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THEORY OF THE MUON $g-2$ *

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We review the present status of the theoretical evaluation of the anomalous magnetic moment of the muon in the Standard Model. We mainly focus on the hadronic contributions due to vacuum polarization effects, light-by-light scattering and higher order electroweak corrections. We discuss some recent calculations together with their uncertainties and limitations and point out possible improvements in the future.

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1. Introduction

For a particle with spin 1/2, the relation between its magnetic moment and its spin reads $\vec{\mu} = g(e/2m)\vec{s}$. The Dirac equation predicts for the gyromagnetic factor $g=2$, but radiative corrections to the vertex can shift the value slightly. The anomalous magnetic moment is then defined as $a \equiv (g-2)/2$. There has been a fruitful interplay between experiment and theory over many decades. The electron anomalous magnetic moment a_e provides a stringent test of QED and leads to the most precise determination of the fine structure constant α. The anomalous magnetic moment of the muon a_μ, on the other hand, allows to test the Standard Model as a whole, since all sectors contribute. Furthermore, a_μ is very sensitive to new physics beyond the Standard Model. Since a_l is dimensionless, one expects in general $a_l \sim (m_l/M_{\rm NP})^2$, therefore, a_μ is about $(m_\mu/m_e)^2 \sim 4\times 10^4$ times more sensitive to the scale of new physics, $M_{\rm NP}$, than a_e. The current experimental world average is dominated by the recent measurements of the $g-2$ collaboration at Brookhaven [1]

$$a_\mu^{\rm exp} = 11\ 659\ 203(8) \times 10^{-10} \,. \qquad (1)$$

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The final goal is to reach an experimental precision of 4×10^{-10}. Unfortunately, the hadronic contributions lead to a large error in the Standard Model prediction for a_μ, about 8×10^{-10}, and they are very difficult to control theoretically. The problem is that at low energies, relevant for the muon $g-2$, quarks are bound by strong gluonic interactions into hadrons. In particular for the light quarks u, d, s one cannot use perturbative QCD. This hinders at present all efforts to extract a clear sign of new physics from a_μ, but the muon $g-2$ puts already some strong bounds on potential new physics contributions. We will not discuss at all here such new contributions to a_μ, *e.g.* from supersymmetry, but refer instead to the article [2] and references therein. Constraints from the muon $g-2$ on some particular two-Higgs doublets models were discussed at this conference [3]. For more details on the subject of $g-2$ in general, we refer to the reviews [4].

The Standard Model contributions are usually split into three parts: $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$. We will now discuss in turn the three types of contributions, with the main emphasis on the hadronic corrections and their uncertainties. We will also take into account some important developments that occurred shortly after this conference.

2. QED and electroweak contributions

A general feature of the QED contribution is that loops with electrons are enhanced due to logarithms $\ln(m_\mu/m_e) \sim 5.3$. These are short-distance logarithms from vacuum polarization and infrared logarithms, for instance in the light-by-light scattering contribution. The latter effect completely dominates the contribution at order $(\alpha/\pi)^3$. In contrast, loops with τ-leptons are suppressed. The result in QED up to 5-loops reads [4] [1]

$$\begin{aligned} a_\mu^{\rm QED} &= 0.5 \times \frac{\alpha}{\pi} + 0.765\ 857\ 399(45) \times \left(\frac{\alpha}{\pi}\right)^2 + 24.050\ 509\ 5(23) \times \left(\frac{\alpha}{\pi}\right)^3 \\ &\quad + 124.84(41) \times \left(\frac{\alpha}{\pi}\right)^4 + 930(170) \times \left(\frac{\alpha}{\pi}\right)^5 \\ &= 11\ 658\ 470.28(28) \times 10^{-10} \,. \end{aligned} \tag{2}$$

The errors given in the second and third term on the right-hand side are due to the uncertainty in the experimental values of $m_\mu/m_{e,\tau}$, the one in the fourth term from the numerical integration and the one in the last term from a renormalization group estimate of that coefficient. We used the value [5]

$$\alpha^{-1}(a_e) = 137.035\ 998\ 75(52)\,, \tag{3}$$

[1] Thanks to M. Passera (private communication) for pointing out an additional small change in the fourth-order coefficient in Eq. (2), that was overlooked in Ref. [5].

obtained from the electron anomalous magnetic moment. Note that this value has shifted recently by about 1.6 standard deviations, due to an error found in the 4-loop QED contribution [6]. Fortunately, this numerical error has not a very big effect for a_μ, changing it by about -0.29×10^{-10} in total.

The electroweak correction to a_μ lies in between the QED and hadronic contribution. At one loop, the result is reliably calculable [4]

$$a_\mu^{\mathrm{EW},(1)} = 19.5 \times 10^{-10}\,, \tag{4}$$

with a Higgs boson contribution that is very small for $M_H \geq 114.5$ GeV (LEP 2 bound).

Two-loop corrections, see Fig. 1, are potentially large due to factors

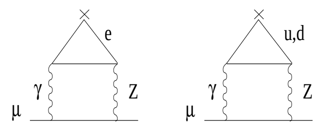

Fig. 1. Two-loop electroweak corrections to a_μ from the first family. The light quark loop is to be understood symbolically, representing a hadronic "blob".

$\ln(M_Z/m_\mu) \sim 6.8$. Furthermore, as noted in Ref. [7], one cannot separate leptons and quarks anymore, but must treat each generation together because of the cancellation of the triangle anomalies. Therefore, earlier estimates [8] were incomplete. A first full two-loop calculation was done in Ref. [9] and recently revisited by two groups [10, 11] to improve on the treatment of the hadronic contributions. Instead of using a simple constituent quark loop, short-distance constraints from the OPE on the relevant QCD three-point function $\langle VVA\rangle$ have been imposed. There is still some disagreement in the details, but the numerical values are very close. Adding the one-loop result from Eq. (4), Ref. [10] obtains $a_\mu^{\mathrm{EW}} = 15.2(0.1)\times 10^{-10}$. The error reflects hadronic uncertainties and the variation of M_H. No resummation[2] has been performed in that reference. Ref. [11] gets $a_\mu^{\mathrm{EW}} = 15.4(0.1)(0.2)\times 10^{-10}$, where the first error corresponds to the hadronic uncertainty and the second to an allowed Higgs boson mass range of 114 GeV $\leq M_H \leq$ 250 GeV, the current top mass uncertainty, and unknown three-loop effects. Averaging the two estimates, we obtain

$$a_\mu^{\mathrm{EW}} = 15.3(0.2) \times 10^{-10}\,, \tag{5}$$

[2] The resummation of leading logarithms has been discussed in Ref. [12] and corrected in Ref. [11]. There are large cancellations in the resummation and the final shift after the resummation is very small.

which corresponds to a large two-loop correction of $a_\mu^{\mathrm{EW},\,(2)} = -4.2(0.2) \times 10^{-10}$. Although not all details have been resolved, the electroweak contribution seems well under control.

3. Hadronic contributions

3.1. Hadronic vacuum polarization

In the hadronic vacuum polarization contribution $a_\mu^{\text{had. v.p.}}$, depicted in Fig. 2, one cannot simply equate for light quarks the hadronic "blob" with a quark loop as it is possible for leptons. In the present case there

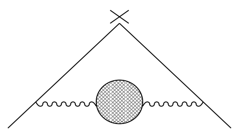

Fig. 2. Hadronic vacuum polarization contribution to a_μ.

is, however, a way out by using the optical theorem (unitarity) to relate the imaginary part of the blob to the measurable scattering cross section $\sigma(e^+e^- \to \gamma^* \to \text{hadrons})$. From a dispersion relation one then obtains the spectral representation [13]

$$a_\mu^{\text{had. v.p.}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s} K(s) R(s)\,, \tag{6}$$

$$R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}\,. \tag{7}$$

The kernel $K(s)/s$ is a known, slowly varying, positive function peaked at low energy. Information on the $I = 1$ vector part of the spectral function $\mathrm{Im}\Pi(s) \sim \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$ in Eq. (6) can also be obtained from hadronic τ decays, like $\tau^- \to \nu_\tau \pi^- \pi^0$. One has, however, to apply corrections due to isospin violations, since $m_u \neq m_d$ and because of electromagnetic radiative corrections, see Refs. [14, 15]. Until very recently, the major sources of isospin breaking seemed to be identified and well under control. We will come back to this point shortly.

Recently, the CMD-2 collaboration has found an error in their Monte Carlo program for Bhabha scattering which was used to determine the luminosity. As a result, the hadronic cross sections went up by 2–3 % [16]. The new contribution to a_μ from the $\pi^+\pi^-$ intermediate state is $a_\mu^{\pi^+\pi^-}(610.50 < E_{\text{c.m.}} < 961.52 \text{ MeV}) = \left(378.6 \pm 2.7_{\text{stat}} \pm 2.3_{\text{syst}}\right) \times 10^{-10}$. This value is confirmed by a preliminary result by the KLOE collaboration [17], using the radiative return method [18], $a_\mu^{\pi^+\pi^-}(610.50 < E_{\rm c.m.} < 961.52~{\rm MeV}) = \left(378.4 \pm 0.8_{\rm stat} \pm 4.5_{\rm syst} \pm 3.0_{\rm theo} \pm 3.8_{\rm FSR}\right) \times 10^{-10}$, although the effect of final state radiation from the pions still has to be analyzed more thoroughly.

Based on the corrected CMD-2 data, recently several estimates for the hadronic vacuum polarization contribution to a_μ appeared, see Table I. The e^+e^- based evaluations shifted upwards, by about $+11\times10^{-10}$, compared to earlier estimates [15,22], [3] *i.e.* towards the τ-based evaluations. Nevertheless, there still remains a large discrepancy of about 10 % between the spectral functions from e^+e^- and τ data, in particular above the ρ-peak, much larger than any known isospin violation [14, 15]. The puzzle is that the e^+e^- data from different experiments, *e.g.* from CMD-2 and KLOE, are compatible, but also the different τ-data sets from ALEPH, CLEO and OPAL are consistent among each other (maybe with the exception of OPAL, which has larger errors however). However, as pointed out very recently, independently by Davier and Jegerlehner [24], one can map the corresponding spectral functions into each other, if one assumes that $M_{\rho^+} - M_{\rho^0} \sim 3-4$ MeV, together with a corresponding change in the width, according to $\Gamma_\rho \sim M_\rho^3/F_\pi^2$. Although there are theoretical arguments which point to a much smaller mass difference [25], $-0.7~{\rm MeV} < M_{\rho^+} - M_{\rho^0} < 0.4$ MeV, one can *a priori* not rule out a larger mass difference. In any case, some independent experimental information on the masses and widths of the neutral and charged ρ-meson would be very useful and seems even necessary, if one still wants to use the τ-data for $a_\mu^{\rm had.v.p.}$.

TABLE I

Recent evaluations of $a_\mu^{\rm had.~v.p.}$

Authors	Contribution to $a_\mu^{\rm had.v.p.} \times 10^{10}$
Davier *et al.* [19] ($e^+e^- + \tau$)	$711.0 \pm 5.0_{\rm exp} \pm 0.8_{\rm rad} \pm 2.8_{\rm SU(2)}$
Davier *et al.* [19] (e^+e^-)	$696.3 \pm 6.2_{\rm exp} \pm 3.6_{\rm rad}$
Jegerlehner [20] (e^+e^-)	694.8 ± 8.6
Teubner [21] (e^+e^-, inclusive)	$691.8 \pm 5.8_{\rm exp} \pm 2.0_{\rm rad}$

Averaging the results that use e^+e^- data only, we obtain

$$a_\mu^{\rm had.~v.p.}(e^+e^-) = 694.3(7.5) \times 10^{-10}\,. \tag{8}$$

Finally, there are higher order vacuum polarization effects, if additional photonic corrections or fermion loops (leptons and hadrons) are included in the diagram in Fig. 2. They have been evaluated in Ref. [26] with the result

$$a_\mu^{\rm h.o.-h.v.p.} = -10.0(0.6) \times 10^{-10}\,. \tag{9}$$

[3] Note that the group of Teubner and collaborators uses another averaging procedure, described in Ref. [23], therefore, they get a rather different central value, despite the use of essentially the same input data.

3.2. Hadronic light-by-light scattering

The present picture of hadronic light-by-light scattering, as reviewed recently in Ref. [27], is shown in Fig. 3 and the corresponding contributions

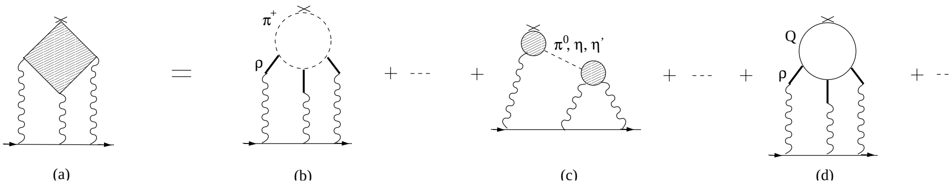

Fig. 3. The hadronic light-by-light scattering contribution to the muon $g-2$.

to a_μ are listed in Table II, taking into account the corrections made in the two full evaluations [28, 29], after we had discovered the sign error in the pion–pole contribution [30, 31].

TABLE II

Contributions to $a_\mu(\times 10^{10})$ according to Fig. 3.

Type	Ref. [28]	Ref. [29]	Ref. [30]	No form factors
(b)	$-0.5(0.8)$	$-1.9(1.3)$		-4.5
(c)	$8.3(0.6)$	$8.5(1.3)$	$8.3(1.2)$	$+\infty$
f_0, a_1	0.174[a]	$-0.4(0.3)$		
(d)	$1.0(1.1)$	$2.1(0.3)$		~ 6
Total	$9.0(1.5)$	$8.3(3.2)$	$8(4)$[b]	

[a] Only a_1 exchange.
[b] Our estimate, using Refs. [28–30].

There are three classes of contributions to the hadronic four-point function [Fig. 3(a)], which can best be understood according to an effective field theory (EFT) analysis of hadronic light-by-light scattering [31]: (1) a charged pion loop [Fig. 3(b)], where the coupling to photons is dressed by some form factor (ρ-meson exchange, *e.g.* via vector meson dominance (VMD)), (2) the pseudoscalar pole diagrams [Fig. 3(c)] together with the exchange of heavier resonances ($f_0, a_1, \ldots$) and, finally, (3) the irreducible part of the four-point function which was modeled in Refs. [28, 29] by a constituent quark loop dressed again with VMD form factors [Fig. 3(d)]. The two groups [28, 29] used similar, but not identical models which explains the slightly different results for the dressed charged pion and the dressed constituent quark loop, although their sum seems to cancel to a large extent and the final result is essentially given by the pseudoscalar exchange diagrams. We take the difference of the results as indication of the error due to the model dependence and have added the corresponding errors linearly.

Pion-pole contribution

The contribution from the neutral pion intermediate state is given by a two-loop integral that involves the convolution of two pion–photon–photon transition form factors $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$, see Fig. 3(c). We refer to Ref. [30] and references therein for all the details. Since no data on the doubly off-shell form factor $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ is available, one has to resort to models. We considered a certain class of form factors which includes the ones based on large-N_C QCD that we had studied in Ref. [32]. These form factors include either one or two narrow vector resonances and are matched at low-energies with chiral perturbation theory and at high momenta with the operator product expansion (OPE). Using two vector resonances, one can also reproduce, for large space-like momenta $Q^2 = -q^2$, the observed $1/Q^2$ behavior of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q^2,0)$, when one photon is on-shell. For comparison, we have also used a VMD form factor and a (unrealistic) constant form factor, derived from the Wess–Zumino–Witten (WZW) term.

For the form factors discussed above one can perform *all* angular integrations in the two-loop integral analytically [30]. The pion-exchange contribution can then be written as a two-dimensional integral representation, where the integration runs over the moduli of the Euclidean momenta

$$a_\mu^{\mathrm{LbyL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \, \sum_i w_i(Q_1, Q_2) \, f_i(Q_1, Q_2) \,, \tag{10}$$

with universal [for the above class of form factors] weight functions w_i [30]. The dependence on the form factors resides in f_i. In this way we could separate the generic features of the pion–pole contribution from the model dependence and thus better control the latter.

The weight functions w_i in the main contribution are positive and peaked around momenta of the order of 0.5 GeV. There is, however, a tail in one of these functions, which produces for the constant WZW form factor a divergence of the form $(\alpha/\pi)^3 \mathcal{C} \ln^2 \Lambda$ for some UV-cutoff Λ, with an universal[4] coefficient $\mathcal{C} = N_C^2 m_\mu^2/(48\pi^2 F_\pi^2)$, that can also be derived within an EFT approach to hadronic light-by-light scattering [31]. Unfortunately, the $g-2$ enters in the EFT as a local counterterm and one can therefore not obtain a prediction for $a_\mu^{\mathrm{LbyL;had}}$ within the pure EFT approach. Furthermore, for the form factors discussed above, there is a strong cancellation between the $\ln^2$ and the ln term and information about the constant term is needed. However, the EFT analysis shows for instance that the modeling of hadronic light-by-light scattering by a constituent quark loop is not a reliable descrip-

[4] The behavior, $(\alpha/\pi)^3 \mathcal{C} \ln^2 M_V$, with the same coefficient $\mathcal{C}$ is observed, if one sends the vector meson mass M_V in the form factors to infinity.

tion of the dominant contribution, since the $\ln^2$ term is not reproduced in this way.

All form factors lead to very similar results (apart from WZW). Judging from the shape of the weight functions described above, it seems more important to correctly reproduce the slope of the form factor at the origin and the available data at intermediate energies. On the other hand, the asymptotic behavior at large Q_i seems not very relevant. The results for the form factors based on the large-$N_{\rm C}$ QCD framework are rather stable under the variation of the model parameters. For all pseudoscalars π^0, η, η' together we obtain the estimate

$$a_\mu^{\rm LbyL;PS} = +8.3(1.2) \times 10^{-10}\,, \tag{11}$$

where the error includes the variation of the parameters in the form factors and an estimate of the intrinsic model dependence. An error of 15 % seems reasonable, since we impose many theoretical constraints from long and short distances on the form factors. Furthermore, we use experimental information whenever available.

The analysis within the EFT and large-$N_{\rm C}$ framework, together with the numerical results for *all* contributions depicted in Fig. 3 and listed in Table II leads us to the following (conservative) estimate for the hadronic light-by-light scattering contribution

$$a_\mu^{\rm LbyL;had} = +\,8(4) \times 10^{-10}\,. \tag{12}$$

In view of the uncontrolled model dependencies for the dressed charged pion loop and the dressed constituent quark loop, it will be very difficult to reduce the error significantly, *e.g.* to much below of 3×10^{-10}.

4. Summary and conclusions

We now collect the results for the different contributions in the Standard Model from Eqs. (2), (5), Table I, Eqs. (8), (9) and (12):

$$a_\mu^{\rm SM}(e^+e^-) \;=\; (11\;659\;177.9 \pm 7.5 \pm 4.0 \pm 0.35) \times 10^{-10}\,, \tag{13}$$

$$a_\mu^{\rm SM}(\tau) \;=\; (11\;659\;194.6 \pm \underbrace{5.8}_{\rm v.p.} \pm \underbrace{4.0}_{\rm LbyL} \pm \underbrace{0.35}_{\rm EW}) \times 10^{-10}\,, \tag{14}$$

where we kept the results based on e^+e^- and τ data separately. Comparison with the experimental value from Eq. (1) leads to

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}(e^+e^-) \;=\; (25.1 \pm 11.7) \times 10^{-10}, \qquad [2.1\ \sigma]\,, \tag{15}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}(\tau) \;=\; (8.4 \pm 10.7) \times 10^{-10}, \qquad [0.8\ \sigma]\,. \tag{16}$$

As mentioned above, the discrepancy between the e^+e^- and τ-data based evaluations could perhaps be resolved, if the masses and widths of the charged and neutral ρ-mesons are different [24]. Although there are arguments [25] for a very small mass difference, one cannot really exclude this possibility. Independent experimental information on the masses and widths is therefore needed, if one wants to use both e^+e^- and τ-data to estimate the hadronic vacuum polarization contribution in a_μ. If the τ-data simply shift towards the e^+e^- data, we seem to be back to a 2σ deviation from the Standard Model. Could this finally be the long-awaited sign of new physics? Let me just caution that at the level of precision required, some of the hadronic uncertainties, *e.g.* connected with modeling of final state radiation [33], might be underestimated. There still remains a lot of work to be done to better control these hadronic contributions, if we want to use the muon $g-2$ to search for signs of new physics.

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