - (2q+1)(2q+1)A_2\left(\frac{4q}{3}+3\right) + (q+1)^2 A_2\left(\frac{4q}{3}+4\right),
\end{aligned}
$$

$$
\begin{aligned}
U_1 &= -2B_{d3} + 2B_{d2}A_1(q) + B_{d1}\left[A_2(q) - 3A_1^2(q)\right], \\
U_2 &= A_2(q+1) - A_2(q) - A_1(q)\left[A_1(q+1) - A_1(q)\right].
\end{aligned}
$$

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