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THE QCD EFFECTIVE STRING*

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QCD can be described in a certain kinematical regime by an effective string theory. This string must couple to background chiral fields in a chirally invariant manner, thus taking into account the true chirally non-invariant QCD vacuum. By requiring conformal symmetry of the string and the unitarity constraint on chiral fields we reconstruct the equations of motion for the latter ones. These provide a consistent background for the propagation of the string. By further requiring locality of the effective action we recover the Lagrangian of non-linear sigma model of pion interactions. The prediction is unambiguous and parameter-free. The estimated chiral structural constants of Gasser and Leutwyler fit very well the phenomenological values.

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1. Introduction

I report here on work done jointly with J. Alfaro, A. Andrianov, L. Balart and A. Dobado. I should thank them for a long and enjoyable collaboration. I shall try to convince you that the collaboration has been fruitful too.

The history of attempts to describe hadrons in the framework of a string theory encompasses already more than 30 years (see [1–8] as well as the reviews [9–11]). Commonly cited arguments to justify the string description of QCD are the dominance of planar gluon diagrams in the large N_c limit [12], being interpreted as the world-sheet of a string, the expansion in terms of surfaces built out of plaquettes in strong-coupling lattice QCD [13], and the manifest success of Regge phenomenology [10, 14].

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There is a well motivated agreement, based on arguments of universality and renormalization group ideas, that in a certain kinematic regime the Nambu–Goto or, equivalently, the Polyakov string must provide a satisfactory effective description of the flux-tube linking one quark and an antiquark. Unfortunately this cannot be the whole story as it has been known for a long time that the hadronic amplitudes derived from such type of strings, if taken at face value, are not physically consistent. To remind the reader about their pitfalls we recall the original Veneziano amplitude [1], which can be derived from Nambu–Goto string and supposedly describes the scattering amplitude of four pions (once conveniently embellished by Chan–Paton group factors). One can show that in this amplitude the scalar resonance is a tachyon and the vector state (which we should identify with the rho particle) is massless. Finally, such an amplitude does not have the appropriate Adler zero, *i.e.* the property that at $s = t = 0$ the pion scattering amplitude vanishes, as required by chiral symmetry. The Veneziano amplitude completely fails to provide an accurate description of the lowest-lying resonances in QCD, although it reproduces a good deal of the higher resonance behavior (for instance, the linear Regge behaviour for large values of the principal quantum number n is compatible with the partonic model and asymptotic freedom).

It seems very reasonable to assume that the main reason for the presence of a tachyon in the spectrum, the wrong chiral properties, and in fact many of the undesirable properties of the Veneziano amplitude lies in a wrong choice of the vacuum [15]. Finding the 'correct' vacuum of string theory in four dimensions seems however a hopelessly difficult task that can be tackled only with the techniques of string field theory — if at all. Even more difficult it seems to find the 'correct' vertex operators implementing the different excitations, assuming that they exist at all. Therefore we have to be a lot more modest and resort to some indirect methods.

A way to take into account all the non-perturbative properties of the QCD vacuum and excitations was suggested in [16] and developed in [17]. One can *assume* that in QCD chiral symmetry breaking takes place and the massless (in the chiral limit) pseudoscalar mesons form the background of the QCD vacuum in which the string propagates. The string itself is assumed to contain all the other (massive) excitations of QCD. The massless pion fields can be collected in a unitary matrix $U(x)$ belonging to SU(2) group (here we consider non-strange Goldstone mesons only). It describes excitations around the non-perturbative vacuum induced by chiral symmetry breaking. From the string point of view $U(x)$ is nothing but a collection of couplings involving the string variable $x_\mu(\tau, \sigma)$. It has to be coupled to the boundary of the string where flavor is attached. Our goal is to find a consistent string propagation in this non-perturbative background.

This procedure is not very different from what string practitioners advocate to derive the stringy corrections to, say, Einstein equations. One *assumes* that the condensation of string modes produces some non perturbative solution for the background metric $G_{\mu\nu}(x)$ and require — and this is the essential ingredient — conformal invariance of the string propagation around this background. The background solution is then found self-consistently and perturbatively as an expansion in powers of the inverse string tension. Standard vertex operator methods work only in linear perturbations around flat space, on the contrary. Indeed an essential property of string theory is conformal invariance. Since it must hold when perturbing the string around any vacuum we demand the new coupling to chiral fields, living on the boundary, which will be described in the next section to preserve this invariance.

Our proposal is thus to introduce the general reparametrization-invariant boundary interaction to chiral fields and derive all the divergences induced by this interaction. We shall need additional dimensional operators in the boundary action to renormalize divergences. From the condition of vanishing β functions for $U(x)$ the equations of motion for chiral fields are obtained in the low-momentum (derivative) expansion. We consistently implement the unitarity constraint on the chiral fields and locality of the chiral Lagrangian and finally calculate the $O(p^4)$ terms of the Gasser and Leutwyler [18] effective Lagrangian without any additional assumptions or free parameters to adjust. A strikingly good correspondence with the phenomenological values is found when this procedure is followed to the end.

2. Attaching pions to the QCD string

The hadronic string in the conformal gauge is described by the following conformal field theory action which has four dimensional Euclidean spacetime as target space

$$\mathcal{W}_{\rm str} = \frac{1}{4\pi\alpha'}\int d^{2+\epsilon}\sigma \left(\frac{\varphi}{\mu}\right)^{-\epsilon} \partial_i x_\mu \partial_i x_\mu \,, \tag{2.1}$$

where for $\epsilon = 0$ one takes $x_\mu = x_\mu(\tau,\sigma)$, $-\infty < \tau < \infty, 0 < \sigma < \infty$, $i = \tau,\sigma$, $\mu = 1,...,4$. The conformal factor $\varphi(\tau,\sigma)$ is introduced to restore the conformal invariance in $2+\epsilon$ dimensions. The Regge trajectory slope (related to the inverse string tension) is known to be universal $\alpha' \simeq 0.9$ GeV^{-2} [19].

We would like to couple in a chiral invariant manner the matrix in flavor space $U(x)$ containing the meson fields to the string degrees of freedom while preserving general covariance in the two dimensional coordinates and conformal invariance under local scale transformations of the two-dimensional metric tensor.

Since the string variable x does not contain any flavor dependence, we introduce two dimensionless Grassmann variables ('quarks') living on the boundary of the string sheet: $\psi_{\rm L}(\tau), \psi_{\rm R}(\tau)$. They transform in the fundamental representation of the light flavor group (SU(2) in the present paper). A local Hermitean action $S_b = \int d\tau L_f$ is then introduced on the boundary $\sigma = 0$ to describe the interaction with background chiral fields $U(x(\tau)) = \exp(i\pi(x)/f_\pi)$, where the normalization scale is set to $f_\pi \simeq 93$ MeV, the weak pion decay constant.

The boundary Lagrangian is chosen to be reparameterization invariant and in its minimal form reads

$$\begin{aligned} L_f \ = \ & \frac{1}{2} i \Big(\bar{\psi}_{\rm L} U(1-z)\dot{\psi}_{\rm R} - \dot{\bar{\psi}}_{\rm L} U(1+z)\psi_{\rm R} \\ & + \bar{\psi}_{\rm R} U^+(1+z^*)\dot{\psi}_{\rm L} - \dot{\bar{\psi}}_{\rm R} U^+(1-z^*)\psi_{\rm L} \Big) , \end{aligned} \tag{2.2}$$

herein and further on a dot implies a τ derivative: $\dot{\psi} \equiv d\psi/d\tau$. It can be seen that this is the most general form of the boundary Lagrangian compatible with all the symmetries.

A further restriction is obtained by requiring CP invariance,

$$U \leftrightarrow U^+, \quad \psi_{\rm L} \leftrightarrow \psi_{\rm R}. \tag{2.3}$$

The above Lagrangian is CP symmetric for $z = -z^* = ia$. The fulfillment of this symmetry happens to be crucial to preserve conformal symmetry in the presence of the added boundary interaction.

Now we expand the function $U(x)$ in powers of the string coordinate field $x_\mu(\tau) = x_{0\mu} + \tilde{x}_\mu(\tau)$ around a constant x_0,

$$U(x) = U(x_0) + \tilde{x}_\mu(\tau)\partial_\mu U(x_0) + \frac{1}{2}\tilde{x}_\mu(\tau)\tilde{x}_\nu(\tau)\partial_\mu\partial_\nu U(x_0) + \dots , \tag{2.4}$$

and look for the potentially divergent one particle irreducible diagrams. The two-fermion, N-boson vertex operators are generated by the expansion (2.4), from the generating functional $Z_b = \langle \exp(iS_b)\rangle$ and Eq. (2.2). Each additional loop comes with a power of α'. One can find a resemblance to the familiar derivative expansion of chiral perturbation theory [18].

The free fermion propagator is

$$\langle \psi_{\rm R}(\tau)\bar{\psi}_{\rm L}(\tau')\rangle = \langle \psi_{\rm L}(\tau)\bar{\psi}_{\rm R}(\tau')\rangle^\dagger = U^{-1}(x_0)\theta(\tau-\tau'). \tag{2.5}$$

The free boson propagator projected on the boundary is

$$\langle x_\mu(\tau)x_\nu(\tau')\rangle = \delta_{\mu\nu}\Delta(\tau-\tau') = -2\delta_{\mu\nu}\alpha'\ln(|\tau-\tau'|\mu). \tag{2.6}$$

The normalization of the string propagator is inferred [17] from the definition of the kernel of the N-point tachyon amplitude for the open string [9]. In dimensional regularization one uses $\Delta(0)\sim\alpha'/\epsilon$ and $\Delta'(0)=0$.

To implement the renormalization process we perform a loop (equivalent to a derivative) expansion, we then proceed to determine the counterterms required to make the theory finite and, finally, we impose the condition of a vanishing beta functional for the coupling $U(x)$ to ensure the absence of conformal anomaly.

3. Renormalization at one and two loops

3.1. One-loop renormalization

Using the above set of Feynman rules one arrives at the one-loop divergent part of the propagator

$$-\theta(A-B)U^{-1}\delta U U^{-1}, \quad \delta U \equiv \Delta(0)\left[\frac{1}{2}\partial_\mu^2 U - \frac{3+z^2}{4}\partial_\mu U U^{-1}\partial_\mu U\right]. \tag{3.1}$$

This divergence is eliminated by introducing an appropriate counterterm $U\to U+\delta U$. Conformal symmetry is restored (the beta-function is zero) if the above contribution vanishes, $\delta U=0$.

Let us find out for which value of z this variation of U is compatible with its unitarity.

$$\delta(UU^+) = U\cdot\delta U^+ + \delta U\cdot U^+ = 0. \tag{3.2}$$

A simple calculation shows that this takes place for $z=\pm i$. The local classical action which has $\delta U=0$ as equation of motion is

$$W^{(2)} = \frac{f_\pi^2}{4}\int d^4x \mathrm{tr}\left[\partial_\mu U\partial_\mu U^+\right], \tag{3.3}$$

i.e. the well known non-linear sigma model of pion interactions.

We have thus found the chiral action induced by the QCD string. It has all the required properties of locality, chiral symmetry and proper low momentum behavior (Adler zero) and describes massless pions. However f_π, the overall normalization scale, cannot be predicted from these arguments.

Before proceeding to a full two loop calculation we have to check whether the minimal Lagrangian (2.2) is sufficient to renormalize also the vertices containing the boson legs. It turns out that it is not.

To obtain the divergences for vertices with external boson lines we introduce an external background boson field $\bar{x}_\mu$ and split $x_\mu = \bar{x}_\mu + \eta_\mu$. The free propagator for the fluctuating field η_μ coincides with the one for x_μ.

The total one-loop divergence in the vertex with two fermions and one boson line can be represented by the following vertex operator in the Lagrangian

$$\frac{i}{2}\left(\bar{\psi}_{\mathrm{L}}\Phi^{(1)}\dot{\psi}_{\mathrm{R}} - \dot{\bar{\psi}}_{\mathrm{L}}\Phi^{(2)}\psi_{\mathrm{R}}\right) + \mathrm{h.c.}, \quad \Phi^{(1,2)} \equiv \bar{x}_\mu(\tau)(1 \mp z)\left[\partial_\mu\left(\delta U\right) \mp \phi_\mu\right]. \tag{3.4}$$

The terms proportional to derivatives of δU are automatically eliminated by the renormalization of the one-loop propagator. But the part proportional to ϕ_μ remains and to absorb these divergences new counterterms are required. The latter ones can be parameterized with three bare constants g_1, g_2 and g_3, which are real if the CP symmetry for $z = -z^*$ holds

$$\begin{aligned} \Delta L_{\mathrm{bare}} \;=\; & \frac{i}{8}(1-z^2)\bar{\psi}_{\mathrm{L}}\Big((g_1 - zg_2)\partial_\nu \dot{U}U^{-1}\partial_\nu U - (g_1 + zg_2)\partial_\nu U U^{-1}\partial_\nu \dot{U} \\ & +2zg_3\partial_\nu U U^{-1}\dot{U}U^{-1}\partial_\nu U\Big)\psi_{\mathrm{R}} + \mathrm{h.c.} \end{aligned} \tag{3.5}$$

Renormalization is accomplished by the subtraction

$$g_i = g_{i,r} - \Delta(0). \tag{3.6}$$

The constants $g_{i,r}$ are finite, but in principle scheme dependent. The counterterms are of higher dimensionality than the original Lagrangian (2.2) and the couplings g_i are of dimension α'. Since (2.2) was the most general coupling permitted by the symmetries of the model, one concludes that conformal symmetry seems to be broken by these boundary couplings already at tree level. However in spite of the fact that the new couplings are dimensional, it turns out that their contribution into the trace of the energy-momentum tensor vanishes once the requirements of unitarity of U and CP invariance are taken into account. Therefore conformal invariance at the world-sheet level is not broken at the order we are working. We refer the reader to the original work [17] for a more detailed discussion on this point.

The appearance of new vertices does however change the fermion propagator. One obtains from such terms the following contribution to the propagator

$$\begin{aligned} &\theta(A-B)\frac{1}{16}\Delta(0)(1-z^2)U^{-1}\left\{2(g_{1,r} - z^2 g_{2,r})\partial_\rho U U^{-1}\partial_\mu\partial_\rho U U^{-1}\partial_\mu U\right. \\ &-(1+z)(g_{1,r} + zg_{2,r})\partial_\rho U U^{-1}\partial_\mu U U^{-1}\partial_\rho\partial_\mu U \end{aligned}$$

$$
\begin{aligned}
&-(1-z)(g_{1,r}-zg_{2,r})\partial_\rho\partial_\mu UU^{-1}\partial_\rho UU^{-1}\partial_\mu U \\
&+4z^2 g_{3,r}\partial_\rho UU^{-1}\partial_\mu UU^{-1}\partial_\rho UU^{-1}\partial_\mu U\big\}\, U^{-1} \\
\equiv\; &-\theta(A-B)\Delta(0)U^{-1}\delta^{(4)}UU^{-1}\,. \qquad (3.7)
\end{aligned}
$$

One should add this divergence to the one-loop result, thereby modifying the U field renormalization and equations of motion

$$
\bar{\delta}U = \Delta(0)\left[\frac{1}{2}\partial_\mu^2 U - \frac{3+z^2}{4}\partial_\mu UU^{-1}\partial_\mu U + \delta^{(4)}U\right] = 0. \qquad (3.8)
$$

This is one source of $O(p^4)$ terms and we shall see that there is yet another contribution at the two loop order.

As to the other vertices it can be proven [17] that any diagram with an arbitrary number of external boson lines and two fermion lines, *i.e.* any vertex of those generated by the perturbative expansion of (2.2) is rendered finite by the previous counterterms. This completes the renormalization program at one loop.

3.2. Two-loop renormalization of the propagator

There are 10 two-loop one-particle irreducible diagrams contributing to the fermion propagator that have been analytically calculated in [17]. The divergences in the propagator consist of the double divergent part, proportional to $\Delta^2(0)$, and of the single divergent contributions, proportional to $\Delta(0)$. A substantial part of these divergences are eliminated by performing the one-loop renormalization. The structure of the double pole divergences is compatible with renormalization group arguments.

Some single-pole divergences remain however. Namely, there are divergences linear in $\Delta(0)$ which come from irreducible two-loop diagrams with maximal number of vertices. These divergences are

$$
\begin{aligned}
-\Delta(0)U^{-1}\delta^{(4)}_{2-l}U^{-1} \;\equiv\; & c\Delta(0)\big[U^{-1}\partial_\rho UU^{-1}\partial_\mu UU^{-1}\partial_\mu UU^{-1}\partial_\rho UU^{-1} \\
& -U^{-1}\partial_\rho UU^{-1}\partial_\mu UU^{-1}\partial_\rho UU^{-1}\partial_\mu UU^{-1}\big]\,, \qquad (3.9)
\end{aligned}
$$

with $c=\alpha'(1-z^2)^2/8=\alpha'/2$ for $z=\pm i$. This term survives after adding all the counterterms. It must therefore modify the equation of motion (3.8) at the next order in the α' expansion, $\delta^{(4)}U \to \delta^{(4)}U + \delta^{(4)}_{2-l}$. Its presence allows for non zero solutions for the coupling constants g_i and therefore for nonzero values for the Gasser–Leutwyler $O(p^4)$ coefficients.

In addition all one-loop counterterms, when inserted into one-loop diagrams contribute to order $(\alpha')^2$, of course. Their contribution has been discussed in the previous subsection.

4. Local integrability of the equations of motion

The equation of motion, $\delta U = 0$, can be derived from the dimension-two local action (3.3), involving a unitary matrix $U(x)$, only for $z = \pm i$. If the four-derivative part of the equations of motion can be derived from dimension-four operators in a local effective Lagrangian then certain constraints are to be imposed on constants $g_{i,r}$.

Such a Lagrangian has only two terms compatible with the chiral symmetry

$$\mathcal{L}^{(4)} = f_\pi^2 \mathrm{tr}\left(K_1 \partial_\mu U \partial_\rho U^+ \partial_\mu U \partial_\rho U^+ + K_2 \partial_\mu U \partial_\mu U^+ \partial_\rho U \partial_\rho U^+\right). \tag{4.1}$$

Other terms containing $\partial_\mu^2 U$ are reduced to the set (4.1) with the help of the lowest-order equations of motion.

The variation of the previous Lagrangian must saturate the dimension-four component of the equations of motion. From this we identify the low-energy constants with the coupling constants arising from the equations of motion (3.8) supplemented with (3.9) and after applying the $O(p^2)$ equations of motion. Then one obtains the following set of coefficients for the various chiral field structures

$$\begin{aligned}
-2(2K_1 + K_2) &= \frac{1}{16}(1-z^2)(1 \pm z)(g_{1,r} \pm z g_{2,r}),\\
-4K_2 &= \frac{1}{8}(1-z^2)(-g_{1,r} + z^2 g_{2,r}), \quad 2[(1-z^2)K_1 + K_2] = -c;\\
-2z^2 K_2 &= 0, \quad 4[K_1 + K_2] = -\frac{1}{4}(1-z^2)z^2 g_{3,r} + c.
\end{aligned} \tag{4.2}$$

For $z^2 = -1$ only one solution is possible,

$$K_2 = 0, \quad K_1 = -\frac{1}{4}c = -\frac{\alpha'}{8}, \qquad g_{1,r} = -g_{2,r} = -g_{3,r} = 4c. \tag{4.3}$$

Thus, comparing Eq. (4.1) with the usual parameterization of the Gasser and Leutwyler Lagrangian [18],

$$L_1 = \frac{1}{2}L_2 = -\frac{1}{4}L_3 = -\frac{1}{2}K_1 f_\pi^2 = \frac{f_\pi^2 \alpha'}{16}. \tag{4.4}$$

For $\alpha' = 0.9$ GeV^{-2} and $f_\pi \simeq 93$ MeV it yields $L_2 \simeq 0.9 \cdot 10^{-3}$ which is quite a satisfactory result [20].

The relation $L_1 = 1/2 L_2 = -1/4 L_3$ was established earlier in bosonization models [21] and in the chiral quark model [22] by means of a derivative expansion of quark determinant. However at that time its possible connection with a string description of QCD was not recognized. The first attempt to derive the chiral coefficients from the Veneziano-type dual amplitude was undertaken in [23] where a similar relation was found but with different numerical values for the L_i. However the specific choice of dual amplitude in [23] cannot be derived from any known string theory.

Another check comes from the compatibility of the unitarity of U and the equations of motion at the two-loop level. It turns out that if one accepts arbitrary real coefficients in the set of dimension-four operators then the only solution compatible with the unitarity is given by the parameterization with constants K_1 and K_2.

In conclusion, there is only one solution that is compatible with the requirements of unitarity of the U matrix, locality of the action, chiral invariance, CP symmetry and conformal invariance, and this solution is the one presented above. We find this unicity quite remarkable.

5. Further developments

We are considering at present the coupling to external left and right gauge fields. The issue is not totally straightforward because gauge invariance is hard to control since the Taylor expansion of the U field does not correspond to an expansion in covariant derivatives.

The direct generalization of our action (2.2) would amount to the replacement

$$\frac{d}{dt} \to \frac{d}{dt} + i\dot{x}_\mu A_\mu, \tag{5.1}$$

where A_μ is actually a matrix in flavor space. This approach has the problem that we have mentioned: the new divergences that appear have coefficients that are not gauge invariant. Interpreting these coefficients of the beta function as equations of motion of the corresponding sigma model, they are gauge-fixed equations.

This is not too surprising; the propagator — which is the object we are renormalizing — is a bilocal object and thus is not gauge invariant. To solve this technical difficulty we shall work with 'dressed' propagators, involving fields ψ_{L} and ψ_{R} that do not actually transform under electromagnetic gauge transformations. This is actually equivalent, after a field redefinition, to work with the action (2.2) with the replacement

$$U \to \tilde{U} \equiv \mathrm{e}^{-i\Phi} U \mathrm{e}^{i\Phi}\,, \tag{5.2}$$

where

$$\Phi = \int_{t_0}^{t} dt' \dot{x}_\mu(t') A_\mu(x(t')). \tag{5.3}$$

Then

$$\tilde{U}(x) = \tilde{U}(x_0) + \tilde{x}_\mu [D_\mu, \tilde{U}] + \frac{1}{2}\tilde{x}_\mu \tilde{x}_\nu [D_\mu, [D_\nu, \tilde{U}]] + ... \\ + \frac{i}{2} \int_{t_0}^{t} dt' \dot{x}_\mu x_\nu [F_{\mu\nu}, \tilde{U}] + ... \tag{5.4}$$

In this way it is quite simple to reproduce the covariant equations of motion at the leading order, which derive from the Lagrangian

$$W^{(2)} = \frac{f_\pi^2}{4} \int d^4x \mathrm{tr} \left[D_\mu U D_\mu U^+ \right] . \tag{5.5}$$

We are actively working in the determination of L_9 and L_{10} which again involves a two-loop calculation. One could even hope to determine spectral functions, such as Π_{VA}.

Another issue of interest is the extension to the odd-parity sector of the effective QCD action, that is to say the anomaly sector. In order to obtain the parity-odd WZW Lagrangian relevant for the case of three flavors one possibility would be to extend the boundary fermion action supplementing one-dimensional fermions with true spinor degrees of freedom. Another possibility is to include some topological terms in the world-sheet action (such as the self-intersection number that involves the ϵ-symbol). This issue is under active investigation now. We have results for 2D QCD that seem to provide a satisfactory answer; the extension to four dimensions appears trickier.

6. Conclusions

In our talk we have reported on a simplified, but hopefully not unrealistic, model of the QCD string. Requiring its conformal invariance around a chirally non-invariant vacuum leads to the Gasser and Leutwyler Lagrangian. However the bosonic string action used here is of course not totally satisfactory. For instance, it does not prevent large Euclidean world sheets from crumpling [24], something that looks very unphysical. It does not also describe correctly the high-temperature behavior of large N QCD [25] either. To correct some of these shortcomings a proper QCD-induced string must be modified [24,26] suitably by including operators breaking manifestly conformal symmetry on the world-sheet for large strings. Nevertheless we are concerned here with the low-energy string properties and therefore do not expect that the strategy and technique to derive the chiral field action needs any significant changes to be adjusted to a modified QCD string action.

A further modification to the model would be to include the quark masses. This has to be done in a way that the conformal invariance of the string is preserved.

Many other open questions can be formulated in connection with this work. For instance, we are obtaining the pion scattering amplitude in a momentum expansion. Is there any way of getting a closed expression similar to the Veneziano amplitude? If the present approach is correct we already know that it does not correspond to the expansion of any Γ function, since we get rational coefficients. What function then? What is the role of crossing in this approach? Can the conformal factor (φ) dependence be related to $\Lambda_{\rm QCD}$? What is the connection to the parton model?

Clearly a lot of work remains to be done. This novel approach seems quite promising to us. So far we have not encountered any ambiguities, up to overall normalization scale (actually f_π), the approach is quite unique and yields an unexpected good agreement with phenomenology so we believe it certainly deserves further investigation.

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