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SPACE-TIME FOAM IN 2D AND THE SUM OVER TOPOLOGIES*

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It is well-known that the sum over topologies in quantum gravity is ill-defined, due to a super-exponential growth of the number of geometries as a function of the space-time volume, leading to a badly divergent gravitational path integral. Not even in dimension 2, where a non-perturbative quantum gravity theory can be constructed explicitly from a (regularized) path integral, has this problem found a satisfactory solution. In the present work, we extend a previous 2d Lorentzian path integral, regulated in terms of Lorentzian random triangulations, to include space-times with an arbitrary number of handles. We show that after the imposition of physically motivated causality constraints, the combined sum over geometries and topologies is well-defined and possesses a continuum limit which yields a concrete model of space-time foam in two dimensions.

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1. Quantum gravity and the sum over topologies

Many attempts of constructing a non-perturbative path integral for gravity start from the premise that this should also contain a sum over space-time topologies, formally written as

$$Z = \sum_{\text{topol.}} \int \mathcal{D}g_{\mu\nu}\ \mathrm{e}^{iS[g_{\mu\nu}]}, \tag{1}$$

with the action

$$S = \int d^4x \sqrt{|\det g|}(\kappa R - \lambda), \tag{2}$$

where each term in the sum (1) is given by the functional integral over equivalence classes $[g_{\mu\nu}]$ of metrics on a space-time of a particular topology.

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This assertion is usually followed by immediately dropping the sum again, since no way can be found to enumerate the different topologies, let alone perform the sum explicitly.

Needless to say that this state of affairs is highly unsatisfactory. Whether or not a sum over topologies should be included is connected to the nature of the fundamental degrees of freedom governing quantum gravity at the very shortest scale, about which little is known. Topological excitations seem a natural enough candidate, and pictures of a multiply-connected space-time foam[1] may be suggestive to the imagination, but there is so far little direct or indirect evidence that such structures are realized in nature.

Is there then anything we can say about the issue of topology change[2], in the absence of a full-fledged non-perturbative theory of quantum gravity? If we managed to make sense, mathematically and physically, of the sum over topologies, how would the final theory be affected by the inclusion? Any theory predicting finite probabilities for *macroscopic* topology changes is likely to be already in contradiction with observational data.

There are to our mind strong indications — at least within the realm of *Euclidean* quantum gravity — that the topological sum cannot be made meaningful, simply because *it results in too many configurations contributing to the path integral.* This is true even in dimension two, where toy models of quantum gravity (in the form of generally covariant non-perturbative Euclidean path integrals over geometries) can be defined and solved exactly. In this case, no difficulty arises with the labeling of topologies, which amounts to a single parameter g, the genus or number of handles (holes) of the 2d geometry. The topological expansion in g was the subject of intense study in the early 1990s, because it is an example of the non-perturbative sum over worldsheets of a bosonic string, in a zero-dimensional target space. The problem in making the sum well-defined stems from the factorial growth in V of the number of inequivalent 2d surfaces of a given volume V. Moreover, the coefficients in the g-expansion are positive, obstructing Borel-summability, and no way has been found to define the non-perturbative sum unambiguously (see [4, 5] for detailed discussions and references).

Given the recent successes in obtaining quantum gravity theories from state sums over *Lorentzian* geometries in dimensions two [6–11] and three [12–15] (see also [16] for a recent set of lecture notes), the question arises of how a topological sum can be incorporated in these models and whether any progress can be made in performing the sum. We have shown in [17] that for quantum gravity in two space-time dimensions the problem is indeed ame-

[1] See [1] for a review and bibliography.

[2] Performing a sum over (space-time) topologies in a path integral with fixed initial and final boundary conditions implies configurations whose spatial topology changes in time. For reviews of the issue of topology change in gravity, see [2, 3].

liorated by going to a Lorentzian signature: consideration of their causal properties leads to a natural restriction on the topology-changing geometries entering the regularized path integral, as will be explained below. The combined sum over topologies and geometries can be performed exactly, and possesses a well-defined double-scaling limit, involving both the cosmological and the gravitational coupling constants, Λ and G. For $G \to 0$, standard Lorentzian quantum gravity without holes is recovered, whereas for values larger than zero, the presence of holes leads to an observable and non-local scattering of light rays traversing the space-time. At $G = G_{\rm max}$, the system undergoes a transition to a phase of "handle condensation". In addition to a further instance of how Lorentzian-ness and causality lead to path integrals that are better behaved than their Euclidean counterparts, this opens up a new playground for gravity-inspired 2d statistical models. The remainder of this contribution describes the construction and solution of 2d Lorentzian gravity with holes obtained from the discrete one-step propagator [17]. A more detailed analysis is contained in a forthcoming publication [18].

2. Implementing topology change

There is some freedom in how to include topology-changing 1+1 dimensional space-time configurations in the gravitational path integral. Our implementation will be minimal in the sense that each hole will be allowed to exist for an infinitesimal proper-time interval only. In our discrete, triangulated framework this will mean that a hole will come into existence at some integer time t and disappear again at $t+1$. The number of allowed holes per time step $\Delta t = 1$ (in the continuum limit) will be arbitrary. As in Lorentzian quantum gravity for fixed topology, all configurations possess a globally defined proper time variable. For the sake of definiteness, we will work with spatially compact slices. Therefore, by construction a spatial slice at some integer t will have the topology S^1 of a circle, whereas for all times in the open interval $]t, t+1[$ it will be split into a constant number $g+1$ of S^1-components.

Although this seems the very mildest form of topology change imaginable in two dimensions, we will see that generic space-times of this kind are extremely ill-behaved in their geometric and causal properties, even if there is only a single hole in the entire space-time. The essential difference with the Euclidean case is that the presence (almost everywhere) of a Lorentzian structure allows us to quantify how badly causality is violated (as it necessarily must be in a topology-changing geometry). We will then argue for a restriction of the state sum to geometries whose causality violations are relatively mild. This is motivated by the search for continuum limits which do *not necessarily* exhibit macroscopic acausal and therefore physically unacceptable behaviour (adopting a similar line of argument as one would in 4d).

What we find is an exactly soluble model of 2d gravity with dynamical topology, with a well-defined double-scaling limit involving both the gravitational and cosmological coupling constants. Its acausal properties can be probed by light rays, and get larger with increasing (renormalized) gravitational constant G. This means that for $G \neq 0$ there is always a non-trivial effect coming from the "infinitesimal" holes, which may still be compatible with observation if the measuring instruments that could detect the acausality were not sufficiently sensitive. However, as we will see, for sufficiently large G any experimental detection threshold will eventually be exceeded. We interpret this behaviour as an a posteriori justification for having restricted the allowed space-time histories in the first place, in the sense that it seems unlikely that a model with significantly more general types of holes will possess any physically acceptable ground state whatsoever.

We will first give a qualitative description of space-time geometries with "bad" and "not-so-bad" topology changes, and then present a concrete realization of the latter within the framework of 2d Lorentzian dynamical triangulations. The generation of holes of either type is illustrated in Fig. 1. At time t, an initial spatial slice S^1 splits into $g+1$ components, (a1), giving rise to g saddle points. The components evolve in time until $t + \Delta t$, where they re-unite to a single S^1. A difference now arises, depending on which pairs of points are identified in the process of merging. In the not-so-bad topology change, the upper saddle point of a hole is by definition time- or light-like related to the lower saddle point of the same hole, in either component, as indicated in (a2), (a3) (for simplicity of illustration, we perform the merger only for two of the components). A merger which is not of this type is illustrated in Fig. 1, (b1). The marked point at time $t + \Delta t$ on the right-most cylinder component is supposed to lie outside the light cone of the lower saddle point[3].

To illustrate the qualitative difference between the two cases, we follow a set of parallel light rays through the resulting space-times, as indicated in Fig. 1, (a4) and (b2)[4]. In both cases, a light ray which "hits the hole" is scattered non-trivially to a different part of the manifold. However, in the case where an additional relative twist of at least one of the cylinder

[3] This is a slight abuse of language, because there is of course no light cone right *at* the saddle point. The geometry of the neighbourhoods of such saddle points plays an important part in the analysis of the causal properties of the associated space-times in the continuum [19–22].

[4] To keep the argument simple, we do not represent any effects of the (generically non-vanishing) curvature on the light propagation, which is of a quite different (namely, local) nature.

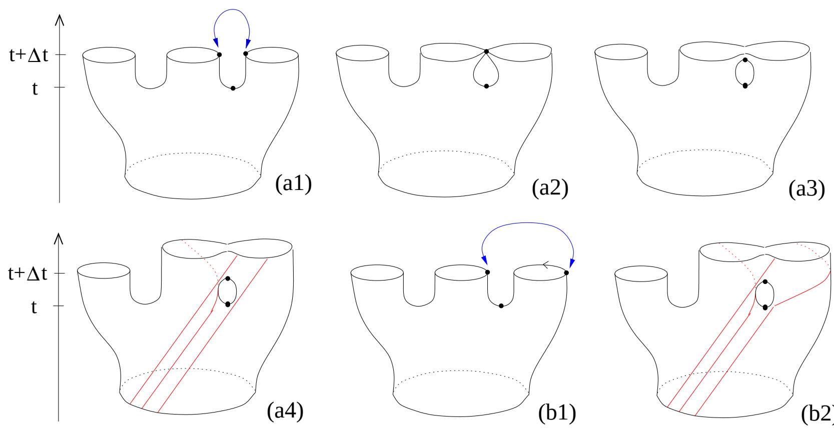

Fig. 1. A connected spatial slice splits into three components at time t, (a1). In the first example, two circles at time $t + \Delta t$ are merged into one by identifying two points which are time-like separated from the original branching point, (a2) and (a3). Parallel light rays passing between t and $t + \Delta t$ are unaffected unless they are scattered non-locally by the hole to another part of the manifold, as is the central light ray in (a4). If one of the merging points is space-like separated from the branching point at t, a twist (indicated by the arrow in the embedded picture, (b1)) is required before the regluing. In the resulting geometry, the distance between two parallel light rays which pass on either side of the hole has jumped discontinuously after the merger, (b2).

components is present (in Fig. 1, (b1) and (b2), the right cylinder has been twisted by an angle π), there is another non-local effect, consisting in a permutation of different finite sections of the propagating light front, which will persist after the hole has disappeared. The effect on the two outer parallel light rays depicted in Fig. 1, (b2), is that they are still parallel after time $t + \Delta t$, but their mutual distance will have jumped.

Note that while the effect of the direct scattering by a single hole will vanish in the limit as $\Delta t \to 0$ (corresponding to the continuum limit in the discretized model)[5], the effect of globally rearranging parts of space-time with respect to each other for the "bad" topology changes will persist in the same limit, and represents an observable, macroscopic violation of causality. We will discard such configurations from the path integral, since we do not think that these large-scale causality violations can cancel out in any superposition of such geometries. Moreover, they completely outnumber the

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[5] This does not imply that the presence of more than one hole, coming from different time slices, cannot lead to observable effects. Indeed, this is exactly what we will find below in the analysis of the continuum limit of the model.

geometries with "not-so-bad" topology changes. The precise definition of the resulting 2d quantum gravity model, its continuum limit, and its physical properties will be the subject of the following section.

3. Lorentzian quantum gravity with holes

We will now discuss how to implement "baby holes" of the type introduced in the previous section explicitly in the framework of piecewise linear two-manifolds. Recall that space-time geometries in Lorentzian dynamical triangulations are constructed by gluing together strips of height $\Delta t = 1$, where t is a discrete analogue of proper time. A given strip between integer times t and $t+1$ consists of N_t Minkowskian triangles (each with one space-like and two time-like edges), and is periodically identified in the spatial direction, as illustrated in Fig. 2.

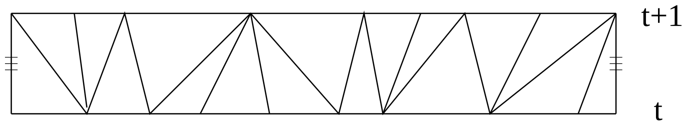

Fig. 2. A strip $[t, t+1]$ of a 2d Lorentzian triangulated space-time. The ends of the strip should be identified as indicated, leading to a compact spatial geometry S^1.

We create a hole of minimal time extension $\Delta t = 1$, and associated with a "not-so-bad" topology change, by identifying two time-like links in the same strip $[t, t+1]$ (these are links interpolating between the slices of constant time t and $t+1$), and cutting them open in the perpendicular direction, thereby creating two cylinders and a minimal hole in between (Fig. 3). This process generates two curvature singularities, at the beginning and end of the hole, which after the Wick rotation will be of the standard conical type,

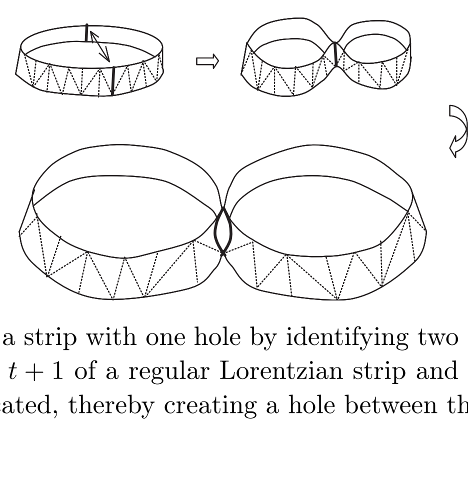

Fig. 3. Constructing a strip with one hole by identifying two of the time-like edges between times t and $t+1$ of a regular Lorentzian strip and separating them perpendicularly as indicated, thereby creating a hole between the two integer times.

and we will choose their Boltzmann weights accordingly. As anticipated earlier, the number of possible strip geometries of this type as a function of the total strip volume scales exponentially, and both the state sum and its continuum limit are completely well defined.

As in the original Lorentzian model [6], it suffices to examine the combinatorics of a single strip to determine the bulk behaviour of the model in the continuum limit (as well as the associated quantum Hamiltonian [18]). After the Wick rotation, the relevant partition function is

$$Z(\lambda,\kappa)=\sum_{l_{\rm in}}\sum_{l_{\rm out}} G_{\lambda,\kappa}(l_{\rm in},l_{\rm out};t=1)\,, \tag{3}$$

where we have performed a sum over both the initial and final boundary geometries of length $l_{\rm in}$ and $l_{\rm out}$, and where the propagator $G_{\lambda,\kappa}$ is given by

$$G_{\lambda,\kappa}(l_{\rm in},l_{\rm out};t=1)={\rm e}^{-\lambda(l_{\rm in}+l_{\rm out})}\sum_{T|_{l_{\rm in},l_{\rm out}}}{\rm e}^{-\kappa g(T)}\,. \tag{4}$$

The sum in (4) is taken over all triangulated strip geometries with boundary lengths $l_{\rm in}$ and $l_{\rm out}$, and $g\geq 0$ denotes the number of holes of the strip. In writing Eq. (4) we have used that the discrete volume of a strip is given by $N\equiv N_t=l_{\rm in}+l_{\rm out}$, as in Lorentzian gravity without topology change. Fixing $l_{\rm in}$ and $l_{\rm out}$ (and for convenience putting a mark on the entrance loop), the number

$$\tilde{G}(l_{\rm in},l_{\rm out})=\binom{l_{\rm in}+l_{\rm out}-1}{l_{\rm in}-1} \tag{5}$$

of distinct (marked) interpolating strip triangulations without holes gives rise to an overall factor 2^{N-1}. For a given triangulated strip of volume $N=l_{\rm in}+l_{\rm out}$, holes are created according to the prescription given above and as shown in Fig. 3. An alternative, planar representation of the creation of a single hole is given in Fig. 4, which shows a cut through the strip halfway between times t and $t+1$. The N time-like links of the strip appear as dots on the circle (Fig. 4(a)). The procedure for several holes is completely analogous. The only restriction one needs to impose in order to obtain a well-defined two-geometry with $g+1$ cylindrical components is that the g arrows identifying pairs of points in the corresponding planar diagram should not cross each other[6]. Also, we will impose the regularity condition that there should be at most one arrow per point. This avoids some double counting of identical geometries and eliminates a few geometries with cylinders of the size of the cutoff, and is not expected to affect the continuum limit in any

[6] The *planarity* of "arrow diagrams" like Fig. 5 is responsible for an exponential, as opposed to a factorial counting.

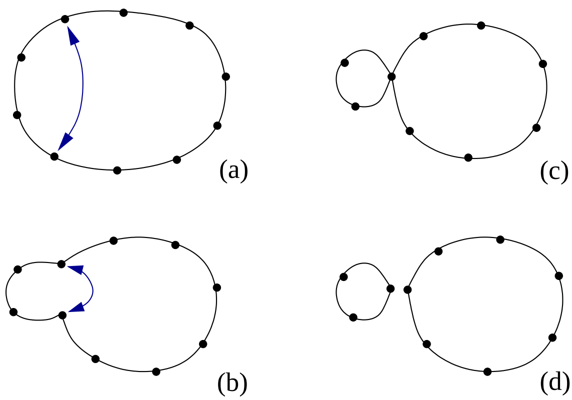

Fig. 4. Inserting a single hole into a regular strip of length $N = 10$ (a), as it appears in a slice of constant time half-way between the two boundaries. The strip is pinched along a pair of time-like links (appearing as dots), as indicated by the arrows (b), until a figure eight is obtained (c), after which the strip is separated into two cylinders as indicated in (d).

way. Note also that we are including some geometries where one or more cylinders degenerate to a point either at time t or $t+1$; this is merely to simplify some of the combinatorial formulas and again will not have any consequences for the continuum limit. Our next task is therefore to count the number of ways of inserting g holes into a strip of volume N, which is equivalent to counting graphs with N points and g arrows, an example of which is shown in Fig. 5. This is readily done by noting that the number of ways to pick $2g$ out of N points, $2g \leq N$, is given by

$$\binom{N}{2g}, \tag{6}$$

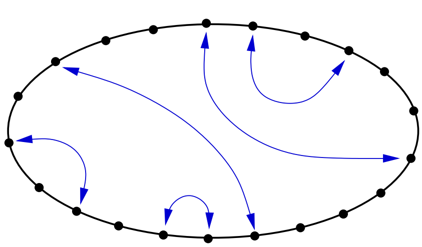

Fig. 5. A typical graph showing the pairwise identification of 10 out of $N = 21$ points, giving rise to a strip with 5 holes.

since the N points can be regarded as distinguishable (at large volumes N, a generic triangulated strip without holes will not have any symmetries). For a given set of $2g$ points, we then have to count the number of ways of connecting them by non-intersecting arches. Fortunately, this is a well-known combinatorial problem whose resolution is given by the so-called Catalan numbers

$$A(2g) = \frac{(2g)!}{g!(g+1)!}\,. \tag{7}$$

The complete formula for the partition function (3) is therefore

$$Z(\lambda,\kappa) = \frac{1}{2}\sum_{N=0}^{\infty}\sum_{g=0}^{[N/2]}\binom{N}{2g}\frac{(2g)!}{g!(g+1)!}\,\mathrm{e}^{-2\kappa g}\mathrm{e}^{-(\lambda-\log 2)N}. \tag{8}$$

After an exchange of the two sums, both of them can be performed explicitly, leading to

$$Z(\lambda,\kappa) = \frac{1}{2(1-\mathrm{e}^{-(\lambda-\log 2)})}\,\frac{1-\sqrt{1-4z}}{2z}\,, \tag{9}$$

where the second term on the right-hand side[7] depends only on the specific combination

$$z := \mathrm{e}^{-2\kappa}(\mathrm{e}^{\lambda-\log 2}-1)^{-2} \tag{10}$$

of the bare coupling constants κ and λ. The partition function (8) is convergent for (real) $\lambda > \log 2$ and $z < 1/4$. We are now interested in constructing a continuum limit of Z. This will necessarily involve an infinite-volume limit $N \to \infty$. It is straightforward to compute the expectation value of the discrete volume,

$$\langle N\rangle = -\frac{1}{Z}\frac{\partial Z}{\partial\lambda} = \frac{\mathrm{e}^{\lambda}}{(\mathrm{e}^{\lambda}-2)\sqrt{1-4z}} - 1\,, \tag{11}$$

from which we deduce that the infinite-volume limit can be obtained by letting λ approach $\log 2$ from above, just like in standard 2d Lorentzian gravity. However, this is only consistent if one stays inside the combined region of convergence of both λ *and* z. From the explicit form (10) of z this is only possible if one scales the bare inverse gravitational coupling κ in such a way as to counterbalance the divergence coming from the inverse powers of $(\mathrm{e}^{\lambda-\log 2}-1)$. More specifically, if we make a standard ansatz of canonical scaling for the cosmological coupling constant,

$$\lambda = \lambda^{\mathrm{crit}} + a^2\Lambda + O(a^3) \equiv \log 2 + a^2\Lambda + O(a^3), \tag{12}$$

[7] This term is recognized as the generating function for the Catalan numbers, and has previously appeared in a statistical model of certain "decorated" 2d Lorentzian geometries without topology changes [11].

where Λ denotes the renormalized, dimensionful cosmological constant, we obtain for any fixed value $z = c < 1/4$ of z an equation for κ, namely,

$$\kappa = -\frac{1}{2}\log\Big(c\,(a^2\Lambda)^2\Big) + O(a), \tag{13}$$

which determines the leading-order behaviour of κ as a function of the cutoff a. This relation can now be read as the defining equation for the renormalized inverse gravitational coupling K,

$$K = \kappa - 2\log\frac{1}{a\sqrt{\Lambda}} + O(a)\,, \quad \text{with} \quad K := \frac{1}{2}\log\frac{1}{c}\,. \tag{14}$$

The logarithmic subtraction is what one would expect for the renormalization of a dimensionless coupling constant. Introducing the renormalized Newton's constant $G = 1/K$, and substituting the expansions (12) and (14) into (9), one obtains to lowest order in a

$$Z = \frac{1}{a^2}\,\frac{\mathrm{e}^{2/G}}{4\Lambda}\left(1 - \sqrt{1 - 4\,\mathrm{e}^{-2/G}}\right) =: \frac{1}{a^2}\;Z^{\mathrm{R}}(\Lambda, G)\,, \tag{15}$$

where we have performed a wave function renormalization to arrive at a finite renormalized partition function $Z^{\mathrm{R}}(\Lambda, G)$. In summary, we have been led to (15) by taking a well-defined *double-scaling limit* of both the gravitational and the cosmological coupling constants, very similar to what has always been hoped for in 2d *Euclidean* quantum gravity (see, for example, [5], p.186ff).

The physical properties of the resulting continuum theory are governed by the values of these two couplings. The expectation value of the space-time volume $V := a^2 N$, computed in analogy with (11), is given by the inverse cosmological constant,

$$\langle V\rangle = -\frac{1}{Z^{\mathrm{R}}}\frac{\partial Z^{\mathrm{R}}}{\partial\Lambda} = \frac{1}{\Lambda}\,, \tag{16}$$

as one may have anticipated. The role of the gravitational constant is exhibited by computing the expectation value of the number of holes per time interval,

$$\langle g\rangle = \frac{1}{2}G^2\frac{1}{Z^{\mathrm{R}}}\frac{\partial Z^{\mathrm{R}}}{\partial G} = -\frac{1}{2}\left(1 - \frac{1}{\sqrt{1 - 4\mathrm{e}^{-2/G}}}\right). \tag{17}$$

This function is plotted in Fig. 6 in the range $G \in [0, 2/\log 4]$. For small coupling $G \approx 0$, there are basically no holes, up to $G \approx 1.33$, where there is an average of a single hole in the entire strip. Beyond this value, the number

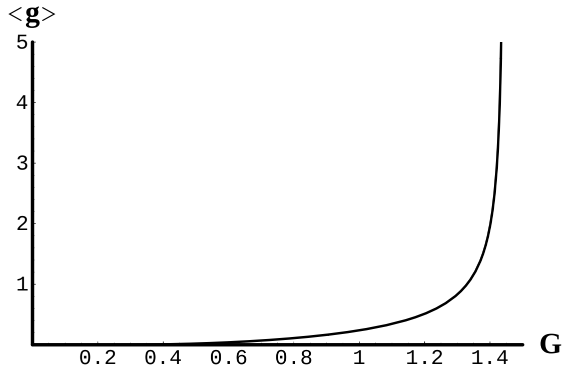

Fig. 6. The average number $\langle g\rangle$ of holes as a function of the renormalized Newton constant G.

of holes increases rapidly and diverges at the boundary $G = 1/\log 2$ of the allowed interval. The average genus is an interesting quantity because it relates in a direct way to an "observable", namely, the part of a light beam that undergoes scattering when passing through a Lorentzian space-time with baby holes. In first approximation, this is given by

$$\text{scattered portion of light beam} \propto \langle g\rangle \frac{T}{L}\,, \tag{18}$$

where L is the characteristic linear spatial extension of the "quantum universe", and T is the (continuum) time of propagation of the light beam.

4. Conclusions

We have formulated a regularized path integral over two-dimensional Lorentzian space-times, including a sum over topologies, and shown that it possesses a non-trivial continuum limit. The Lorentzian structure of the individual discretized geometries was used to classify topology changes as "bad" and "not-so-bad" in terms of their causal properties. The former were excluded from the sum over space-times because they implied large-scale causality violations, even for isolated holes of minimal duration. At the same time, this prescription turned out to eliminate a factorial divergence in the entropy part of the state sum, which in the past has proved troublesome in analogous, purely Euclidean gravitational path integrals.

The sum over the remaining topology-changing geometries can be performed explicitly, and converges for suitable choices of the two bare coupling constants. The infinite-volume is obtained by tuning the cosmological constant to its critical value. In order not to run out of the region of convergence of the wick-rotated partition function, a simultaneous scaling of the gravitational coupling is required. This double-scaling limit leads to an unambiguously defined non-perturbative theory of 2d quantum gravity, which one may think of as a concrete realization of space-time foam in two dimensions. Its physical properties depend on the values of the two renormalized couplings, where the cosmological constant Λ sets the overall scale of the universe and the renormalized Newton constant G determines the abundance of microscopic holes.

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