

# FROM 4D REDUCED SYM INTEGRALS TO BRANCHED-POLYMERS\*

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We derive analytically one-loop corrections to the effective Polyakov-line operator in the branched-polymer approximation of the reduced four-dimensional supersymmetric Yang–Mills integrals.

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## 1. Introduction

The IKKT matrix model, called also IIB matrix model, was proposed [1] as a candidate for a non-perturbative formulation of superstring theory. Its partition function is defined by reduced ten-dimensional supersymmetric Yang–Mills integrals, which generate a quantum extension of the sum over topologies of the string world-sheet. In the large  $N$  limit the action of the IKKT model approaches the action of type IIB superstrings. The one-loop approximation of the model can be rewritten as a geometrical model of graphs [2]. The vertices of these graphs, which are dressed with ten-dimensional vectors, contain information about the space-time location of the world-sheet points of the corresponding string. The graphs are closely related to branched-polymers which are known to have fractal dimension equal four. This means that the scaling of physical observables within the ten-dimensional model of graphs is expected to be given by that of the four-dimensional theory [2, 3]. This fact is treated as an indication of the spontaneous symmetry breaking of the ten-dimensional Lorentz symmetry to its four-dimensional subgroup.

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Counterparts of the IKKT model can be defined in four and six dimensions. In four dimensions, the one-loop approximation simplifies, the corresponding graph model reduces to a model of tree graphs called also branched-polymers. Contrary to the corresponding graphs in six and ten dimensions, where the integration over fermionic degrees of freedom leads to a sign problem, each graph in the branched-polymer ensemble of the four-dimensional model has a positive definite weight. Therefore the four-dimensional model is frequently treated as a test-bed for the IKKT model. In fact many features of the model were first discovered in four dimensions, where one could explicitly carry out calculations [4–9].

In this paper we continue the studies of the four-dimensional case. We address the problem of the approximation of operators. In particular we derive the form of the Polyakov-line operator for the corresponding branched-polymer model.

### 2. One-loop approximation

The partition function of the 4D IIB matrix model is given by a supersymmetric Yang–Mills reduced integral [1]:

$$Z = \int \mathcal{D}A \mathcal{D}\bar{\Psi} \mathcal{D}\Psi e^{-S[A, \bar{\Psi}, \Psi]} \tag{1}$$

with the action

$$S[A, \bar{\Psi}, \Psi] = -\frac{1}{4g^2} \text{Tr} [A^\mu, A^\nu]^2 - \frac{1}{2g^2} \text{Tr} \bar{\Psi} \Gamma_\mu [A^\mu, \Psi], \tag{2}$$

where  $A^\mu (\mu = 1, \dots, 4)$  are traceless  $N \times N$  Hermitian matrices and  $\bar{\Psi}, \Psi$  are  $N \times N$  traceless matrices of Grassmann variables, which transform as Weyl-spinors. For convenience the spinor indices of  $\bar{\Psi}^a, \Psi^a, a = 1, 2$  and matrix indices  $ij$  of the matrices  $A_{ij}^\mu, \bar{\Psi}_{ij}^a$  and  $\Psi_{ij}^a$  are suppressed in (1) and (2). The measures of integration are flat in the space of traceless matrices:

$$\begin{aligned} \mathcal{D}A &\equiv \prod_{\mu=1}^4 \left( \prod_{i=1}^N dA_{ii}^\mu \delta \left( \sum_{i=1}^N A_{ii}^\mu \right) \prod_{i<j} d\text{Re} A_{ij}^\mu d\text{Im} A_{ij}^\mu \right), \\ \mathcal{D}\bar{\Psi} \mathcal{D}\Psi &\equiv \prod_{a=1}^2 \left( \prod_{i=1}^N d\bar{\Psi}_{ii}^a d\Psi_{ii}^a \delta \left( \sum_{i=1}^N \bar{\Psi}_{ii}^a \right) \delta \left( \sum_{i=1}^N \Psi_{ii}^a \right) \right) \\ &\times \prod_{a=1}^2 \left( \prod_{i<j} d\text{Re} \bar{\Psi}_{ij}^a d\text{Re} \Psi_{ij}^a d\text{Im} \bar{\Psi}_{ij}^a d\text{Im} \Psi_{ij}^a \right). \end{aligned}$$

The model is invariant with respect to Lorentz transformations. It has also  $\mathcal{N}=2$  supersymmetry and  $SU(N)$  invariance, being a remnant of the gauge invariance of the full non-reduced model.

Since the model has not yet been solved analytically, one way of getting insight into its behaviour is to use approximate methods such as, for example, the one-loop approximation [2, 3]. The idea behind the one-loop approximation is to split the fields  $A^\mu$ ,  $\bar{\Psi}$  and  $\Psi$  into the classical part and quantum fluctuations and further to integrate out the quantum fluctuations using the Gaussian approximation. In this way one derives an effective one-loop action, which depends only on the classical fields. One can see that a classical vacuum for the action (2) is given by a set of diagonal matrices  $A^\mu$ ,  $\bar{\Psi}$  and  $\Psi$ , or by any set of matrices, which can be obtained from it by a symmetry transformation. The one-loop approximation was worked out in paper [2]. Starting point of the approximation is to divide the matrices into the diagonal (classical) and off-diagonal (quantum) parts:

$$\begin{aligned} A_{ij}^\mu &= x_i^\mu \delta_{ij} + a_{ij}^\mu, \\ \bar{\Psi}_{ij}^a &= \bar{\xi}_i^a \delta_{ij} + \bar{\psi}_{ij}^a, \\ \Psi_{ij}^a &= \xi_i^a \delta_{ij} + \psi_{ij}^a, \end{aligned} \tag{3}$$

where  $\sum_i x_i^\mu = 0$  and  $\sum_i \bar{\xi}_i^a = 0$  ( $\sum_i \xi_i^a = 0$ ), respectively. Let us denote the one-loop effective action by  $S_1[x, \bar{\xi}, \xi]$ . The partition function (1) is approximated by:

$$Z_1 = \int \mathcal{D}x \mathcal{D}\bar{\xi} \mathcal{D}\xi e^{-S_1[x, \bar{\xi}, \xi]}.$$

For large separations between the eigenvalues  $|x_i^\mu - x_k^\mu| \gg \sqrt{g}$  the effective one-loop action can be shown to admit the following form:

$$S_1[x, \bar{\xi}, \xi] = -\frac{1}{2} \sum_{i < j} \text{tr} C_{ij}^2, \tag{4}$$

where the trace  $\text{tr}$  is taken in the Lorentz indices of  $C_{ij}^{\mu\nu}$

$$C_{ij}^{\mu\nu} \equiv (\bar{\xi}_i - \bar{\xi}_j) \Gamma^{\mu\sigma\nu} (\xi_i - \xi_j) \frac{(x_\sigma^i - x_\sigma^j)}{\|x^i - x^j\|^4}. \tag{5}$$

The symbol  $\|.\|$  denotes the length of a vector:  $\|x_i - x_j\| = \sqrt{\sum_\mu (x_i^\mu - x_j^\mu)^2}$  and  $\Gamma^{\mu\sigma\nu} = \frac{1}{3!} \Gamma^{[\mu} \Gamma^\sigma \Gamma^{\nu]}$ . In four dimensions this symbol can be simplified to  $\Gamma^{\mu\sigma\nu} = \varepsilon^{\mu\sigma\nu\tau} \Gamma_\tau \Gamma_5$ .

One can further simplify the problem by the explicit integration of the remaining fermionic zero modes  $\bar{\xi}, \xi$ . For large separations between the eigenvalues  $x_i^\mu$  one obtains:

$$\int \mathcal{D}\bar{\xi}\mathcal{D}\xi e^{-S_1[x,\bar{\xi},\xi]} = \sum_T \prod_{(ij)\in T} \|x_i - x_j\|^{-6}, \tag{6}$$

where the sum runs over all tree graphs consisting of  $N$  vertices and  $(N-1)$  links. Each vertex is dressed with a vector  $x_i^\mu$ ,  $\mu = 1, \dots, 4$ . The product runs over all links  $(ij)$  of the tree graph, giving the total weight, which is equal to a product of link-weights depending on the link length.

If the eigenvalues come close to each other, one has to introduce corrections to the approximation [2, 3]. One can explicitly calculate within the matrix model, that a strong repulsion appears in the effective potential for  $\|x_i - x_j\| < \sqrt{g}$ , which prevents the eigenvalues from approaching each other. Since one is interested in the large distance behaviour of the model, the details of the model on short distances are not important. One can therefore model this repulsion by adding a hard core to the potential, which keeps each pair of eigenvalues away from each other. Having done this, one eventually arrives at the partition function [2]:

$$Z_{\text{bp}} = \sum_T \int \prod_i' d^4x_i \left( \prod_{ab} \theta(\|x_a - x_b\| - c\sqrt{g}) \right) \prod_{(ij)\in T} \|x_i - x_j\|^{-6},$$

which describes an ensemble of branched-polymers embedded in four dimensions with a hard core. The core is modelled by the Heaviside step function  $\theta(x)$  and prevents any two vertices from being closer to each other than the core size, which is proportional to  $\sqrt{g}$  with some irrelevant constant of proportionality  $c$ . As above the sum runs over all trees. The integral is taken over all positions of tree vertices, such that the center of mass of the polymer is at zero:  $\delta^4(\sum_j x_j)$ , which is indicated as a prime at the product in the integration measure. This constraint is a remnant of the tracelessness of the matrices. It removes the translational zero mode of the integral, which would otherwise make the integral divergent. The product over  $ab$  runs over all pairs of vertices, while the one over  $(ij)$  only over pairs of vertices joined by a link of the tree graph  $T$ . One can write the partition function as follows:

$$Z_{\text{bp}} = \sum_T \int \prod_i' d^4x_i e^{-S_{\text{bp}}[T,x]} \tag{7}$$

with the action

$$S_{\text{bp}}[T, x] = 6 \sum_{(ij)\in T} \ln \|x_i - x_j\| + H \sum_{ab} \theta(c\sqrt{g} - \|x_a - x_b\|), \tag{8}$$

where  $H$  is a huge positive constant  $H \rightarrow \infty$ , which takes care of the short distance repulsion between any two points. The configuration space changes in the course of approximating the theory. At the beginning, it is given by  $N \times N$  matrices  $(A, \bar{\Psi}, \Psi)$ , then by diagonal matrices  $(x, \bar{\xi}, \xi)$  and eventually by branched-polymers dressed with position vectors  $(T, x)$ . The partition function and the effective action change correspondingly:

$$Z \rightarrow Z_1 \rightarrow Z_{\text{bp}} ,$$

$$S[A^\mu, \bar{\Psi}, \Psi] \rightarrow S_1[x^\mu, \bar{\xi}, \xi] \rightarrow S_{\text{bp}}[T, x^\mu] .$$

To complete the approximation scheme, one has also to find out how the operators change. For any operator  $\mathcal{O}[A^\mu, \bar{\Psi}, \Psi]$  defined in the original theory, one has to find its counterpart in the reduced ensembles:

$$\mathcal{O}[A, \bar{\Psi}, \Psi] \rightarrow \mathcal{O}_1[x, \bar{\xi}, \xi] \rightarrow \mathcal{O}_{\text{bp}}[T, x] .$$

The operators have to fit into the approximation scheme:

$$\langle \mathcal{O} \rangle \rightarrow \langle \mathcal{O}_1 \rangle \rightarrow \langle \mathcal{O}_{\text{bp}} \rangle ,$$

where the averages are defined as follows:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\Psi} \mathcal{D}\Psi e^{-S[A, \bar{\Psi}, \Psi]} \mathcal{O}[A^\mu, \bar{\Psi}, \Psi] ,$$

$$\langle \mathcal{O}_1 \rangle = \frac{1}{Z_1} \int \mathcal{D}x \mathcal{D}\bar{\xi} \mathcal{D}\xi e^{-S_1[x, \bar{\xi}, \xi]} \mathcal{O}_1[x^\mu, \bar{\xi}, \xi] ,$$

$$\langle \mathcal{O}_{\text{bp}} \rangle = \frac{1}{Z_{\text{bp}}} \sum_T \int \prod_i' d^4x_i e^{-S_{\text{bp}}[T, x]} \mathcal{O}_{\text{bp}}[T, x] .$$

### 3. Polyakov-line operator

The most fundamental operator in the IIB matrix model is the Polyakov-line operator. The operator is defined as [7, 9]:

$$P_k[A] = \frac{1}{N} \text{Tr} e^{ik_\mu A^\mu} , \tag{9}$$

where  $k$  is some external momentum. We now intend to derive the form of this operator in the one-loop approximation  $P_{1k}$  and for the branched-polymer ensemble  $P_{\text{bp},k}$ , using the approximation scheme described in the

previous section:  $P_k \rightarrow P_{1k} \rightarrow P_{\text{bp},k}$ . As before we shall perform the calculations for classical backgrounds, such that  $\|x_i - x_j\| \gg \sqrt{g}$ . Finally we will add an effective hard core for small separations of the classical fields  $x_i^\mu$ .

In the one-loop calculation we have to expand the operator up to second order in quantum corrections and integrate them out. The first two terms of the expansion are:

$$P_k[x + a] = \frac{1}{N} \left( \sum_i e^{ik_\mu x_i^\mu} + \sum_{i < j} \frac{e^{ik_\mu x_i^\mu} - e^{ik_\nu x_j^\nu}}{ik_\rho(x_i^\rho - x_j^\rho)} (k_\sigma a_{ij}^\sigma)(k_\tau a_{ij}^{*\tau}) \right) + o(a^4). \tag{10}$$

This result is derived in the appendix. The corrections of order  $o(a^4)$  can be neglected within the one-loop approximation. The operator is purely bosonic. The integration over  $\bar{\psi}, \psi$  can be carried out identically to the calculations of the partition function [2]. Using this result and adding the operator to the integrand we obtain:

$$\langle P_k \rangle = \mathcal{N} \int \mathcal{D}x [\Delta^2(x)]^4 \mathcal{D}a \mathcal{D}\xi P_k[x + a] e^{-\frac{1}{g^2} \sum_{i < j} \|x^i - x^j\|^2 (\delta_{\mu\nu} + C_{\mu\nu}^{ij}) a_{ij}^{*\mu} a_{ij}^\nu}, \tag{11}$$

where  $\Delta^2(x) \equiv \prod_{i < j} (x_i - x_j)^2$ . The normalization factor  $\mathcal{N}$  is the ratio of the factor, which arises from the integration over  $\bar{\psi}, \psi$ , and of  $Z$ , or alternatively it is a factor which takes care of the proper normalization  $\langle P_{k=0}[x + a] \rangle = 1$ .

We can now insert the first two terms of the expansion  $P_k[x + a]$  (10) into the integral (11). The integrand is a product of a second order polynomial in the field  $a$  and the Gaussian function. Performing the Gaussian integration yields:

$$\begin{aligned} \langle P_{1k} \rangle &= \frac{1}{Z_1} \int \mathcal{D}x \mathcal{D}\bar{\xi} \mathcal{D}\xi \frac{1}{N} \left( \sum_i e^{ik_\mu x_i^\mu} + g^2 \sum_{i < j} f_{ij} k_\mu k_\nu (\delta^{\mu\nu} + C_{ij}^{\mu\nu})^{-1} \right) \\ &\times \prod_{k < l} \det_{\mu\nu}^{-1} (\delta^{\mu\nu} + C_{kl}^{\mu\nu}), \end{aligned}$$

where the coefficients  $f_{ij}$  are given by:

$$f_{ij} = \frac{e^{ik_\mu x_i^\mu} - e^{ik_\nu x_j^\nu}}{ik_\rho(x_i^\rho - x_j^\rho) \|x_i - x_j\|^2}. \tag{12}$$

For a given pair  $(ij)$ , the matrix  $C_{ij}$  (5) is antisymmetric in Lorentz indices  $C_{ij}^{\mu\nu} = -C_{ij}^{\nu\mu}$ . Thus all odd powers of the matrix  $C_{ij}$  vanish. The matrix  $C_{ij}$  effectively depends on one Weyl multiplet  $(\bar{\xi}_{ij}^a, \xi_{ij}^a)$ , where  $\xi_{ij}^a = \xi_i^a - \xi_j^a$ . For a

given pair  $(ij)$  this multiplet has four independent Grassmannian variables  $(\bar{\xi}_{ij}^a, \xi_{ij}^a)$ ,  $a = 1, 2$ . Any higher than the second power of the matrix  $C_{ij}$  contains more than four members of the multiplet and therefore vanishes, since at least one of the Grassmannian variables enters the product more than once. The only non-trivial contribution comes from the second power  $C_{ij}^2$ . We have  $\text{tr} \ln(\mathbb{1} + C_{ij}) = \frac{1}{2} \text{tr} C_{ij}^2$  and  $k_\mu k_\nu (\delta^{\mu\nu} + C_{ij}^{\mu\nu})^{-1} = k_\mu k_\nu (\delta^{\mu\nu} + (C^2)_{ij}^{\mu\nu})$ . If we insert this to the last equation we obtain:

$$\langle P_{1k} \rangle = \frac{1}{Z_1} \int \mathcal{D}x \mathcal{D}\bar{\xi} \mathcal{D}\xi P_1[x, \bar{\xi}, \xi] e^{-S_1[x, \bar{\xi}, \xi]}, \tag{13}$$

where the one-loop action is given by (4) and the Polyakov-line operator by:

$$P_1[x, \bar{\xi}, \xi] = \frac{1}{N} \left( \sum_i e^{ik_\mu x_i^\mu} + g^2 k^2 \sum_{i < j} f_{ij} + g^2 \sum_{i < j} f_{ij} k_\mu (C_{ij}^2)^{\mu\nu} k_\nu \right). \tag{14}$$

As one can see, the effective Polyakov-line operator, which in the original model depends only on bosonic fields, depends in the one-loop approximation also on the fermionic zero modes  $\bar{\xi}, \xi$ . The dependence is hidden in the matrix  $C_{ij}$  (5).

The last step is to integrate out the fermionic zero modes  $\bar{\xi}$  and  $\xi$  from the expression (13) with the operator given by (14). The first two terms of (14) do not depend on  $\bar{\xi}$  and  $\xi$ . Let us integrate  $\bar{\xi}$  and  $\xi$  for these two terms. The only dependence on  $\bar{\xi}$  and  $\xi$  is now in the action. Expanding  $e^{-S_1[x, \bar{\xi}, \xi]}$  in the integrand of (13), we obtain a sum of products over links of graphs  $G$  constructed out of links  $(ij)$  joining all pairs of vertices  $x_i$  and  $x_j$ ,  $i, j = 1, \dots, N$ :

$$\exp \left( \frac{1}{2} \sum_{i < j} \text{tr} C_{ij}^2 \right) = \prod_{i < j} (1 + \frac{1}{2} \text{tr} C_{ij}^2) = \sum_G \prod_{(ij) \in G} \frac{1}{2} \text{tr} C_{ij}^2. \tag{15}$$

Each link contributes an entire Weyl multiplet  $(\bar{\xi}_{ij}, \xi_{ij})$  to the product. It is easy to see what happens if one integrates a product (15) for the given graph  $G$  with the measure  $\mathcal{D}\bar{\xi} \mathcal{D}\xi$ . The measure contains  $(N-1)$  independent Weyl multiplets. Therefore all integrals for graphs, which have the number of links different than  $(N-1)$  vanish. Among the diagrams with  $(N-1)$  links, only those which have a tree structure survive. To see this, it is enough to notice that if a graph has a loop one of the Grassmannian variables enters the integrand twice and the corresponding product vanishes. Graphs which have  $(N-1)$  links and visit all  $N$  vertices are called maximal trees. Thus the integration over zero modes cuts-out only products of  $\frac{1}{2} \text{tr} C_{ij}^2$  for  $(ij)$  being links of a maximal tree. When integrating over the  $(N-1)$  independent

multiplets  $\bar{\xi}_i, \xi_i$  for the given maximal tree it is convenient to change the integration variables to  $\bar{\xi}_{ij}, \xi_{ij}$ , where  $(ij)$  are the links of the tree. The Jacobian of the variable change is equal one. The integration  $\prod_{(ij)} d\bar{\xi}_{ij} d\xi_{ij}$  factorises in the link variables and can be done independently for each link of the tree resulting in the following form:

$$W_{ij} = \frac{1}{2} \int d\bar{\xi}_{ij} d\xi_{ij} \bar{\xi}_{ij} \Gamma^{\mu\alpha\nu} \xi_{ij} \cdot \bar{\xi}_{ij} \Gamma^{\nu\beta\mu} \xi_{ij} \frac{x_{ij}^\alpha x_{ij}^\beta}{\|x_{ij}\|^8}, \tag{16}$$

where  $x_{ij} = x_i - x_j$ . The integration gives the product of  $W_{ij}$  over all links of the tree.

One can similarly integrate the third term of  $P_1[x, \bar{\xi}, \xi]$  (14). The term  $f_{ij} k_\mu (C_{ij}^2)^{\mu\nu} k_\nu$  adds a link to each graph in the expansion (15). As before the integration over fermionic zero modes selects only maximal trees, the integrals over link variables factorise and can be done for each link separately. The new link, which comes from the third term of the operator (14) has now a slightly different fermionic dressing. The integration of  $f_{ij} k_\mu (C_{ij}^2)^{\mu\nu} k_\nu$  for this link gives:

$$U_{ij} = f_{ij} \int d\bar{\xi}_{ij} d\xi_{ij} \bar{\xi}_{ij} \Gamma^{\mu\alpha\nu} \xi_{ij} \cdot \bar{\xi}_{ij} \Gamma^{\nu\beta\sigma} \xi_{ij} \frac{x_{ij}^\alpha x_{ij}^\beta k^\mu k^\sigma}{\|x_{ij}\|^8}, \tag{17}$$

which differs from the factors  $W_{ij}$  (16) of all remaining links of the tree. For the entire tree  $T$  the integration of the third term gives:

$$\sum_{(ab) \in T} \left( U_{ab} \prod_{(ij) \neq (ab)} W_{ij} \right), \tag{18}$$

where the sum goes over all links of  $T$ , and the product over all links of the tree except  $(ab)$ . The link-weights  $W_{ij}$  and  $U_{ij}$  are calculated in the appendix. The result reads:

$$W_{ij} = \frac{6}{\|x_i - x_j\|^6},$$

$$U_{ij} = 4f_{ij} \frac{\|x_i - x_j\|^2 k^2 - ((x_i - x_j) \cdot k)^2}{\|x_i - x_j\|^8}.$$

The expression (18) can be rewritten in a form:

$$\left( \prod_{(ij)} W_{ij} \right) \sum_{(ab)} \frac{U_{ab}}{W_{ab}} = \left( \prod_{(ij)} W_{ij} \right) \sum_{(ab)} f_{ab} \frac{2(x_{ab}^2 k^2 - (x_{ab} k)^2)}{3\|x_{ab}\|^2}$$

in which the product of  $W_{ij}$  runs over all links exactly as in the expression for the weight of the tree.

Putting all together, we can write down the result of the integration of the fermionic zero modes as follows:

$$\langle P_{\text{bp},k} \rangle = \frac{1}{Z_{\text{bp}}} \sum_T \int \prod_i' d^4x_i \left( \prod_{(ij) \in T} W_{ij} \right) P_{\text{bp},k}[x],$$

where the operator is given by

$$P_{\text{bp},k}[x] = \frac{1}{N} \left\{ \sum_a e^{ik_\mu x_a^\mu} + g^2 k^2 \sum_{a < b} f_{ab} + g^2 \sum_{(ab) \in T} f_{ab} \frac{2(x_{ab}^2 k^2 - (x_{ab} k)^2)}{3||x_{ab}||^2} \right\}. \tag{19}$$

Everything in this section was done under the assumption that  $||x_i - x_j|| \gg \sqrt{g}$ . The product of  $W_{ij}$  describes the weight of the model for large distances. It is equivalent to the effective action  $S_{\text{bp}} = 6 \sum_{(ij) \in T} \ln ||x_i - x_j||$  for large distances between  $x_i$ 's. For small distances, we have to introduce an effective repulsion core as described in the previous section, eventually obtaining the effective action (8). The Polyakov-line operator is given in the branched-polymer model by (19).

### 4. Discussion

It is quite surprising that the partition function (1), a model defined by supersymmetric Yang–Mills integrals, can be approximated by the partition function of a very simply statistical model of branched-polymers. However, as we demonstrated in this paper, observables which are given by simple expressions in the supersymmetric Yang–Mills model become rather complicated in the corresponding statistical model of branched-polymers. With the explicit form of the Polyakov-line operator, which was derived analytically in this paper in the one-loop approximation, one should be able in the future to study numerically its scaling properties and to discuss limits of applicability of the one-loop approximation.

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### Appendix A

In this part of the appendix, we calculate the second order terms in off-diagonal quantum corrections  $a_{ij}$  (3) to the Polyakov-line operator (10).

$$P_k[A] = \frac{1}{N} \text{Tr} e^{ik_\mu A^\mu} = \frac{1}{N} \text{Tr} e^{\chi + \alpha}, \tag{20}$$

where the matrix  $\chi_{ij} = \chi_i \delta_{ij} = ik_\mu x_i^\mu \delta_{ij}$  is diagonal and  $\alpha_{ij} = ik_\mu a_{ij}^\mu$  is off-diagonal. The zeroth order term of the expansion in  $\alpha$  of the right-hand side of (20) is  $\frac{1}{N} \sum_i e^{ik_\mu x_i^\mu}$ , while the first order vanishes. The second order terms can be collected to the expression:

$$\sum_{n=2}^{\infty} \frac{1}{n!} \sum_{r+q=n-2} (r+1) \text{Tr} (\chi^r \alpha \chi^q \alpha). \tag{21}$$

Because  $\chi$  is diagonal and  $\alpha$  is off-diagonal we can write:

$$\text{Tr} (\chi^r \alpha \chi^q \alpha) = \sum_{ij} \chi_i^r \chi_j^q \alpha_{ij} \alpha_{ji} = \sum_{i < j} (\chi_i^r \chi_j^q + \chi_i^q \chi_j^r) \alpha_{ij} \alpha_{ji}.$$

We thus have to evaluate

$$\sum_{r+q=n-2} (r+1) (\chi_i^r \chi_j^q + \chi_i^q \chi_j^r).$$

This can be rewritten as

$$\left( \frac{\partial}{\partial \chi_i} \chi_i + \frac{\partial}{\partial \chi_j} \chi_j \right) \left[ \sum_{r+q=n-2} \chi_i^r \chi_j^q \right],$$

while the term in square brackets is just

$$\frac{\chi_i^{n-1} - \chi_j^{n-1}}{\chi_i - \chi_j}.$$

Acting on it with the differential operator we get

$$\left( \frac{\partial}{\partial \chi_i} \chi_i + \frac{\partial}{\partial \chi_j} \chi_j \right) \left[ \frac{\chi_i^{n-1} - \chi_j^{n-1}}{\chi_i - \chi_j} \right] = \frac{n\chi_i^{n-1} - n\chi_j^{n-1}}{\chi_i - \chi_j}.$$

Performing the summation over  $n$  as in the equation (21) we can write the sum of the second order terms in a compact form:

$$\sum_{i < j} \frac{e^{\chi_i} - e^{\chi_j}}{\chi_i - \chi_j} \alpha_{ij} \alpha_{ji} = \sum_{i < j} \frac{e^{ik_\mu x_i^\mu} - e^{ik_\nu x_j^\nu}}{ik_\rho (x_i^\rho - x_j^\rho)} k_\sigma k_\tau a_{ij}^\sigma a_{ji}^\tau,$$

which leads to the equation (10).

### Appendix B

We present here an explicit calculation of the link-weights  $W_{ij}$  and  $U_{ij}$ . Throughout this appendix, we denote the members of the multiplet  $(\bar{\xi}_{ij}, \xi_{ij})$  by  $(\bar{\xi}, \xi)$ , to simplify the notation. This does not lead to a confusion, because the integrals  $W_{ij}$  and  $U_{ij}$  are one-link integrals and involve only integration over one Weyl multiplet.

From this point of view  $\xi_{ij}$  plays merely the role of the integration variable and can be renamed.

For a Weyl spinor  $\bar{\xi}_a, \xi_a$ ,  $a = 1, 2$  in  $D = 4$  we have

$$\int d\bar{\xi}_1 d\bar{\xi}_2 d\xi_1 d\xi_2 \bar{\xi}_a \xi_b \bar{\xi}_c \xi_d = -\varepsilon_{ac} \varepsilon_{bd} = \delta_{ad} \delta_{cb} - \delta_{ab} \delta_{cd}.$$

In the Weyl representation, the gamma matrices are given by

$$\Gamma^k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & -i\mathbb{1} \\ i\mathbb{1} & 0 \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix},$$

where  $\sigma_k, k = 1, 2, 3$  are the Pauli matrices. We can write

$$\Gamma^\mu = \begin{pmatrix} 0 & \Gamma^\mu_- \\ \Gamma^\mu_+ & 0 \end{pmatrix}, \quad \Gamma^\mu \Gamma_5 = \begin{pmatrix} 0 & -\Gamma^\mu_- \\ +\Gamma^\mu_+ & 0 \end{pmatrix}.$$

We will need a second expression in the calculation involving the symbol:

$$\Gamma^{\mu\alpha\nu} = \varepsilon^{\mu\alpha\nu\sigma} \Gamma^\sigma \Gamma_5.$$

Now we can calculate  $W_{ij}$ :

$$\begin{aligned} W_{ij} &= \frac{1}{2} \int d\bar{\xi}_1 d\bar{\xi}_2 d\xi_1 d\xi_2 \bar{\xi} \Gamma^{\mu\alpha\nu} \xi \cdot \bar{\xi} \Gamma^{\mu\beta\nu} \xi \frac{x_{ij}^\alpha x_{ij}^\beta}{\|x_{ij}\|^8} \\ &= -\frac{1}{2} \varepsilon_{ac} \varepsilon_{bd} \varepsilon^{\mu\alpha\nu\sigma} \varepsilon^{\mu\beta\nu\tau} \Gamma_{-ab}^\sigma \Gamma_{-cd}^\tau \frac{x_{ij}^\alpha x_{ij}^\beta}{\|x_{ij}\|^8} \\ &= \frac{1}{2} \underbrace{(\Gamma_{-ab}^\sigma \Gamma_{-ba}^\tau - \Gamma_{-cc}^\sigma \Gamma_{-dd}^\tau)}_{2\delta^{\sigma\tau}} \varepsilon^{\mu\alpha\nu\sigma} \varepsilon^{\mu\beta\nu\tau} \frac{x_{ij}^\alpha x_{ij}^\beta}{\|x_{ij}\|^8} \\ &= \underbrace{\varepsilon^{\mu\alpha\nu\sigma} \varepsilon^{\mu\beta\nu\sigma}}_{3!\delta^{\alpha\beta}} \frac{x_{ij}^\alpha x_{ij}^\beta}{\|x_{ij}\|^8} \\ &= \frac{6}{\|x_{ij}\|^6}. \end{aligned}$$

Similarly for  $U_{ij}$

$$\begin{aligned}
 U_{ij} &= f_{ij} \int d\bar{\xi}_1 d\bar{\xi}_2 d\xi_1 d\xi_2 \quad \bar{\xi} \Gamma_-^{\mu\alpha\nu} \xi \cdot \bar{\xi} \Gamma_-^{\sigma\beta\nu} \xi \frac{x_{ij}^\alpha x_{ij}^\beta k^\mu k^\sigma}{\|x_{ij}\|^8} \\
 &= -\varepsilon_{ac} \varepsilon_{bd} \varepsilon^{\mu\alpha\nu\rho} \varepsilon^{\sigma\beta\nu\tau} \Gamma_{-ab}^\rho \Gamma_{-cd}^\tau f_{ij} \frac{x_{ij}^\alpha x_{ij}^\beta k^\mu k^\sigma}{\|x_{ij}\|^8} \\
 &= \underbrace{(\Gamma_{-ab}^\rho \Gamma_{-ba}^\tau - \Gamma_{-cc}^\rho \Gamma_{-dd}^\tau)}_{2\delta^{\rho\tau}} \varepsilon^{\mu\alpha\nu\rho} \varepsilon^{\sigma\beta\nu\tau} f_{ij} \frac{x_{ij}^\alpha x_{ij}^\beta k^\mu k^\sigma}{\|x_{ij}\|^8} \\
 &= 4f_{ij} (\delta^{\mu\sigma} \delta^{\alpha\beta} - \delta^{\mu\beta} \delta^{\alpha\sigma}) \frac{x_{ij}^\alpha x_{ij}^\beta k^\mu k^\sigma}{\|x_{ij}\|^8} \\
 &= 4f_{ij} \frac{x_{ij}^2 k^2 - (x_{ij} k)^2}{\|x_{ij}\|^8}.
 \end{aligned}$$

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