THE DIJKGRAAF–VAFA CORRESPONDENCE FOR THEORIES WITH FUNDAMENTAL MATTER FIELDS*

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In this talk I describe some applications of random matrix models to the study of $\mathcal{N}=1$ supersymmetric Yang–Mills theories with matter fields in the fundamental representation. I review the derivation of the Veneziano–Yankielowicz–Taylor/Affleck–Dine–Seiberg superpotentials from constrained random matrix models (`hep-th/0211082`), a field theoretical justification of the *logarithmic* matter contribution to the Veneziano–Yankielowicz–Taylor superpotential (`hep-th/0306242`) and the random matrix based solution of the complete factorization problem of Seiberg–Witten curves for $\mathcal{N}=2$ theories with fundamental matter (`hep-th/0212212`).

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1. Introduction

One of the most important and challenging problems in physics is the understanding of the non-perturbative properties of gauge theories. In the last 10 years a series of significant breakthroughs were made in the study of *supersymmetric* gauge theories, where the additional symmetry properties allowed to obtain exact non-perturbative information at the same time unraveling various unexpected mathematical structures.

The first wave of research was based on exploiting the holomorphic properties of various $\mathcal{N}=1$ non-perturbative quantities (see *e.g.* [1]). The second breakthrough, this time giving very complete information on $\mathcal{N}=2$ theories, showed that the low energy dynamics is encoded in properties of certain elliptic (or hyperelliptic) curves — the Seiberg–Witten curves [2, 3].

The most recent breakthrough arose in the wake of the gauge theory/string correspondence [4]. Using D-brane constructions of gauge theories and performing 'geometric transitions' to a dual geometric setup, superpotentials

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in supersymmetric gauge theories where related to the geometry of the dual Calabi–Yau manifold [5, 6]. In a subsequent crucial development Dijkgraaf and Vafa interpreted the geometrical formulas in terms of a saddle point solution of an associated random matrix model [7, 8]. Subsequently the resulting random matrix prescription for calculating the superpotential was proven purely in the field-theoretical context by using diagrammatic analysis [9] and anomaly considerations [10].

In this talk, after recalling some basic facts about $\mathcal{N}=1$ SYM theories, I would like to review three developments [11–13] in the extension of the original Dijkgraaf–Vafa proposal to supersymmetric gauge theories with matter fields in the fundamental representation.

The original proposal involved only superpotentials expressed in terms of the glueball (gaugino bilinear) superfield S. In [11] we showed how one could incorporate into the framework mesonic superfields, and how to obtain the classical Veneziano–Yankielowicz–Taylor/Affleck–Dine–Seiberg superpotentials directly from the matrix model.

The structure of the superpotentials involve typically logarithmic and power-series terms. The latter are quite well understood [9, 10] while the former remain somewhat mysterious. In [12] we gave a field-theoretic diagrammatic derivation of the *matter induced* logarithmic part of the VYT potential.

Finally I would like to review the use of random matrix models to obtain an explicit complete factorization [13] of the Seiberg–Witten curve of an $\mathcal{N}=2$ $U(N_c)$ SYM theory with $N_f < N_c$ fundamental flavours. This is equivalent to finding the submanifold of $\mathcal{N}=2$ moduli space of vacua where all monopoles in the theory become massless.

2. Physics of $\mathcal{N}=1$ SYM theories with fundamental matter

In order to study the dynamics of the theory at low energies one is interested in obtaining the low energy effective action for the relevant degrees of freedom. In the case of $\mathcal{N}=1$ gauge theories most of such degrees of freedom may be described by chiral (and anti-chiral) superfields. Then the constraints of supersymmetry restrict the effective action to be of the form:

$$S_{\text{eff}} = \int d^2\theta d^2\bar{\theta}\, K(\Phi,\tilde{\Phi}) + \int d^2\theta\, W_{\text{eff}}(\Phi) + \int d^2\bar{\theta}\, \tilde{W}_{\text{eff}}(\tilde{\Phi})\,, \tag{1}$$

where Φ stands here for a generic chiral superfield. In general little is known about the non-chiral part $K(\Phi,\tilde{\Phi})$, but a lot of information can be obtained on the effective superpotential $W_{\text{eff}}(\Phi)$. Its knowledge allows us to determine vacuum expectation values (VEV) of Φ from the equation

$$\frac{\partial W_{\text{eff}}(\Phi)}{\partial \Phi} = 0\,. \tag{2}$$

Let us briefly recall what is known about effective superpotentials first for pure $\mathcal{N} = 1$ SYM with gauge group $\mathrm{SU}(N_c)$, and then for the theory with N_f fundamental flavours.

Pure $\mathcal{N} = 1$ SYM

A natural chiral superfield which can be built from the gauge field is

$$S = -\frac{1}{32\pi^2} \mathrm{tr}\, \mathcal{W}_\alpha \mathcal{W}^\alpha \tag{3}$$

whose lowest component is the gaugino bilinear. The form of the effective superpotential $W_{\mathrm{eff}}(S)$ has been determined by Veneziano and Yankielowicz based on anomaly considerations [14]:

$$W_{\mathrm{VY}}(S) = -S \log \frac{S^{N_c}}{\Lambda^{3N_c}} + 2\pi i \tau_0 S\,. \tag{4}$$

The second piece is just the tree level YM coupling ($\tau_0 = \theta/2\pi + 4\pi i/g^2_{\mathrm{YM}}$) and can be absorbed into the first one through a redefinition of Λ. The power S^{N_c} is determined by the anomaly, while the exponent of Λ is just the coefficient of the 1-loop β function, in order for the superpotential to be RG invariant. $W_{\mathrm{VY}}(S)$ leads to a nonzero VEV for S-gaugino condensation.

$\mathcal{N} = 1$ SYM with $N_f < N_c$ flavours

When we add to the theory matter fields in the fundamental representation there appear additional mesonic chiral superfields

$$X_{ij} = \tilde{Q}_i Q_j\,. \tag{5}$$

Veneziano, Yankielowicz and Taylor determined the appropriate superpotential again using anomaly considerations [15]

$$W_{\mathrm{VYT}}(S,X) = (N_f - N_c) S \log \frac{S}{\Lambda^3} - S \log \frac{\det X}{\Lambda^{2N_f}} = -S \log \left(\frac{S^{N_c - N_f} \det X}{\Lambda^{3N_c - N_f}} \right). \tag{6}$$

Note the modification of the anomaly coefficient $(N_c - N_f)$ and the 1-loop β function $3N_c - N_f$ due to the matter fields. Later we will show how these modifications arise from the matrix model framework. After one integrates out S one is left with the Affleck–Dine–Seiberg superpotential [16] for the mesonic superfield only:

$$W_{\mathrm{ADS}}(X) = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det X} \right)^{\frac{1}{N_c - N_f}}\,. \tag{7}$$

Intriligator–Leigh–Seiberg (ILS) linearity principle

Suppose that the gauge theory is deformed by some tree level potential $V_{\mathrm{tree}}(X)$ (*e.g.* mass terms for the flavours and some self-interactions). Intriligator, Leigh and Seiberg [17] conjectured that the full dynamics is described just by the sum

$$W_{\mathrm{VYT}}(S,X)+V_{\mathrm{tree}}(X) \tag{8}$$

i.e. the dynamical potential is not modified by the deformation.

This concludes the very brief review of classical results on $\mathcal{N}=1$ SYM theories. In the next section we will present the main new ingredient — the link with random matrix models proposed by Dijkgraaf and Vafa, and proceed to describe its applications.

3. The Dijkgraaf–Vafa correspondence

Dijkgraaf and Vafa proposed an effective way of calculating superpotentials for $\mathcal{N}=1$ theories with adjoint fields and arbitrary tree level superpotentials [8] (later generalized also to include fields in the fundamental representation [18]):

$$W_{\mathrm{eff}}(S)=W_{\mathrm{VY}}(S)+N_c\frac{\partial\mathcal{F}_{\chi=2}(S)}{\partial S}+\mathcal{F}_{\chi=1}(S)\,, \tag{9}$$

where the $\mathcal{F}_i$'s are defined through a matrix integral

$$\mathrm{e}^{-\sum_\chi \frac{1}{g_s^\chi}\mathcal{F}_\chi(S)}=\int D\varPhi DQ_iD\tilde{Q}_i\,\exp\left\{-\frac{1}{g_s}W_{\mathrm{tree}}(\varPhi,Q_i,\tilde{Q}_i)\right\}. \tag{10}$$

Here $\varPhi$ is an $N\times N$ matrix, while the Q_i's are N component (complex) vectors. In this expression one takes the limit $N\to\infty$, $g_s\to 0$ with $g_sN=S=$ const in order to isolate graphs with the topology of a sphere ($\mathcal{F}_{\chi=2}$) and of a disk ($\mathcal{F}_{\chi=1}$).

The above expression gives a prescription for the superpotential only in terms of the glueball superfield S. However it is interesting to ask if one could obtain directly from matrix models the superpotential which involves also mesonic superfields. A proposal for doing that was given in [11] and will be described in the next section.

4. Mesonic superpotentials from Wishart random matrices

In order to express the effective superpotential in terms of mesonic superfields $X_{ij}=Q_i^\dagger\tilde{Q}_j$, it was proposed in [11] to perform only a partial integration over the $Q's$ in (10) and impose the constraint $X_{ij} = Q_i^\dagger \tilde{Q}_j$ directly in the matrix integral *i.e.*

$$\mathrm{e}^{-\sum_\chi \left(\frac{N}{S}\right)^\chi \mathcal{F}_\chi(S,X)} = \int DQ_i D\tilde{Q}_i\, \delta(X_{ij} - Q_i^\dagger \tilde{Q}_j) \exp\left\{-\frac{N}{S} V_{\mathrm{tree}}(X)\right\} . \quad (11)$$

Then the effective superpotential involving both S and X is obtained from

$$W_{\mathrm{eff}}(S,X) = W_{\mathrm{VY}}(S) + N_c \frac{\partial \mathcal{F}_{\chi=2}(S,X)}{\partial S} + \mathcal{F}_{\chi=1}(S,X) . \quad (12)$$

For theories with matter fields only in the fundamental representation (as written in (11)) the above simplifies since $\mathcal{F}_{\chi=2} = 0$.

From (11) we see that $V_{\mathrm{tree}}(X)$ contributes directly to $\mathcal{F}_{\chi=1}(S,X)$. This is in complete agreement with the ILS principle where the 'dynamical' Veneziano–Yankielowicz–Taylor superpotential is not influenced by the tree level deformation.

The 'dynamical' contribution to $\mathcal{F}_{\chi=1}(S,X)$ will come from the constrained integral over N_f vectors of length N

$$\int DQ_i D\tilde{Q}_i\, \delta(X_{ij} - Q_i^\dagger \tilde{Q}_j) . \quad (13)$$

Up to an inessential term $\exp(-\mathrm{tr}\, X)$ this is just the probability distribution of (complex) Wishart random matrices [19]. The result is known[1] and for the case $N > N_f$ reads

$$\int DQD\tilde{Q}\, \delta(\tilde{Q}Q - X) = \frac{(2\pi)^{\frac{N(N+1)}{2}}}{\prod_{j=N-N_f+1}^{N} (j-1)!} (\det X)^{N-N_f} . \quad (14)$$

Extracting now the large N asymptotics according to (11) one sees that the normalization factor gives rise to a $N_f S \log S$ term, which reflects the contribution of the matter fields to the anomaly. The $\det X$ term leads to the correct dependence on the mesonic superfield. The full superpotential is then

$$W_{\mathrm{VY}}(S) + N_f S \log \frac{S}{\Lambda^3} - S \log\left(\frac{\det X}{\Lambda^{2N_f}}\right) + V_{\mathrm{tree}}(X) = W_{\mathrm{VYT}} + V_{\mathrm{tree}}(X) \quad (15)$$

as expected from anomaly considerations [15]. It is quite surprising that both terms arise from the classical random matrix Wishart distribution.

[1] See *e.g.* [20] for a general proof and Eq. (15) in [21] for the numerical coefficient.

The case with $N_f = N_c$

It is interesting to consider the case $N_f = N_c$ with $V_{\rm tree}(X) = 0$. From the general form of (15) we see that the logarithmic term vanishes and S appears only *linearly* as

$$W_{\rm eff} = S \log\left(\det \frac{X}{\Lambda^{2N_c}}\right) \tag{16}$$

thus it generates in a natural way a constraint surface (moduli space) satisfying the equation $\det X = \Lambda^{2N_c}$. On this surface the effective potential vanishes. This is the correct behaviour for $N_f = N_c$ and agrees with Seiberg's quantum constraint

$$\det X - B\bar{B} = \Lambda^{2N_c} \tag{17}$$

when the baryonic fields are integrated out [22]. However a direct inclusion of the baryonic superfields into the random matrix or a more general combinatorial framework remains an open problem [23, 24].

5. Perturbative justification of mesonic superpotentials

For adjoint matter fields the non-logarithmic part of the Dijkgraaf–Vafa prescription has been verified by a perturbative diagrammatic calculation [9] and arguments based on generalized Konishi anomalies [10]. However the origin of the logarithmic Veneziano–Yankielowicz term remains from this point of view quite obscure — as it should arise from Feynman graphs which involve 'dynamical' vector superfield loops.

Since a logarithmic term also appeared from a random matrix calculation in the constrained matrix integral it is interesting to check if the matter induced part of the Veneziano–Yankielowicz superpotential may be also obtained diagrammatically directly in the gauge theory. This calculation was done in [12].

The matter contribution to the effective potential in S (with all matter superfields integrated out) is given by [9]

$$\int DQD\tilde{Q}\; e^{\int d^4x d^2\theta\, \left(-\frac{1}{2}\tilde{Q}(\Box - i\mathcal{W}^\alpha\partial_\alpha)Q + W_{\rm tree}(\tilde{Q},Q)\right)}\,, \tag{18}$$

where $\mathcal{W}^\alpha$ is the external field related to S through (3) and $\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha}$. In [12], in order to obtain the effective superpotential involving also mesonic superfields we introduce the superspace constraint

$$X = \tilde{Q}Q \tag{19}$$

by inserting into (18) a Lagrange multiplier chiral superfield α. Using the fact, exploited in [9], that the antichiral sector does not influence chiral superpotentials we introduce also an antichiral partner $\bar{\alpha}$ with a tree level potential $\bar{\alpha}^2$ *i.e.*

$$\int d^4xd^4\theta\ \bar{\alpha}\alpha + \int d^4xd^2\theta\ \bar{\alpha}^2. \tag{20}$$

The path integral over $\bar{\alpha}$ can be carried out exactly and yields (*cf.* [9])

$$-\frac{1}{2}\int d^4xd^2\theta\ \alpha\Box\alpha. \tag{21}$$

Therefore the final path integral which one has to evaluate is

$$\int D\alpha D\tilde{Q}DQ\ \mathrm{e}^{\int d^4xd^2\theta\left(-\frac{1}{2}\tilde{Q}(\Box - i\mathcal{W}^\alpha\partial_\alpha)Q-\frac{1}{2}\alpha\Box\alpha-\alpha X+\alpha\tilde{Q}Q\right)}\,. \tag{22}$$

This is a nontrivial interacting field theory, but since we want to extract only the $\mathrm{tr}\,\mathcal{W}^2$ terms we can allow only at most two $\mathcal{W}$ insertions in a $\tilde{Q}Q$ loop. The structure of the integration over fermionic momenta then significantly reduces the number of contributing graphs. In particular we are left only with graphs coming from (22) which have the structure of $\tilde{Q}Q$ loops connected by at most one α propagator and α propagators connected to the external field X as shown in Fig. 1. Since the α propagators are then evaluated necessarily at zero momentum one has to include an IR cut-off Λ_{IR}.

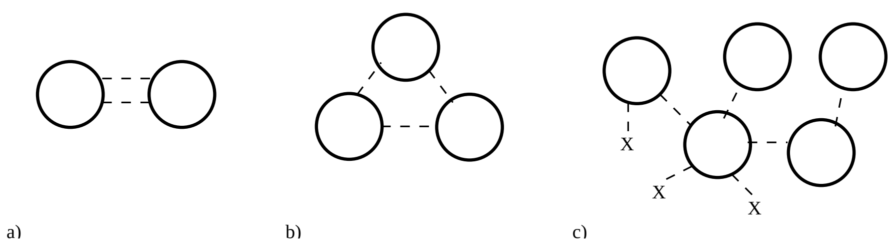

Fig. 1. Only tree level graphs survive (like the one shown in Fig. 1(c)).

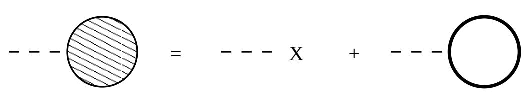

Fig. 2. The Schwinger–Dyson equation for F.

The form of the superpotential then follows from (for details see [12])

$$S \log\det\frac{F}{\Lambda}\,, \tag{23}$$

where F is the α 1-point function determined by a Schwinger–Dyson equation (Fig. 2):

$$F = -\frac{1}{\Lambda_{\rm IR}} X + \frac{1}{\Lambda_{\rm IR}} \frac{S}{F} \,. \tag{24}$$

The only IR-finite solution is $F = SX^{-1}$ which when inserted into (23) yields the expected result (compare with (15))

$$N_f S \log \frac{S}{\Lambda^3} - S \log \det \frac{X}{\Lambda^2} \tag{25}$$

including the matter generated logarithmic term.

6. Random matrices and Seiberg–Witten curves

In the preceeding sections we described the rederivation using either random matrix or diagrammatic methods of the Veneziano–Yankielowicz–Taylor/Affleck–Dine–Seiberg superpotentials. It is interesting to ask whether one can use the close relation between random matrix models and SYM theories with matter fields to obtain some new nontrivial information about the latter. An example of this kind of result is the effective factorization of Seiberg–Witten curves from random matrix considerations [13]. Before we describe it in the following section let us first recall some basic facts about $\mathcal{N} = 2$ SYM theories.

At low energies $\mathcal{N} = 2$ $\mathrm{U}(N_c)$ SYM theories develop an N_c-dimensional moduli space of vacua parametrized by vacuum expectation values

$$u_p = \left\langle \frac{1}{p} \mathrm{tr}\, \Phi^p \right\rangle \tag{26}$$

with $1 \leq p \leq N_c$. The IR dynamics of the theory around each such vacuum is described by the Seiberg–Witten curve

$$y^2 = P_{N_c}(x, u_k)^2 - 4\Lambda^{2N_c - N_f} \prod_{i=1}^{N_f} (x + m_i) \,, \tag{27}$$

where the polynomial $P_{N_c}(x, u_k) = \langle \det(xI - \Phi) \rangle \equiv \sum_{\alpha=0}^{N_c} s_\alpha x^{N_c - \alpha}$ depends in an explicit way on the u_k's:

$$\alpha s_\alpha + \sum_{k=0}^{\alpha} k s_{\alpha-k} u_k = 0 \,, \tag{28}$$

$$s_0 = 1, \qquad u_0 = 0 \,. \tag{29}$$

Of particular interest (*cf.* [25]) are the vacua where all the monopoles of the theory are massless. Then the Seiberg–Witten curve *completely factorizes*, *i.e.* the right hand side of (27) has only *two* single zeroes, all the remaining zeroes are double. The Seiberg–Witten curve then has to have the form

$$y^2 = (x-a)(x-b)H_{N_c-1}(x)^2 \,. \tag{30}$$

The problem is to find the moduli $\{u_k\}$'s for which this happens. One expects a 1-parameter family of solutions. For pure $\mathcal{N}=2$ theory ($N_f=0$) the explicit form of this submanifold of vacua has been found using special properties of Chebyshev polynomials [25]. Unfortunately these methods cannot be extended to the case of $N_f>0$. In [13], using random matrix model techniques, we obtained formulas for u_k when the general form of the Seiberg–Witten curve (27) completely factorizes for $N_f<N_c$ and arbitrary masses m_i.

We note that the general problem of factorizing (27) is highly nonlinear and involves coupled sets of polynomial equations. In fact we would not expect *a-priori* that a closed-form analytic solution would exist at all.

In the following section we will briefly describe the results of [13]. Other studies of the close link between Seiberg–Witten curves and random matrices include [26–30].

7. Factorization solution of Seiberg–Witten curves for theories with fundamental matter

The main tool which allows to obtain the factorization of SW curves is the study of the $\mathcal{N}=2$ theory deformed by a tree-level superpotential

$$W_{\mathrm{tree}} = \sum_{p=1}^{N_c} g_p \cdot \frac{1}{p}\mathrm{tr}\,\Phi^p. \tag{31}$$

Then once the factorization solution $u_p^{\mathrm{fact.}}$ is known, the *effective* superpotential is given by

$$W_{\mathrm{eff}} = \sum_{p=1}^{N_c} g_p u_p^{\mathrm{fact.}}(\Lambda, m_i, T), \tag{32}$$

which should then be minimized with respect to the parameter T of $u_p^{\mathrm{fact.}}$. If we integrate in S by performing a Legendre transformation with respect to $\log \Lambda^{2N_c-N_f}$ we obtain the superpotential

$$\begin{aligned} W_{\mathrm{eff}}(S,u_1,\Omega,\Lambda) &= S\log\Lambda^{2N_c-N_f} + W_{\mathrm{eff}}(S,u_1,\Omega) \\ &= S\log\Lambda^{2N_c-N_f} - S\log\Omega^{2N_c-N_f} + \sum_{p=1}^{N_c} g_p u_p^{\mathrm{fact.}}(\Omega,u_1)\,. \end{aligned} \tag{33}$$

In order to obtain the factorization solution $u_p^{\text{fact.}}$ from random matrix models we should first use the Dijkgraaf–Vafa prescription to obtain $W_{\text{eff}}(S)$ from a random matrix expression and then recast it in the gauge-theoretic form $W_{\text{eff}}(S, u_1, \Omega)$ defined by (33) which is *linear* in the couplings g_p — this is the most difficult part of the computation. Then one can directly read off the factorization solution from the coefficients of the couplings.

The structure of the solution obtained in [13] is the following:

$$u_p^{\text{fact.}} = N_c \mathcal{U}_p^{\text{pure}}(R,T) + \sum_{i=1}^{N_f} \mathcal{U}_p^{\text{matter}}(R,T,m_i)\,, \tag{34}$$

where the two random matrix parameters R and T are related to the physical parameters Λ, u_1 by the constraints

$$\Lambda^{2N_c-N_f} = \frac{R^{N_c}}{\prod_{i=1}^{N_f} \frac{1}{2}\left(m_i + T + \sqrt{(m_i+T)^2-4R}\right)}\,, \tag{35}$$

$$u_1 = N_c T - \frac{1}{2}\sum_{i=1}^{N_f} m_i + T - \sqrt{(m_i+T)^2-4R}\,. \tag{36}$$

In (34) $\mathcal{U}_p^{\text{pure}}(R,T)$ is the factorization solution for *pure* $\mathcal{N}=2$ theory

$$\mathcal{U}_p^{\text{pure}}(R,T) = \frac{1}{p}\sum_{q=0}^{[p/2]} \begin{pmatrix} p \\ 2q \end{pmatrix} \begin{pmatrix} 2q \\ q \end{pmatrix} R^q T^{p-2q} \tag{37}$$

while the explicit form of $\mathcal{U}_p^{\text{matter}}(R,T,m_i)$ is given in Section 7 of [13]. Here we just cite the result for $p \leq 3$:

$$\mathcal{U}_1^{\text{matter}}(R,T,m) = \frac{1}{2}\left(-m - T + \sqrt{(m+T)^2-4R}\right), \tag{38}$$

$$\mathcal{U}_2^{\text{matter}}(R,T,m) = \frac{1}{4}\left(m^2 - 2R - T^2 + (T-m)\sqrt{(m+T)^2-4R}\right), \tag{39}$$

$$\mathcal{U}_3^{\text{matter}}(R,T,m) = \frac{1}{6}\left(-m^3 - 6RT - T^3 + (m^2 + 2R - mT + T^2) \times\sqrt{(m+T)^2-4R}\right). \tag{40}$$

Let us note some striking features of the solution (34). Firstly, it has an extremely simple dependence on the number of colours. Secondly, each flavour contributes linearly. Thirdly, the solution for pure SYM theory appears as a part of the expression. All of these features are quite surprising if we keep in mind the very much nonlinear character of the factorization problem for the curve (27). Also from the physical point of view such unexpected linearization seems to suggest some hidden structure of the SYM theory with fundamental flavours.

Recently, the factorization solution (34) was used in [31] to rederive from the Seiberg–Witten curve perspective the Affleck–Dine–Seiberg superpotential (7).

8. Discussion

The link of random matrices to supersymmetric gauge theories is one of the most exciting theoretical developments in the last year. It is notable as giving a completely new *exact* application of random matrix models. The interest lies both in reinterpreting classical random matrix results/ensembles in a new language and context, and in using the novel random matrix methods to obtain new results in supersymmetric gauge theories. It raises also numerous questions of a more mathematical nature. In particular it would be interesting to understand the precise interrelation between Calabi–Yau constructions and random matrices in the most general setting. This link was in fact at the origin of the Dijkgraaf–Vafa proposal.

Last but not least it would be interesting to understand the mathematical structures which allowed to use random matrix model calculations to factorize Seiberg–Witen curves. The solution obtained in [13] is based on a reinterpretation of a random matrix formula using input from gauge-theoretical reasonings involving such non-perturbative concepts as 'integrating-in' etc. . It would be fascinating to link directly the underlying geometry of factorization of Seiberg–Witten curves with random matrix considerations. Another interesting open problem would be to generalize the solution to the case of non-complete factorization *i.e.* with at least one monopole staying massive. This is of direct relevance to the study of the global structure of $\mathcal{N} = 1$ vacua [32–34].

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