HYPOTHETICAL FIRST ORDER $|\Delta S| = 2$ TRANSITIONS IN THE $K^0 - \overline{K^0}$ SYSTEM

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The influence of a hypothetical CP violating $|\Delta S| = 2$ interaction on the masses and lifetimes of neutral mesons K^0 and $\overline{K^0}$ is investigated. It is shown, that the assumption of the existence of this superweak interaction does not significantly affect these parameters if phenomenological constraints based on recent experiments are imposed. To establish this result we use a computer simulation of a parameter corresponding to the difference between the diagonal elements of the effective Hamiltonian governing the time evolution in the $K^0 - \overline{K^0}$ system. Instead of the widely used Lee, Oehme and Yang approximation which is insensitive to the $|\Delta S| = 2$ interaction we use a formalism based on the Królikowski–Rzewuski equation.

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1. Introduction

Until recently there were two types of models of interactions which were considered plausible while investigating the source of the CP violation [1]. The first of them is the class of miliweak models which assume that a part of order $10^{-3}G_{\mathrm{F}}$ in the weak interaction is responsible for the observed CP violation effects. One of the most important predictions of this class of models is that the CP violation should also be observed in other than $K^0 \rightleftarrows \overline{K^0}$ processes, and that it should be of the same order. The CKM model is an example of such miliweak models, at the same time being the most successful one. The recent experimental results concerning the measurement of ϵ'/ϵ and CP violation in the neutral B-meson system show, that the CKM model correctly describes CP violation. There is, however, a small possibility that a superweak-like interaction (the terms "superweak" and "superweak-like" will be used interchangeably) does exist in addition to the CKM mechanism, and some authors consider its implications (see [2] and references therein).

One of the most important parameters of the $K^0 - \overline{K^0}$ system is the difference between the diagonal elements of the effective Hamiltonian governing the time evolution. The standard approximation put forward by Lee, Oehme and Yang (the LOY approximation) [3] leads to the conclusion that the above difference is exactly zero. The real parts of the diagonal elements are interpreted as the masses of the particles and the imaginary parts are the decay widths. Consequently, in the LOY approximation the masses and the decay widths of K^0 and $\overline{K^0}$ are equal. This result is corroborated by the experimental result $|(m_{K_0} - m_{\bar{K}_0})/m_{K_0}| \leq 10^{-18}$ [4]. However, in the course of derivation of the LOY approximation the elements which correspond to the hypothetical superweak interaction are neglected [3, 5]. In [6–10] it was shown with the use of a method based on the Królikowski–Rzewuski equation [11, 12], that if such elements are present the masses of the kaon and anti-kaon need not be equal. It is, therefore, interesting to see what effect such a superweak interaction might have on the masses of neutral kaons if the limitations following from the most recent experiments are taken into account. The result of this calculation may be used to test the accuracy of the LOY approximation.

The paper is organised as follows: In the second section we review the most important (for our purposes) features of the Standard Model and the Superweak Model and their present experimental status. The third section reviews the standard phenomenological approach to the neutral kaon system which is based on the Weisskopf–Wigner approach. Also, basing on [6–10], we review the alternative formalism and analyse its relevance to the superweak interaction. The fourth section contains a computer simulation of the time dependence of the parameter describing the difference between the diagonal elements in the alternative model, which in the presence of the superweak interaction turns out to be different from zero. In this section, by imposing the phenomenological constraints following from the most recent experiments, we find the upper bound on the difference between the masses of the K^0 and $\overline{K^0}$ mesons. This bound turns out to be extremely small, which shows that the LOY approximation is very good, even in the presence of the superweak interaction. The summary and conclusions are contained in the last section.

2. $K^0 \rightleftarrows \overline{K^0}$ mixing in the Standard Model and the Superweak Model

In this section we review the Standard Model approach to the $K^0 \rightleftarrows \overline{K^0}$ system. We also briefly describe the salient features of the superweak scenario of CP violation.

2.1. $K^0 \rightleftarrows \overline{K^0}$ mixing and CP violation in the Standard Model

The flavour transitions allowed in the Standard Model are specified by the CKM matrix, which allows the following flavour mixing

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{2.1}$$

Consequently, in the CKM theory there are no direct, first order $K^0 \rightleftarrows \overline{K^0}$ transitions. In other words there are no first order $|\Delta S| = 2$ transitions, or, in yet another equivalent formulation, which we will be using in the remaining part of the paper

$$\langle \mathbf{1}|H^{(1)}|\mathbf{2}\rangle = 0 \,, \tag{2.2}$$

where $|K_0\rangle \equiv |\mathbf{1}\rangle$ and $|\overline{K}_0\rangle \equiv |\mathbf{2}\rangle$ and $H^{(1)}$ is the flavour-changing part of the weak Hamiltonian [5].

Matrix (2.1) is unitary and contains 9 parameters. Three of these parameters may be chosen to be real angles θ_{12}, θ_{13}, θ_{23} and the remaining six are phases. The number of phases can be reduced by using the fact, that spinors are defined up to a phase, so we may redefine the quark eigenstates. After doing this we notice, that in the procedure there are only five independent phase differences, whereas there are six phases in (2.1), so there is one physically meaningful phase in this unitary matrix. This is the crucial point of the CKM theory because this phase allows for CP violation [1].

2.2. The hypothetical superweak interaction

The Superweak Model postulates the existence of a new $|\Delta S| = 2$, CP violating interaction. The coupling constant of this interaction should be smaller than second order weak interaction. Thus, the Superweak Model assumes a non-vanishing first order transition matrix element

$$\langle \mathbf{1}|H^{(1)}|\mathbf{2}\rangle \sim gG_{\mathrm{F}} \neq 0 \,, \tag{2.1}$$

where g is the superweak coupling constant. It is widely accepted that this interaction can only be detected in the $K_{\mathrm{L}} - K_{\mathrm{S}}$ system, because it is the only known pair of states with the energy difference so small, that it is sensitive to interactions weaker than second order weak interaction [1].

2.3. The status of the Standard Model and the Superweak Model

The recent experimental results from the CPLEAR and KTeV Collaborations and others have given the decisive answer to the question whether the CP violation effects are correctly described by the CKM miliweak theory.

The measured value of $\epsilon'/\epsilon = (1.72 \pm 0.18) \times 10^{-3}$ [13] proves that there is a direct CP violating effect, and that CP violation cannot only be ascribed to mass mixing in the $K^0 \rightleftarrows \overline{K^0}$ process. On the contrary: the CKM mechanism must be the dominant source of CP violation (in low-energy flavour-changing processes) [13]. Additionally, the measured value is perfectly consistent with the world average for the value ϵ'/ϵ [14]. Another experimental argument for the miliweak CKM theory are the two recent measurements of CP violation in B decays [13] (and references therein). In other words, the Standard Model alone is able to correctly predict the value of ϵ'/ϵ and no improvements or extensions are in fact necessary.

However, even if the CP violation effects are correctly described by the CKM mechanism the idea of a $|\Delta S| = 2$ interaction has not been abandoned entirely. Indeed, some authors consider the implications of such an interaction. For example the question of the existence of the superweak interaction turns out to be of some importance in tagged experiments in which flavour is determined for the initial meson and then for the meson at the time of decay. The existence of the $|\Delta S| = 2$ superweak interaction might cause the production of "wrong" neutral meson states [2]. The effect of such a hypothetical interaction is, however, believed to be negligibly small.

3. The standard phenomenological description of the $K^0 - \overline{K^0}$ system

In this section we briefly describe the phenomenology which is currently used to describe the time evolution of the $K^0 - \overline{K^0}$ system.

3.1. The Lee, Oehme and Yang approximation

This formalism is based on the formalism of particle mixture introduced by Gell-Mann and Pais [15]. The most important modification was introduced to this formalism by Lee, Oehme and Yang [3], who, using the Weisskopf–Wigner approximation arrived at the widely known formula (3.5) — see below. Further extensions were introduced by many other authors, *e.g.* Bell and Steinberger [16].

In the standard approach the full Hamiltonian is divided into two parts

$$H = H^{(0)} + H^{(1)}\,, \tag{3.1}$$

where $H^{(0)}$ is the flavour-conserving part of the Hamiltonian, and $H^{(1)}$ is the flavour-changing part. The complete state vector which has evolved from $|1\rangle$ or $|2\rangle$ is projected onto the subspace spanned by $|1\rangle$ and $|2\rangle$. Therefore, we define this projected state vector as

$$|\Psi;t\rangle_{||} = \alpha_1(t)|1\rangle + \alpha_2(t)|2\rangle\,. \tag{3.2}$$

Lee, Oehme and Yang, by modifying the Weisskopf–Wigner method for the single line, showed that the time dependence of the vector $\begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix}$ can be described by the following Schrödinger-like equation

$$i\frac{d}{dt}\begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix} = \begin{pmatrix} H_{11}^{\rm LOY} & H_{12}^{\rm LOY} \\ H_{21}^{\rm LOY} & H_{22}^{\rm LOY} \end{pmatrix}\begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix}, \qquad (3.3)$$

where we have adopted $\hbar = c = 1$. The operator on the right hand side of the equation is the LOY effective Hamiltonian, and its matrix elements are matrix elements of the weak interaction transition operator. In the case of CPT conserved it can be shown, that for this effective Hamiltonian we have $H_{11}^{\rm LOY} = H_{22}^{\rm LOY}$ [5].

The effective Hamiltonian can be split into two parts, each of them with a definite physical meaning

$$H^{\rm LOY} = M - i\frac{\Gamma}{2}, \qquad (3.4)$$

or

$$i\frac{d}{dt}\begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix} = \begin{pmatrix} M_{11} - i\dfrac{\Gamma_{11}}{2} & M_{12} - i\dfrac{\Gamma_{12}}{2} \\ M_{21} - i\dfrac{\Gamma_{21}}{2} & M_{22} - i\dfrac{\Gamma_{22}}{2} \end{pmatrix}\begin{pmatrix} \alpha_1(t) \\ \alpha_2(t) \end{pmatrix}. \qquad (3.5)$$

For our purpose, which is the analysis of the influence of the hypothetical $|\Delta S| = 2$ interaction on the time evolution of the $K^0 - \overline{K^0}$ system, the LOY method is not suitable. Indeed, in [9, 17] it was shown, that the LOY formulae may only be correct if we assume $\langle \mathbf{1}|H^{(1)}|\mathbf{2}\rangle = 0$ and take $t \to \infty$. This obviously excludes the possibility of using the Lee, Oehme and Yang approximation in studying the hypothetical superweak interaction

3.2. The alternative approach

One alternative to the approach described above is the formalism developed in [6–9]. We will briefly review this approximation and its basic findings.

The starting point of the derivation of the alternative effective Hamiltonian carried out in [9–11] is the Królikowski–Rzewuski equation [11, 12]. In this approach the time evolution is not studied in the total space of states $\mathcal{H}$ but rather in a closed subspace $\mathcal{H}_{||} \subset \mathcal{H}$. If we define the following projector

$$P \stackrel{\rm def}{=} |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}|,$$

then the subspace $\mathcal{H}_{||}$ may be defined as $\mathcal{H}_{||} = P\mathcal{H}$ or $|\psi;t\rangle_{||} = P|\psi;t\rangle \in \mathcal{H}_{||}$. In this way the total state space is split into two orthogonal subspaces $\mathcal{H}_{||}$ and $\mathcal{H}_{\perp} = \mathcal{H} \ominus \mathcal{H}_{||}$, and the Shrödinger equation can be replaced by equations describing each of the subspaces, respectively. The equation for $\mathcal{H}_{||}$ has the following form [9–12]

$$\left(i\frac{\partial}{\partial t} - PHP\right)|\psi;t\rangle_{||} = |\chi;t\rangle - i\int_0^\infty K(t-\tau)|\psi;\tau\rangle_{||}d\tau\,, \tag{3.6}$$

$$Q = I - P\,, \tag{3.7}$$

$$K(t) = \Theta(t)PHQe^{-itQHQ}QHP\,, \tag{3.8}$$

$$|\chi;t\rangle = PHQe^{-itQHQ}|\psi\rangle_{\perp}\,, \tag{3.9}$$

where

$$\Theta(t) = \begin{cases} 1 & \text{for} \quad t \geq 0 \\ 0 & \text{for} \quad t < 0 \end{cases}\,,$$

$$|\psi\rangle_{\perp} = Q|\psi;0\rangle\,.$$

The initial conditions for this problem are

$$|\psi;0\rangle = |\psi;0\rangle_{||}\,, \tag{3.10}$$

which means

$$|\psi\rangle_{\perp} \equiv 0\,.$$

Following [11, 12] we introduce an effective Hamiltonian

$$H_{||}(t) \equiv PHP + V_{||}(t)\,. \tag{3.11}$$

This formula corresponds to (3.4), which also specifies an effective Hamiltonian.

The main difference between the standard Lee, Oehme and Yang approximation and this approach is the effective potential. It can be shown [9, 10] that

$$V_{||}(t) \simeq V_{||}^1(t) = -i\int_0^\infty K(t-\tau)e^{i(t-\tau)PHP}Pd\tau\,. \tag{3.12}$$

To establish notation let us now define the following symbols

$$PHP \equiv \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\,, \tag{3.13}$$

$$
\begin{aligned}
H_{ij} &\equiv \langle \boldsymbol{j}|H|\boldsymbol{i}\rangle\,, \\
H_0 &\equiv \frac{1}{2}(H_{11} + H_{22})\,, \\
\kappa &\equiv \sqrt{|H_{12}|^2 + \frac{1}{4}(H_{11} - H_{22})}\,, \\
H_z &\equiv \frac{1}{2}(H_{11} - H_{22})\,.
\end{aligned}
$$

The matrix elements $v_{ij}(t) = \langle \boldsymbol{j}|V_{||}(t)|\boldsymbol{i}\rangle$ of $V_{||}(t)$ (3.12), without assuming any symmetries, like $[CP, H] = 0$ or $[CPT, H] = 0$ [9, 10] are

$$
\begin{aligned}
v_{j1}(t) = &- \frac{1}{2}\Big(1 + \frac{H_z}{\kappa}\Big)\Xi_{j1}(H_0 + \kappa, t) \\
&- \frac{1}{2}\Big(1 - \frac{H_z}{\kappa}\Big)\Xi_{j1}(H_0 - \kappa, t) \\
&- \frac{H_{21}}{2\kappa}\Xi_{j2}(H_0 + \kappa, t) + \frac{H_{21}}{2\kappa}\Xi_{j2}(H_0 - \kappa, t)\,, \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
v_{j2}(t) = &- \frac{1}{2}\Big(1 - \frac{H_z}{\kappa}\Big)\Xi_{j2}(H_0 + \kappa, t) \\
&- \frac{1}{2}\Big(1 + \frac{H_z}{\kappa}\Big)\Xi_{j2}(H_0 - \kappa, t) \\
&- \frac{H_{12}}{2\kappa}\Xi_{j1}(H_0 + \kappa, t) + \frac{H_{12}}{2\kappa}\Xi_{j1}(H_0 - \kappa, t)\,, \qquad (3.15)
\end{aligned}
$$

where

$$
\Xi(\lambda, t) \stackrel{\mathrm{def}}{=} PHQ\ \frac{e^{-it(QHQ-\lambda)} - 1}{QHQ - \lambda}\ QHP\,, \qquad (3.16)
$$

and $\Xi_{jk}(\varepsilon, t) = \langle \boldsymbol{j} \mid \Xi(\varepsilon, t) \mid \boldsymbol{k}\rangle$, $j, k = 1, 2$. This effective potential, together with the remaining parts of the effective Hamiltonian yields the following matrix elements for the effective Hamiltonian

$$
h_{jk}(t) = \langle \boldsymbol{j}|H_{||}|\boldsymbol{k}\rangle = H_{jk} + v_{jk}(t)\,, \qquad (j, k = 1, 2)\,. \qquad (3.17)
$$

For the $[CPT, H] = 0$ case the formulae simplify as $H_z = 0$ in this case.

Now, it is easy to notice that, in the case of $[CPT, H] = 0$, contrary to the LOY effective Hamiltonian for which we have $H_{11}^{\mathrm{LOY}} - H_{22}^{\mathrm{LOY}} = 0$, the difference between the diagonal elements is non-vanishing

$$
h_z(t) = \frac{1}{2}(h_{11}(t) - h_{22}(t)) \neq 0\,, \quad (t > 0)\,. \qquad (3.18)
$$

It is also obvious that the necessary condition for (3.18) to be true is $H_{12} \neq 0$, that is, the existence of the superweak interaction.

4. Computer simulation of the time evolution of $h_z(t)$ within the Friedrichs–Lee model

In this section we perform a numerical simulation of the $h_z(t)$ parameter, which has proved so important in the present approach. By making some assumptions concerning the scale of the hypothetical superweak interaction we arrive at a form which is convenient for computer analysis. We analyse the time evolution of the module and the real and imaginary part of this parameter.

4.1. The Friedrichs–Lee model

In [9] the Friedrichs–Lee model [18] was used to obtain the following formulae for the matrix elements of the effective Hamiltonian with the $[CPT, H] = 0$ assumption

$$\begin{aligned} h_{j1}(t) \;=\; & m_{j1} - \frac{1}{2}\left(\varGamma_{j1} + \frac{m_{21}}{|m_{12}|}\varGamma_{j2}\right)\varPhi_0(t; m_0 + |m_{12}| - \mu) \\ & - \frac{1}{2}\left(\varGamma_{j1} - \frac{m_{21}}{|m_{12}|}\varGamma_{j2}\right)\varPhi_0(t; m_0 - |m_{12}| - \mu)\,, \end{aligned} \tag{4.1}$$

$$\begin{aligned} h_{j2}(t) \;=\; & m_{j2} - \frac{1}{2}\left(\varGamma_{j2} + \frac{m_{12}}{|m_{12}|}\varGamma_{j1}\right)\varPhi_0(t; m_0 + |m_{12}| - \mu) \\ & - \frac{1}{2}\left(\varGamma_{j2} - \frac{m_{12}}{|m_{12}|}\varGamma_{j1}\right)\varPhi_0(t; m_0 - |m_{12}| - \mu)\,. \end{aligned} \tag{4.2}$$

In these formulae $m_0 \equiv \langle \mathbf{1}|H^{(0)}|\mathbf{1}\rangle = \langle \mathbf{2}|H^{(0)}|\mathbf{2}\rangle$, compare Eq. (3.1), $m_{12} \equiv H_{12}$; $m_0 - \mu$ is the difference between the mass of the mesons considered and the threshold energy of the continuum state, like $K \to 2\pi$. Functions $\varPhi_0(t, m)$ are defined by

$$\varPhi_0(t, m) = F_0(m) - F_0(t, m)\,, \tag{4.3}$$

where

$$\begin{aligned} F_0(t, m) \;=\; & \frac{a}{\sqrt{m}}\left[S(\sqrt{mt}) - C(\sqrt{mt})\right] \\ & - i\frac{a}{\sqrt{m}}\left[C(\sqrt{mt}) + S(\sqrt{mt}) - 1\right]\,, \end{aligned} \tag{4.4}$$

$$F_0(m) \;=\; i\frac{a}{\sqrt{m}}\,, \qquad a = (m_{11} - \mu)^{\frac{1}{2}}\,, \tag{4.5}$$

and finally $S(y)$ and $C(y)$ are the sine and cosine Fresnel integrals

$$C(y) = \frac{1}{\sqrt{2}} \int_0^{y^2} \frac{\cos(\tau)}{\sqrt{\tau}} d\tau \,,$$

$$S(y) = \frac{1}{\sqrt{2}} \int_0^{y^2} \frac{\sin(\tau)}{\sqrt{\tau}} d\tau \,.$$

The parameters $\varGamma_{ij}$ correspond to the matrix elements of the decay matrix in the LOY approximation (3.5).

By setting $j = 1$ in (4.1) and $j = 2$ in (4.2) and then subtracting (4.2) from (4.1) we get

$$\begin{aligned} h_z(t) &= \frac{m_{21}\varGamma_{12} - m_{12}\varGamma_{21}}{4|m_{12}|} \\ &\times \Big[\varPhi_0(t; m_0 - |m_{12}| - \mu) - \varPhi_0(t; m_0 + |m_{12}| - \mu)\Big] . \end{aligned} \tag{4.6}$$

Let us introduce a new independent variable x

$$x \equiv x(t) = (m_0 - \mu)\; t \,. \tag{4.7}$$

If we define $\tau_{\mathrm{L}} = 5.2 \times 10^{-8} s$ as the mean lifetime of K_{L}, the value of x corresponding to this time is $x = 2.8 \times 10^{16}$. Now, using x we can rewrite $h_z(t)$ as

$$h_z(x) = \frac{m_{21}\varGamma_{12} - m_{12}\varGamma_{21}}{4|m_{12}|} r(x) \,, \tag{4.8}$$

where $r(x)$ has the following form (see Appendix A)

$$\begin{aligned} r(x) &= \sqrt{b_-}\,\{i - [S(\sqrt{x_-}) - C(\sqrt{x_-})] - i\,[C(\sqrt{x_-}) + S(\sqrt{x_-})]\} \\ &+ \sqrt{b_+}\,\{-i + [S(\sqrt{x_+}) - C(\sqrt{x_+})] - i\,[C(\sqrt{x_+}) + S(\sqrt{x_+})]\} \,, \end{aligned} \tag{4.9}$$

where

$$b_\pm = \left(1 \pm \frac{|m_{12}|}{m_0 - \mu}\right), \qquad x_\pm = b_\pm x.$$

This expression (4.9) is simple to analyse using computer methods as it contains no other variables but the independent variable x.

To extract any numerical information from (4.9) we need to make some assumptions concerning the strength of the superweak interaction. There are some estimates in the literature — we will accept the one suggested by Lee in [5] (equation 15.138, page 375) $|m_{12}|/(m_0 - \mu) \simeq 10^{-17}$. To be sure, we do not even know if the strength is different from zero, we are assuming a value of $|m_{12}|/(m_0 - \mu)$ which is consistent with the assumptions made in Sec. 2. to see how $h_z(t)$ changes with time.

4.2. Time dependence of $h_z(x(t))$

From (A.2), and (A.3)

$$r(x) = \frac{h_z(x(t))}{i\,|\Gamma_{12}|\,2\sin(\phi-\theta)}\,, \tag{4.10}$$

so we may analyse the following three parameters

$$|r(x)| \sim |h_z(x(t))|, \tag{4.11}$$
$$\Re(ir(x)) \sim \Re(h_z(x)), \tag{4.12}$$
$$\Im(ir(x)) \sim \Im(h_z(x)). \tag{4.13}$$

The computer analysis of the module (4.11) shows that for very small values of x it rapidly oscillates around the value of $|r(x)| \simeq 10^{-16}$, then it becomes basically constant. What are small values of x will become apparent from Fig 1. and the discussion of the imaginary part. The real part of $ir(x)$ is basically constant: $\Re(ir(x)) \simeq 10^{-16}$ for all $x > 0$ except for the immediate neighbourhood of 0, where $r(0) = 0$. The behaviour of the imaginary part of $ir(x)$ is shown in Fig 1. It oscillates rapidly about 0 for very small x, then the imaginary part quickly tends to zero. This, together with the behaviour of the real part of $ir(x)$ means that the oscillations in $|r(x)|$ should be attributed to the oscillations in the imaginary part.

In the standard approach the real parts of the diagonal elements of the effective Hamiltonian are interpreted as the masses of the particles. Therefore, it seems that the existence of the superweak interaction would remove

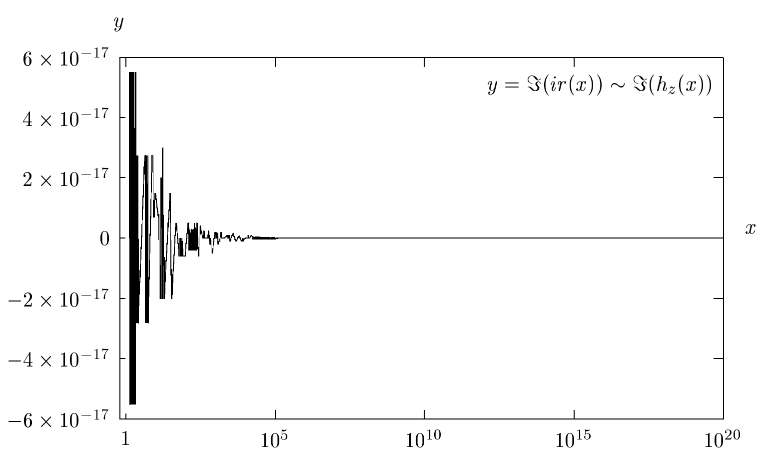

Fig. 1. The x dependence of $\Im(h_z(x))$ for $|m_{12}|/(m_0-\mu) = 10^{-17}$. The value of $x = 2.8 \times 10^{16}$ corresponds to τ_{L} — the lifetime of K_{L}. This is why the region of rapid oscillation is called "small x".

the mass degeneracy between the particle and antiparticle in the neutral kaon system. Correspondingly, the imaginary parts are interpreted as the decay constants, so in the model considered the decay widths of the particle and antiparticle should be equal, which is consistent with the standard result. These results are consistent with the conclusions reached earlier on the basis of the form of $h_z(t)$ for large times [9].

4.3. Order-of-magnitude estimation of the effect introduced by the superweak interaction

In this short subsection we try to estimate the order of magnitude of the effect introduced by the hypothetical superweak-like interaction. To this end we use the assumption, that the dominating contribution to $|\Gamma_{12}|$ is correctly described by the Standard Model. This means that we may assume

$$|\Gamma_{12}| \sim \frac{G_{\mathrm{F}}^2 M_P^4 \sin^2\theta}{(2\pi)^4}\, m_{K_0} \sim 10^{-12}\mathrm{MeV}\,, \tag{4.14}$$

where G_{F} is the Fermi constant, M_P is the proton mass and θ is the Cabbio angle [19]. Using our result from the previous section, $|r(x \to \infty)| \sim 10^{-16}$ and equation (4.10) we get the following upper bound on our parameter

$$\left|\frac{h_z(t \to \infty)}{m_{K_0}}\right| \lesssim \frac{1}{2} \times 10^{-31}\,. \tag{4.15}$$

This value corresponds to the currently measured $|(m_{K_0} - m_{\bar{K}_0})/m_{K_0}| \leq 10^{-18}$ [4]. Obviously, this effect is much too small to be observed with the present, and possibly also future, experiments.

5. Summary and conclusions

In this paper the influence of the hypothetical superweak interaction on the neutral kaon system has been studied. As the LOY approximation is insensitive to such an interaction we chose an alternative formalism, namely the formalism based on the Królikowski–Rzewuski equation.

The analysis performed in Sec. 4 yielded the following results: The difference between the real parts of the diagonal elements of the effective Hamiltonian is different from zero and basically constant for all times. This difference is usually interpreted as the difference between the mass of the kaon and anti-kaon. In the LOY approximation it is equal to zero and experimentally it is bound by $|(m_{K_0} - m_{\bar{K}_0})/m_{K_0}| \leq 10^{-18}$. Result (4.15) shows that the LOY approximation is very good even if the superweak interaction really exists. Another result of Sec. 4 is that the superweak interaction would not affect the difference between the decay widths and they would be equal for K^0 and $\overline{K^0}$ even in the presence of this hypothetical interaction.

Finally, it should be stressed that all the results obtained in the present paper are consistent with the Standard Model and the recent experimental findings. We have been assuming, that even if there is a first order CP violating $|\Delta S| = 2$ interaction, the $K^0 - \overline{K^0}$ system is correctly described by the Standard Model to a high degree of accuracy. This is the reason for assuming $|m_{12}|/(m_0 - \mu) = 10^{-17}$ and $|\Gamma_{12}| \sim 10^{-12}$ MeV in Sec. 4.

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Appendix A

By rewriting the m_{12} and Γ_{12} parameters as

$$m_{21} \ = \ m_{12}^* \equiv |m_{12}|e^{-i\theta}\,, \qquad \Gamma_{12} = \Gamma_{21}^* \equiv |\Gamma_{12}|e^{i\phi}\,, \tag{A.1}$$

we may cast $h_z(t)$ in the following form

$$\begin{aligned} h_z(t) \ &= \ \frac{1}{4}\,|\Gamma_{12}|\left(e^{i(\phi-\theta)} - e^{-i(\phi-\theta)}\right) \\ &\quad \times \Big[\Phi_0(t; m_0 - |m_{12}| - \mu) - \Phi_0(t; m_0 + |m_{12}| - \mu)\Big] \\ &= i\,|\Gamma_{12}|\,2\sin(\phi-\theta)\Big[\Phi_0(t; m_0 - |m_{12}| - \mu) - \Phi_0(t; m_0 + |m_{12}| - \mu)\Big]\,. \end{aligned} \tag{A.2}$$

It is easy to notice, that if $\phi = \theta$ we have $h_z = 0$, but this case corresponds exactly to the CP conserved case — compare [9] page 3743. Let us assume from now on, that we are dealing with the CP violating, CPT conserving case in which $\sin(\phi - \theta) \neq 0$.

To make our formulae simpler, let us define

$$r(t) = [\Phi_0(t; m_0 - |m_{12}| - \mu) - \Phi_0(t; m_0 + |m_{12}| - \mu)]\,. \tag{A.3}$$

So now

$$\begin{aligned} r(t) \ &= \ F_0(m_0 - |m_{12}| - \mu) - F_0(t; m_0 - |m_{12}| - \mu) \\ &\quad - F_0(m_0 + |m_{12}| - \mu) + F_0(t; m_0 + |m_{12}| - \mu)\,. \end{aligned} \tag{A.4}$$

Let us transform the above expression using

$$\sqrt{m_0 - |m_{12}| - \mu} = \sqrt{m_0 - \mu}\,\sqrt{1 - \frac{|m_{12}|}{m_0 - \mu}}\,,$$

and

$$\sqrt{m_0 + |m_{12}| - \mu} = \sqrt{m_0 - \mu}\, \sqrt{1 + \frac{|m_{12}|}{m_0 - \mu}}\,.$$

This justifies introducing the dimensionless variable (4.7).
Now we have

$$\begin{aligned}(m_0 - |m_{12}| - \mu)t &= (m_0 - \mu)\left(1 - \frac{|m_{12}|}{m_0 - \mu}\right) t = \left(1 - \frac{|m_{12}|}{m_0 - \mu}\right) x\,,\\ (m_0 + |m_{12}| - \mu)t &= (m_0 - \mu)\left(1 + \frac{|m_{12}|}{m_0 - \mu}\right) t = \left(1 + \frac{|m_{12}|}{m_0 - \mu}\right) x\,. \qquad \text{(A.5)}\end{aligned}$$

Using the above and equation (A.4) we arrive at (4.9).

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