TCP/IP FLOW DISTRIBUTION IN RANDOM NETWORKS*

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We study the performance of scale free Internet-like networks and compare them to a classical random graph based network. The scaling of the traffic load with the nodal degree is established, and confirmed in a numerical simulation of the TCP traffic. The scaling allows us to estimate the link capacity upgrade required making and extra connection to an existing node.

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1. Introduction

Random graphs were studied since the middle of the 20th century with the initiative works of mathematicians Pál Erdős and Alfréd Rényi. In 1960 they published their paper *On the Evolution of Random Graphs*, with the first thorough study of the graph theory [1]. However, these results were strictly theoretical, since tools to measure real-world random graphs at that time were not available. With the evolution of the personal computers, we have now the possibility to study this topic in practice, and, interestingly, the computers themselves provide one of the most exciting real random graphs: the computer networks.

As these networks evolved, their properties were analysed, and soon it was clear, that the Erdős–Rényi models (ER) were not appropriate for these graphs [2]. The most important difference was the *scale free* property, that the computer networks have, but the classical models lack. It turned out,

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that this feature is shared by other types of networks, like social [3] and metabolic [4] ones. It means that the distribution of the degrees of the nodes follows a power-law distribution, while the classic graphs have a Poisson degree distribution, with an exponential tail. It shows that, in the real-world graphs, nodes with a high number of connections are much more likely than expected. Therefore, new models were necessary to describe these kind of random graphs.

One of these models was developed by László Barabási and Réka Albert (BA model) [5, 6]. Their method has two key features: *incremental growth* and *preferential attachment*. Incremental growth means that the graph is constructed by adding nodes to the existing graph, and connecting them according to a construction rule, contrary to the original Erdős–Rényi picture with a static graph. Preferential attachment means that the likelihood of a connection depends on the degree of a node, again lifting the classical assumption of equal probabilities. This model has more variations, and they are able to describe a wide class of random graphs. There is another class of models study such networks without incremental growing, purely on their statistical properties [7].

In this paper, we investigate network properties of BA-like models with different parameters and raise the question whether non-classical graphs may perform better transmitting data over them. Certain value of parameters yield a graph with an exponential tail degree distribution, hence allowing us to compare these classical type models to the ones with power-law distribution. In Sec. 2 we analyse the properties of the extended BA models, estimating their node distribution. Next, in Sec. 3 we construct networks based on these models, and simulate a network traffic on such a graph in a simplified model estimating the traffic load on the nodes. The scaling of the load with nodal degree is presented, allowing to estimate the proper bandwidth allocation when upgrading a node. In Sec. 4 we study a more realistic setup with TCP dynamics and compare the theoretical result of the previous section to the simulated ones, while in Sec. 5 we discuss the overall performance of the different simulations. Finally, we conclude out analysis.

2. The model

In the original BA-model the graph is constructed as follows. Starting from a small initial graph, we extend it by adding a new node in each step and connecting it to m randomly selected existing nodes. The probability of choosing a particular node is proportional to its degree,

$$p_i = \frac{d_i}{\sum\limits_{j=1}^{n-1} d_j}\,. \tag{1}$$

This model constructs a scale free graph, where the cumulative degree distribution has a power-law decay with an exponent 2 [5].

In the following, we modify the construction law, and use the more general form

$$p_i = \frac{d_i^\alpha}{\sum_{j=1}^{n-1} d_j^\alpha}\,, \tag{2}$$

weighting the probability with a power α of the nodal degree, similarly to Ref. [8]. The BA model corresponds to $\alpha = 1$, while for $\alpha = 0$ the preferential connectivity is cancelled, and we are back to a classical ER-like graph model with a uniform distribution, leading to exponential node distribution. While this model is similar to the original ER model, it differs in some aspects, such as it has a minimal guaranteed degree, and the ordering of the nodes presents nonzero correlations in the degrees [9]. With $\alpha \in (0,1)$, the models provide a smooth transition between the classical and the scale free models.

The degree distribution for these models can be derived following the method introduced in [5]. Here we give a fast estimate on the distribution, for the exact result see [8]. First, we estimate the rate of growth of the degree at each node, assuming that the growth of the degree is continuous in time. At time t there are exactly t nodes and $m\,t$ links between them. Hence, the expected degree value $k_i = E(d_i)$ of node i is growing as

$$\partial_t k_i(t) = \frac{m\,k_i(t)^\alpha}{\sum_{j=0}^{t} k_j(t)^\alpha}\,. \tag{3}$$

For $\alpha = 0$, the denominator is simply counting the number of nodes, and is equal to t, while for $\alpha = 1$, it is (double) counting the number of links, and thus is $2m\,t$, both being a linear function of the time t. Numerical simulations showed that for $\alpha \in (0,1)$ the denominator is well approximated by a linear function $c\,t$, where c is the α dependent measured slope. Hence, generally our differential equation can be written as

$$\partial_t k_i(t) = \frac{k_i(t)^\alpha}{c\,t}\,. \tag{4}$$

Fortunately, these equations are solvable for each α in the chosen $(0,1)$ range. Specifically, for $\alpha = 0$ we get

$$k_i(t) = \frac{\log t}{c}\,, \tag{5}$$

and more generally for $\alpha \in (0,1)$

$$k_i(t) = \left((1-\alpha) \frac{\log t}{c} \right)^{\frac{1}{1-\alpha}} . \tag{6}$$

Finally, for the scale free case $\alpha = 1$ the structure changes, and we arrive at

$$k_i(t) = t^{\frac{1}{c}} . \tag{7}$$

In order to get a formula for the degree distribution, note that for each i, $k_i(i) = m$, since at the time the node is added it has exactly m connections. The growth process for node i has a similarity to the growth of the previous nodes, and this suggests that

$$k_i(t) = k_1 \left(\frac{t}{i} \right) . \tag{8}$$

The cumulative distribution $P(k_i > x)$ counts the number of nodes with $k_i(t) > x$, *i.e.* $k_1 \left(\frac{t}{i}\right) > x$, which leads to

$$\frac{t}{k_1^{-1}(x)} > i . \tag{9}$$

Normalising back with the total number of nodes at time t (being also t) we get the approximate cumulative probability

$$P(k_i > x) \simeq \frac{1}{k_1^{-1}(x)} . \tag{10}$$

Inverting Eq. (6) we find

$$P(k_i > x) \simeq \mathrm{e}^{-\frac{c}{1-\alpha} x^{1-\alpha}} , \tag{11}$$

the Weibull-distribution for $\alpha \in (0,1)$ with scale parameter c, and shape parameter $1-\alpha$. We note, that the exact result is of the form [8]

$$P(k_i > x) \simeq x^{-\alpha} \mathrm{e}^{-\frac{c}{1-\alpha} x^{1-\alpha} \cdots} , \tag{12}$$

where ... denotes higher order correction terms.

For $\alpha = 0$ $(c = 1)$ one recovers the classical limit $P(x) \sim \mathrm{e}^{-x}$. In the region $\alpha \in (0,1)$ the distribution still vanishes exponentially, so strictly speaking no heavy tails are present, however, the probability of finding nodes with large degree increases dramatically. Numerical studies revealed, that the distribution indeed follows the form (11), however, the parameters are slightly different from the predicted ones, due to the approximation used in (4). In the limit $\alpha = 1$ ($c = 2$) we arrive at the genuine scale free heavy tail result

$$P(x) \sim x^{-c} \,. \tag{13}$$

We note, that the analysis can be extended to $\alpha > 1$, however, such a distribution actually tends to the degenerate case with one node being connected with all others in a star-like topology. For $\alpha = 1$ the original BA model results in $c = 2$, however, different growth strategies are able to vary the value of c, hence the decay exponent. In this work we restrict ourselves to the original BA model.

3. Estimation of the network load in a simplified model

In the following, first we generate random graphs on a computer according to the extended BA model (3) and study the load in a simplified network model. We assume, that each node generates a constant traffic (data stream) to all the other nodes. The amount of traffic is the same for all connections, and furthermore, we assume that the link capacity can handle the accumulated traffic. The properties of the network are analysed tuning the geometrical parameter α.

For each value of α we performed measurements on 8 different graphs (generated with the same statistical properties), and then averaged. Each topology was made out of $N = 100\,000$ nodes and $m = 3$ links per node. The traffic was routed using the standard shortest-path strategy, and the number of data streams, passing a node was counted for each node. Since by construction at least $m(N-1)$ data channel are open per node, we define the load l_i, to be the number of data channels divided by N, the number of nodes, at node i. In the large N limit this gives the number of links m, if no "foreign" connection is going through the selected node. One can regard this quantity as "weighted degree", with weights equal to the link loads.

To study the effect of congestion, we assume, that for N nodes a data stream occupies $1/N$ part of the link capacity. With $N(N-1)$ connections there will be certainly "over-used" links, where the link bandwidth capacity constrains the throughput of the node. Thus we define the throughput of a link to be the minimum of the load and the link capacity, while the throughput of the node to be the sum of the link troughputs over the links connected to the given node.

Fig. 1 shows the tails of the cumulative distribution functions (1.0−CDF) for degrees, load and (node) throughput on graphs with $\alpha = 0$, 0.5 and 1. The distribution of the *degrees* show the expected tails, exponential ($\alpha = 0$, ER graph), Weibull ($\alpha = 0.5$), and the power-law ($\alpha = 1$, BA graph),

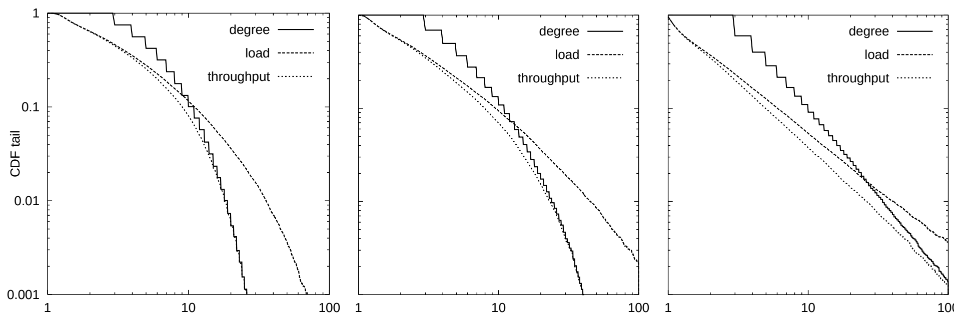

Fig. 1. Tail of cumulative distributions of the nodal degree, load and throughput for geometries with α =0 (ER graph, left), 0.5 (middle) and 1 (BA graph, right).

respectively. However, the distribution of load shows interesting deviations from the one of the nodal degree. For $\alpha = 0$ the weighted nodal degree (load) distribution can be approximated much better by a Weibull distribution than an exponential one, describing the nodal degree distribution. This is a clear indication, that the distribution of the network traffic even in a simulated "uniform" situation does not follow one-by-one the underlying network topology, rather develops a heavier tail. For $\alpha = 0.5$ the load remains Weibull, however, the shape parameter (theoretically $1-\alpha$), changes from the numerical value 0.62 to 0.32. In case of $\alpha = 1$ the power-law decay survives, but with changing CDF tail exponent, decreasing from the numerical value 1.97 to 1.25. We may conclude, that the load distribution has considerably fatter tails than the underlying nodal distribution.

The per-node *throughput* shows a transition between the nodal degree and the load distribution. For low throughput values it follows the load distribution (the bandwidth is enough to hold the traffic), however, at higher values it approaches the nodal degree distribution, simply counting the number of links. The transition is governed by the link capacities, in a real network environment with large available capacity one expects the throughput to follow the load distribution, however, in an underdesigned network it follows the degree distribution, as we show in the next section.

The change of the distribution from exponential to Weibull, and the parameter changes in the Weibull and in the power-law case in the load distribution suggests to examine the correlation between the degree and the load. This is shown in Fig. 2 (left) on a scatter-plot. Since it is log–log scaled, the linear clusters indicate a power-law correlations, $l_i = d_i^\beta$. The dependence of the scaling exponent β, on the network geometry parameter α, is shown in Fig. 2 (right). The load is pushed to have fatter tails than the degree distribution, and the more "classical" is the network the larger the deviation. For the BA geometry the load distribution decays $\sim$ 1.6 times slower than the corresponding degree distribution.

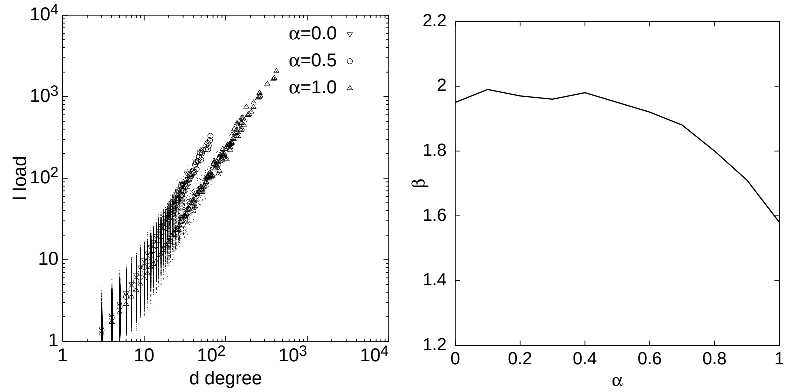

Fig. 2. The degree-load joint distribution for different network geometries (left). Scaling exponent β, of the load with the network geometry parameter α (right).

This means that not only the distribution of the load is more heavy-tailed than the distribution of the degrees, but also means that this dependence is quite strictly a power-law function. It also explains the deviations from the degree distributions, since a power of an exponentially distributed ($\alpha = 0$) random variable is Weibull, while in the case of Weibull or power-law degree distribution the transformation results only in a parameter change for the distribution. It seems that the traffic pushes the distribution into heavier tails, from the exponential distribution, and converging to a power-law through Weibulls. The initial push at $\alpha = 0$ is extremely high, with an exponent $1/2$ in the Weibull distribution. The load distribution is much less sensitive to the underlying network as the degree distribution.

Since the nodal throughput is bounded from above by the degree, it limits the throughput on the high-degree nodes, where the links are already fully utilised. Therefore, the throughput-degree joint distribution is different from the load-degree distribution, the power-law correlation is only valid for low and medium-degree nodes. But in real life the congestion at the overloaded nodes also affects the other parts of the network, since every data flow through these nodes is jammed. Furthermore, the TCP dynamics is also known to be chaotic [10] which may change the scalings observed in a simpler model. To simulate the real life situation, and compare them with the results of the simplified model, next, we simulate a realistic traffic on a computer network, too.

4. Estimation of the network load in a simulated traffic

In order to compare the theoretical results from the previous section to a more realistic setup, we simulated a TCP/IP network with $N = 1024$ nodes, $m = 3$ links per node, and a uniform link bandwidth of 1 Mb/s.

Every node communicated with a randomly selected target node. To study the effect of congestion we modelled three scenarios: a low, a medium, and a high traffic one, using constant bit-rate data flows of 16 Kb/s, 64 Kb/s, and 256 Kb/s, respectively. The simulation was ran using the Berkeley Network Simulator package [11]. The link throughput was calculated as the number of packets sent through that link, and the nodal values as the sum of the throughput of the incident links.

To our surprise, in each scenario the throughput showed a good scaling through the whole degree range (see Fig. 3). Since the throughput at the high-degree nodes is obviously limited, it must mean that the congestion limits the throughput of the other nodes in such way that the power-law throughput-degree correlation remains valid. It also means that the congestion affects each other node proportionally to its degree.

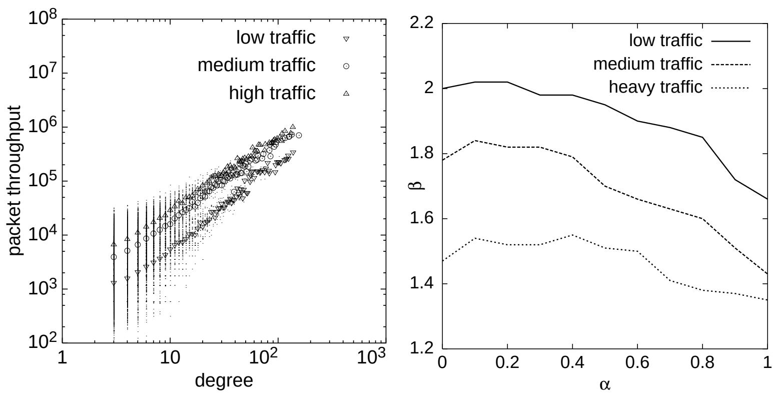

Fig. 3. Simulated traffic on an $\alpha = 1$ geometry network. The degree-load joint distribution for different network traffic (left). Scaling exponent β, of the load with the network geometry parameter α for low, medium and high traffic (right).

The correlation exponent β was, however, different in the three simulation. In the low-traffic scenario the measured distribution is exactly the same as obtained from the numerical simulations of the load in the previous section. The congestion still has not set up, and the throughput is identical to the load. As the traffic intensity grows, exponent β decreases, and flattens. It shows that the heavy traffic is more evenly spreads through the network, but its dependence on the degrees remains a power-law function.

5. Overall performance

The next quantity we studied is the total number of transferred packets in the simulated network. Each node transmits constantly TCP packets to its randomly chosen partner, and if a packet arrives, an acknowledgement is sent back to the originating node. We counted the number of packets for which the acknowledgement was received and the difference between the number of transmitted packets and acknowledged packets is the loss.

The source of the loss of packets in the network is the congestion: whether the link capacity cannot handle the amount of traffic, or the nodes in between cannot cope with the routing of the packages. One would expect, that in a network with smaller shortest paths between two randomly chosen nodes the load on the links and routers is higher, hence using the same devices it would drop the packets more often as a network with larger shortest path.

It is also known, that scale free network have smaller shortest paths connecting two arbitrary nodes [6,12], *i.e.* a scale free network uses less routing devices, however, the load on them is higher. Indeed, numerical simulation, performed in the previous section also showed, that with increasing traffic the performance of the scale free (BA) network downgrades, for example, with a drop rate of 24% already at medium traffic, while the classical (ER) network show a downgrade only of 6% for the same traffic.

TABLE I

The performance of the networks in millions of packets successfully sent.

scenario	$\alpha = 0.0$	$\alpha = 0.5$	$\alpha = 1.0$
low traffic	0.59	0.59	0.57
medium traffic	2.21	2.09	1.74
high traffic	3.42	3.26	2.96

6. Conclusion

In this paper we showed, that the distribution of the network traffic even in a simulated "uniform" situation does not follow one-by-one the underlying network, rather it develops fatter tails than the nodal degree distribution of the network. A scaling between the nodal degree distribution and the load on a node was established, showing a power like patter, $l \sim d^{\beta(\alpha)}$, where the scaling exponent β, is a decreasing function of the network parameter, α. For networks with fatter nodal distribution the exponent is smaller. As a consequence, attaching a new connection to the node requires less upgrade in the bandwidth for a scale free network to keep the performance, as for a classical (ER) one.

The above theoretical result was confirmed by simulation, the per-node throughput is still *scaling* with nodal degree. The scaling depends on the amount of the traffic, for a completely congested situation the throughput distribution by definition agrees with the nodal degree distribution, hence the scaling exponent is 1, however, for partly congested or congestion free networks this scaling approaches the load distribution with scaling exponents in the range 1.4 (partially congested BA network) to 2 (congestion free ER). A scale free network requires less resource upgrade when a new node is added.

The overall performance of a scale free network is decreasing rapidly with the traffic, where the classical network still has almost no loss. It is due to the feature, that the ER network uses more routers along the shortest connection, and hence the traffic is distributed more evenly.

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