ON THE QUANTUM CORRECTIONS OF GONIHEDRIC STRING*

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We present the one and two-loop quantum corrections to the earlier proposed string theory whose world-sheet action measures the linear sizes of the surfaces by the square root of the extrinsic curvature. We find in this model the usual conformal anomaly. Moreover, the one-loop perturbative analysis shows that the dynamics of this model is determined by a reduced number of degrees of freedom compared to the usual string. We point out that this model does not receive any quantum corrections around its classical trajectory. Finally we show that the constraint, relating the induced metric with the string fields, is enforced by radiative corrections and it does not allow the generation of the Polyakov–Kleinert smooth string.

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1. One-loop perturbative analysis

The world-sheet action to be studied is the following:

$$S = m \int d^2\zeta \sqrt{g} \sqrt{(\Delta(g) X_\mu)^2}\,, \tag{1}$$

where m is a constant with the dimension of a mass, $X_\mu(\zeta)$ parameterizes the string in a Minkowskian D dimensional flat space-time with the metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, \ldots)$. The induced metric of the string is

$$g_{ab} = \partial_a X^\mu \partial_b X_\mu\,. \tag{2}$$

In (1) $\Delta(g) X_\mu$ is defined as

$$\Delta(g) X_\mu = \frac{1}{\sqrt{g}} \partial_a \left(\sqrt{g} g^{ab} \partial_b X_\mu \right)\,, \tag{3}$$

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where $g = \det(g_{ab})$. The second fundamental form K is defined through the relations:

$$\begin{aligned} K^i{}_{ab} n^i_\mu &= \partial_a \partial_b X_\mu - \Gamma^c{}_{ab} \partial_c X_\mu = D_a \partial_b X_\mu \,, \\ n^i_\mu n^{j\mu} &= \delta^{ij} \,, \\ n^i_\mu \partial_b X^\mu &= 0 \,, \end{aligned} \tag{4}$$

where n^i_μ are $D-2$ normals, D_a indicates the covariant derivative respect to the connection Γ, $a, b = 1, 2$; $\mu = 0, 1, \ldots, D-1$; $i, j = 1, 2, \ldots, D-2$.

The $D-2$ 2×2 symmetric matrices $K^i{}_{ab}$ are known as extrinsic curvature and they satisfy the Gauss–Codazzi [1] equations which in the flat space becomes

$$R = K^2 - \mathrm{Tr}(K^i K^i) \,, \qquad D_a K^i{}_{bc} = D_b K^i{}_{ac} \,, \tag{5}$$

R being the intrinsic scalar curvature of the surface, $K^i \equiv K^i{}_a{}^a$,$\mathrm{Tr}(K^i K^j) \equiv K^i{}_a{}^b K^j{}_b{}^a$ and the covariant derivative D_a satisfies

$$D_a n^i_\mu = -K^i{}_{ab} \partial^b X^\mu . \tag{6}$$

By these definitions the action (1) takes the following forms up to surface terms

$$S = m \int d^2\zeta \sqrt{g} \sqrt{(D^2 X^\mu)^2} = m \int d^2\zeta \sqrt{g} \sqrt{K^2} . \tag{7}$$

It is called gonihedric model and it was proposed for the first time in [2] and studied on the lattice in [3]. It differs from the models considered in the previous studies [4–7], because the action has the dimension of length. It is proportional to the linear size of the surface similar to the path integral action. This is in contrast with the previous proposals where the extrinsic curvature term is a dimensionless functional.

The model is invariant under two-dimensional general coordinate transformations. The higher derivatives behavior of the action S brings about new ghosts states which are hard to interprete and there is no proof that these ghosts are identical to the ghosts of the usual string. In this work we will perform a pure perturbative analysis of the model around its classical trajectory. The parametrization invariance allows us to consider just normal fluctuations

$$\delta X^\mu = \xi^i n^{i\mu} \,, \tag{8}$$

upon this action.

Our aim will be now the expansion of (1) around the solutions of the classical equations of motion for X up to quadratic fluctuations ξs in order to get the one loop quantum corrections to (1).

Due to (8) the metric changes as

$$\begin{aligned}\delta g_{ab} &= D_a X^\mu D_b \delta X_\mu + D_a \delta X^\mu D_b X_\mu + D_a \delta X^\mu D_b \delta X_\mu \\ &= -2\xi^i K^i{}_{ab} + D_a \xi^i D_b \xi^i + K^i{}_{ac} K^{jc}{}_b \xi^i \xi^j \,,\end{aligned} \tag{9}$$

and the change of the inverse metric up to quadratic fluctuations is

$$\begin{aligned}\delta g^{ab} &= -g^{ac}\delta g_{cd} g^{db} + g^{ac}\delta g_{cd} g^{de} \delta g_{ef} g^{fb} \\ &= 2\xi^i K^{iab} - D^a \xi^i D^b \xi^i + 3K^{ia}{}_l K^{jbl} \xi^i \xi^j \,.\end{aligned} \tag{10}$$

The (quadratic) change of the area element amounts to

$$\begin{aligned}\delta\sqrt{g} &= \sqrt{g}\left[\frac{1}{2}g^{ab}\delta g_{ab} - \frac{1}{4}g^{ab}\delta g_{bc} g^{cd}\delta g_{da} + \frac{1}{8}(g^{ab}\delta g_{ab})^2\right] \\ &= -\xi^i K^i + \frac{1}{2}D^c \xi^i D_c \xi^i + \frac{1}{2}R^{ij}\xi^i\xi^j \,,\end{aligned} \tag{11}$$

where $R^{ij} \equiv K^i K^j - \mathrm{Tr}(K^i K^j)$ has a trace which is the scalar curvature. The variation of the connection up to linear terms is

$$\delta\Gamma^c{}_{ab} = D_a \partial_b \delta X^\mu D^c X_\mu + D_a \partial_b X^\mu D^c \delta X_\mu \,. \tag{12}$$

In fact

$$\delta\Gamma^c{}_{ab} = \delta(g^{cd}\Gamma_{abd}) \,, \tag{13}$$

but from (4)

$$\Gamma_{abd} = \partial_a \partial_b X^\mu \partial_d X_\mu \tag{14}$$

and due to (9) we easily get (12), which amounts to

$$\delta\Gamma^c{}_{ab} = K^i{}_{ab} D^c \xi^i - D_b \xi^i K^{ic}{}_a - D_a \xi^i K^{ic}{}_b - \xi^i D_a K^{ic}{}_b \,. \tag{15}$$

Let us make now the variation of the Laplacian of X^μ. The relevant terms for our aim are the following:

$$\begin{aligned}\delta D^2 X^\mu &= g^{ab} D_a \partial_b \delta X^\mu + \delta g^{ab} D_a \partial_b X^\mu - g^{ab}\delta\Gamma^c_{ab}\partial_c X^\mu \\ &\quad + \delta g^{ab} D_a D_b \delta X^\mu - g^{ab}\delta\Gamma^c_{ab}\partial_c \delta X^\mu \,,\end{aligned} \tag{16}$$

using (5), (8), (9), (10), (12) we obtain

$$\begin{aligned}\delta D^2 X^\mu &= \Big[D^2\xi^i + \xi^j\,\mathrm{Tr}(K^i K^j) + \xi^l \xi^j\,\mathrm{Tr}(K^l K^j K^i) - D^a \xi^j D^b \xi^j K^i_{ab} \\ &\quad + 2\xi^j K^{jab} D_a D_b \xi^i - K^j D^c \xi^j D_c \xi^i + 2D_a \xi^j K^{jac} D_c \xi^i \\ &\quad + \xi^j D^c K^j D_c \xi^i\Big]\, n^{i\mu} - K^i D^d \xi^i D_d X^\mu \,.\end{aligned} \tag{17}$$

Therefore, the variation of the gonihedric action amounts to

$$\delta S = m\int d^2\zeta\sqrt{g}\left[-\xi^i K^i\sqrt{K^2}+\frac{K^i}{\sqrt{K^2}}D^2\xi^i+\frac{K^i}{\sqrt{K^2}}\,\mathrm{Tr}(K^iK^j)\xi^j\right] \tag{18}$$

$$\begin{aligned}&+\frac{m}{2}\int d^2\zeta\sqrt{g}\left[\frac{1}{\sqrt{K^2}}D^2\xi^i\mathit{\Pi}^{ij}D^2\xi^j+\frac{2}{\sqrt{K^2}}\xi^jD^2\xi^i\mathit{\Pi}^{il}\,\mathrm{Tr}(K^lK^j)\right.\\&+\frac{1}{\sqrt{K^2}}\xi^i\,\mathrm{Tr}(K^iK^l)\mathit{\Pi}^{lm}\,\mathrm{Tr}(K^mK^j)\xi^j+\sqrt{K^2}D^c\xi^i\mathit{\Pi}^{ij}D_c\xi^j\\&-\xi^i\frac{K^iK^j}{\sqrt{K^2}}D^2\xi^j+4\xi^j\frac{K^{jab}}{\sqrt{K^2}}K^iD_aD_b\xi^i+\frac{4}{\sqrt{K^2}}K^{jac}K^iD_a\xi^jD_c\xi^iK^i\\&-\frac{2}{\sqrt{K^2}}K^iK^{iab}D_a\xi^jD_b\xi^j+\frac{2}{\sqrt{K^2}}\xi^jK^iD^cK^jD_c\xi^i\\&+\frac{2K^i}{\sqrt{K^2}}\,\mathrm{Tr}(K^iK^lK^j)\xi^l\xi^j-\frac{K^2}{\sqrt{K^2}}\,\mathrm{Tr}(K^iK^j)\xi^i\xi^j\\&\left.+\frac{K^2}{\sqrt{K^2}}K^iK^j\xi^i\xi^j-2\frac{K^iK^l}{\sqrt{K^2}}\xi^i\xi^j\,\mathrm{Tr}(K^lK^j)\right]+O(\xi^3)\,,\end{aligned} \tag{19}$$

where we have defined

$$\mathit{\Pi}^{ij}=\delta^{ij}-\frac{K^iK^j}{K^2}\,, \tag{20}$$

with the following features:

$$\mathit{\Pi}^2=\mathit{\Pi}\,,\qquad \mathit{\Pi}^{ij}K^i=0\,. \tag{21}$$

From (18) one can immediately recognize that the equation of motion for X is given by

$$D^2\left(\frac{K^i}{\sqrt{K^2}}\right)=K^i\sqrt{K^2}-\frac{K^i}{\sqrt{K^2}}\,\mathrm{Tr}(K^iK^j)\,. \tag{22}$$

We will make the following approximation:

$$D_aK^i=0 \tag{23}$$

which reduces (22) to

$$K^i\,\mathrm{Tr}(K^iK^j)=K^2K^j\,,\qquad K^i\neq 0\,. \tag{24}$$

We will consider the short wavelength fluctuations around the classical trajectory given by (24). They are dominated by the term

$$\frac{1}{\sqrt{K^2}}D^2\xi^i\mathit{\Pi}^{ij}D^2\xi^j\,. \tag{25}$$

Because Π^{ij} is not invertible the propagator can be defined only for the $D-3$ components of the ξ field which are not zero modes of Π. In order that the path integral on ξ fields is well defined the zero modes of the ξ fields must decouple. In order to prove this decoupling at this order we make the following decomposition:

$$\xi^i = \xi^i_0 + \xi^i_1 \,, \tag{26}$$

where

$$\xi^i_1 \equiv \Pi^{ij}\xi^j \,, \tag{27}$$

and due to (21)

$$\xi^i_0(\zeta) = K^i f(\zeta) \,, \tag{28}$$

where f is an arbitrary function of the world-sheet. By substituting (27) and (28) in (19) and using (21) we easily get

$$\begin{aligned}
\delta S \;=\; & \frac{m}{2}\int d^2\zeta\sqrt{g}\left[\frac{1}{\sqrt{K^2}}D^2\xi^r_1 D^2\xi^r_1 + \frac{2}{\sqrt{K^2}}D^2\xi^r_1\,\mathrm{Tr}(K^r K^s)\xi^s_1\right.\\
& \left. + \sqrt{K^2}D_c\xi^r_1 D^c\xi^r_1 - \frac{2}{\sqrt{K^2}}K^i K^i{}_{ab}D^a\xi^r_1 D^b\xi^r_1\right]\\
& - \frac{m}{2}\int d^2\zeta\sqrt{g}\sqrt{K^2}\left[K^2 f D^2 f + 2K^i K^i{}_{ab}D^a f D^b f\right]\\
& + m\int d^2\zeta\,\xi^i G^{ij}\xi^j \,,
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
G^{ij} \;=\; & \frac{1}{2\sqrt{K^2}}\,\mathrm{Tr}(K^i K^l)\Pi^{lm}\,\mathrm{Tr}(K^m K^j) + \frac{1}{2\sqrt{K^2}}\,\mathrm{Tr}(K^j K^l)\Pi^{lm}\,\mathrm{Tr}(K^m K^i)\\
& + \frac{K^l}{\sqrt{K^2}}\,\mathrm{Tr}(K^i K^j K^l) + \frac{K^l}{\sqrt{K^2}}\,\mathrm{Tr}(K^j K^i K^l) - \sqrt{K^2}\,\mathrm{Tr}(K^i K^j)\\
& + \sqrt{K^2}K^i K^j - \frac{K^i K^l}{\sqrt{K^2}}\,\mathrm{Tr}(K^l K^j) - \frac{K^j K^l}{\sqrt{K^2}}\,\mathrm{Tr}(K^l K^i) \,,
\end{aligned} \tag{30}$$

and $1 \le r \le D-3$.

It is easy to prove that using (24)

$$K^i G^{ij} = \frac{2K^l K^i\,\mathrm{Tr}(K^j K^i K^l) - 2K^i (K^2)^2}{\sqrt{K^2}} \,. \tag{31}$$

Since three general symmetric matrices A, B, C have the following feature [8]:

$$\begin{aligned}
\mathrm{Tr}(ABC) \;=\; & -\mathrm{Tr}(BAC) + \mathrm{Tr}(A)\,\mathrm{Tr}(BC) + \mathrm{Tr}(B)\,\mathrm{Tr}(AC)\\
& + \mathrm{Tr}(C)\,\mathrm{Tr}(AB) - \mathrm{Tr}(A)\,\mathrm{Tr}(B)\,\mathrm{Tr}(C) \,,
\end{aligned} \tag{32}$$

we get

$$G^{ij}K^j = 0\,. \tag{33}$$

It implies that in (29) we have

$$\xi^i G^{ij} \xi^j = \xi_1^r G^{rs} \xi_1^s\,. \tag{34}$$

Moreover, integrating by part and due to the symmetry under permutation of i and Lorentz invariance we have

$$\int d^2\zeta \sqrt{g} K^i {K^i}_{ab} D^a f D^b f = -\frac{1}{2} \int d^2\zeta \sqrt{g} K^2 f D^2 f. \tag{35}$$

The results (34) and (35) imply that in the quadratic expansion of the gonihedric action the zero modes of Π^{ij} decouple, therefore, only the $D-3$ degrees of freedom ξ_1^r propagate. It could be related to some extra unknown gauge symmetry at the moment under investigations. Therefore, we can assert that the free propagator is

$$\langle \xi^r(-p) \xi^s(p) \rangle = -i \frac{\sqrt{K^2}}{p^4} \delta^{rs}\,. \tag{36}$$

We will use the dimensional regularization and due to [9]

$$\int \frac{d^n p}{(2\pi)^n} \frac{1}{p^4} = 0\,. \tag{37}$$

We can conclude that the terms containing fewer than two derivatives in ξ are irrelevant in the ultraviolet. Thus integrating out the ξ_1 fields leads to the additional one-loop action

$$S_1 = -(D-3)\,\mathrm{Tr}\ln(D^2) - \frac{L}{\sqrt{K^2}} \int d^2\zeta \sqrt{g} R\,. \tag{38}$$

where $L = \int (d^2p/(2\pi)^2)\,(1/p^2)$.

$\mathrm{Tr}\ln(D^2)$ is the usual conformal anomaly [10]. Due to the Gauss–Bonnet theorem we have got a divergency which is field independent, therefore, in agreement with [11] the theory does not get radiative corrections. Moreover, we have got the usual conformal anomaly at quantum level, whose coefficient is proportional to $(D-3)$, the number of degrees of freedom determining the one-loop dynamics of this model.

2. Two-loop perturbative analysis in conformal gauge

The perturbative two-loop expansion following the previous approach will be given in a forthcoming paper. Now we will focus our attention on the two loop calculation in the conformal gauge in which

$$g_{ab} = \rho\eta_{ab} \,. \tag{39}$$

In the following we will quote the results proved in [12].

We consider g and X as independent variables by adding to (1) the term

$$S_\lambda = -m \int d^2\zeta \lambda^{ab}(\partial_a X^\mu \partial_b X_\mu - g_{ab}) \,, \tag{40}$$

where λ^{ab} is a Lagrangian multiplier field for the constraint (2).

We split all fields into a sum of classical solution of their equations of motion plus a quantum correction (fast variables), it amounts to considering

$$X^\mu = X_0^\mu + X_1^\mu \,, \tag{41}$$

$$\rho = \rho_0 + \rho_1 \,, \tag{42}$$

$$\lambda^{ab} = \lambda_0^{ab} + \lambda_1^{ab} \,. \tag{43}$$

The investigation about the saddle point for the Lagrangian multiplier is performed by considering the ansatz

$$\lambda_0^{ab} = \lambda\sqrt{g}g^{ab} = \lambda\eta^{ab} \,, \tag{44}$$

where the conformal gauge has been used and λ is a constant field.

In order to integrate out the fast fluctuations we expand X_1^μ fields in tangential and normal components ϕ^a, ξ^i

$$X_{1\mu} = \phi^a \bar{e}_{a\mu} + \xi^i \bar{n}^i_\mu \,,$$

$$\bar{n}^{i\mu}\bar{e}^a_\mu = 0 \,, \qquad \bar{e}^\mu_a \bar{e}^a_\mu = 2\rho_0 \,.$$

The fluctuations of the Lagrange multiplier is decomposed *à* la Polyakov [4] as

$$\lambda_1^{ab}(p) = \omega(p)\left(\eta^{ab} - \frac{p^a p^b}{p^2}\right) + (p^a f^b + p^b f^a - (p\cdot f)\eta^{ab}) \,. \tag{45}$$

The interested reader can find the detailed integration of the fast variables ξ, φ, ω and f in [12]. Here we would like to remark that

- The longitudinal fields φs do not propagate. Their free propagator is zero and the vertices structure is such that one cannot construct proper bubble vacuum with propagating φs fields.

- The only physical components of X_1 fields are the normal fluctuations ξs. Their short wave-length fluctuations are still dominated by (25) but including the cubic and quartic interactions to perform calculations till two-loop order the decoupling of the zero modes of the operator Π^{ij} does not seem to be evident. We will deserve more about this point. Concerning now we have used the mass-term for ξs fields with mass proportional to λ in order to break the ξ kernel degeneration. The contributions to the effective action coming only from ξs fields at two-loop order are given by

$$-\frac{(D-3)(D-5)}{32\pi^2}\left(\frac{\lambda}{m}\right)^2 \ln^2\left(\frac{\lambda}{M}\right)\int d^2\zeta\sqrt{g}\sqrt{K^2}\,. \tag{46}$$

  From the point of view of the previous discussed extra symmetry this result is not very encouraging because it seems assert that not all $D-3$ degrees of freedom give contributions at two loop order for the renormalization of the parameter m.

- The observed decoupling at one loop between the degrees of freedom ξs and the components of λ_1 is not more present when the cubic vertex $f\xi\xi$ is included. The contribution to the effective action give a term breaking the stability of the model, amounting to

$$\frac{(D-3)}{16\pi^2}\frac{\lambda}{m}\ln^2\left(\frac{\lambda}{M}\right)\int d^2\zeta\sqrt{g}K^2\,, \tag{47}$$

  the Polyakov–Kleinert smooth string.

The finite effective action till two-loop order is given by

$$\begin{aligned} S_{\text{eff}} &= m\left[1+a\frac{\lambda}{m}\left(\ln\left(\frac{\lambda}{M}\right)-1\right)+b\left(\frac{\lambda}{m}\right)^2\ln^2\left(\frac{\lambda}{M}\right)\right] \\ &\quad\times\int d^2\zeta\sqrt{g}\sqrt{K^2}+\frac{a}{4\pi}\frac{\lambda}{m}\ln^2\left(\frac{\lambda}{M}\right)\int d^2\zeta\sqrt{g}K^2\,, \end{aligned} \tag{48}$$

where $a=(D-3)/(4\pi)$ and $b=-[(D-3)(D-5)]/(32\pi^2)$. Eq. (48) has a nontrivial saddle point for $\lambda=M$. Perturbations around this saddle point do not add any quantum corrections to the model and avoid in particular the appearance of the Polyakov–Kleinert term.

3. Conclusion

The one and two-loop perturbative analysis of the gonihedric model has shown that this model does not receive any quantum corrections around its classical trajectory. In particular the finite effective action exhibits a non-trivial saddle point enforcing the relation between g and X at quantum level. Moreover, there are strong indications about the existence of an extra gauge symmetry which reduces the number of the physical degrees of freedom compared to the usual string. The results of the two-loop analysis do not seem to agree with this decoupling, maybe because it was performed by adding extra degrees of freedom as the Lagrangian multiplier. It might be that a pure perturbative analysis could again show the previous decoupling. A non-trivial test in order to understand the number of the physical degrees of freedom of the model could be the calculation of the static quark-antiquark potential in this model. In fact its subleading $1/R$ term has an universal coefficient depending on the number of the degrees of freedom of the string model and not on the details of their interactions. Therefore, for this model it should be proportional to $D - 3$.

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