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EFFECTIVE THEORY AND RENORMALIZATION*

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I discuss the renormalization of the shape-function describing the effective mass distribution of the b quark inside a B meson in semi-inclusive non-charmed B meson decays. An effective theory is required in the soft and collinear kinematical regions. It is enlightened the interplay between the choice of the effective theory and the regularization procedures.

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1. Introduction

The renormalization of the shape function is discussed focusing in particular on the problem of the electron spectrum in the decay

$$B \to X_u + l + \nu \,. \tag{1}$$

As it is well known, this semi-inclusive decay is useful for measuring $|V_{ub}|$. Let us review the basic points. Experimentally

$$\frac{|V_{ub}|}{|V_{cb}|} \;\leq\; O(10^{-1})\,,$$

implying that the above rate is smaller than the background process

$$B \to X_c + l + \nu \tag{2}$$

by at least two orders of magnitude. To kill this huge background, one can take advantage of the fact that the endpoint for the electron spectrum for the decay (2)

$$E_{\max}^{b\to c} = \frac{m_B^2 - m_D^2}{2m_B}$$

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is smaller than the endpoint for the decay (1)

$$E_{\max}^{b\to u} = \frac{m_B^2 - m_\pi^2}{2m_B} \cong \frac{m_B}{2},$$

because of the charm mass. Therefore, there is a window in the electron spectrum around the endpoint accessible only to the $b \to u$ decay, of width

$$\Delta E = \frac{m_D^2}{2m_B} \sim 330 \text{ MeV}.$$

The spectrum inside this window is notoriously difficult to compute because of the presence of non-perturbative phenomena as well as substantial perturbative corrections. To solve this problem, an effective theory approach has been formulated. The main result of the effective theory is that the semi-inclusive decays of a b quark inside a B meson can be treated as decays of a free quark with a (non-perturbative) probability distribution for its mass. The kinematical configuration refers to an hadronic jet X with a very large energy,

$$E_X^2 \ \sim \ O(m_B^2), \tag{3}$$

and invariant mass in an "intermediate region"

$$m_X^2 \ \sim \ O(m_B \Lambda_{\mathrm{QCD}}), \tag{4}$$

such that the ratio vanishes as an inverse power of the heavy quark mass:

$$\frac{m_X^2}{E_X^2} \ \sim \ O\left(\frac{\Lambda_{\mathrm{QCD}}}{m_B}\right).$$

In physical terms, we can say that the disintegration of the b quark in a B meson is modified with respect to the free case ($\alpha_{\mathrm{s}} = 0$) by two distinct processes: the momentum exchanges with the valence quark (meson cloud) and the emission of radiation (real and virtual quanta) which accompanies the hard process.

By setting the effective limit $m_b \to \infty$, new ultraviolet singularities arise. While in full QCD the scale function evolves changing m_b, in the effective theory it evolves with the renormalization point μ, because the evolution in the former is controlled by terms of the form

$$C\alpha \log^2\left(\frac{m_b^2}{Q^2}\right),$$

while the evolution in the latter is controlled by terms of the form

$$C\alpha \log^2\left(\frac{\mu^2}{Q^2}\right).$$

The full and the effective theory shape functions should match at the scale $\mu = m_b$. Afterwards, it is possible to evolve the shape function in the effective theory up to the desired value of μ with an evolution equation. The fundamental point is that the constant C must be the same in both theories: that assures the cancellation of the infrared contributions (*i.e.* of those contributions which diverge as $Q^2 \to 0$) in the matching constant or, equivalently, assures the complete factorization of these effects in the effective theory shape function. We will show that, contrary to naïve expectations, this does not happen in all effective theories; in fact, the approximations of the effective theory can interfere with ultraviolet momenta regions, and therefore, with ultraviolet regularization procedures.

2. The effective theory

Let us consider the inclusive B decay

$$B \to X_u + l + \nu\,,$$

where X_u is any hadronic state containing the fragmentation *up* quark. In order to calculate the rate, one has to evaluate the hadronic tensor

$$W_{\mu\nu} \equiv \sum_X \langle B|J^\dagger_\nu(0)|X\rangle\langle X|J_\mu(0)|B\rangle\delta^4(p_B - q - p_X)\,, \qquad (5)$$

where $J_\mu(x) = \overline{q}(x)\gamma_\mu\,(1-\gamma_5)\,Q(x)$, with $q(x)$ being a light quark field and $Q(x)$ a heavy quark field. p_B is the momentum of the B meson, q is the momentum of the leptonic pair and p_X the momentum of the final hadronic jet. According to the optical theorem, we have

$$W_{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu}\,, \qquad (6)$$

where

$$T_{\mu\nu} \equiv -i\int d^4x\,\mathrm{e}^{-iqx}\langle B|T\left(J^\dagger_\mu(x)J_\nu(0)\right)|B\rangle\,. \qquad (7)$$

As it is well known, in HQET:

$$Q(x) \simeq \mathrm{e}^{-im_B v\cdot x}\;h_v(x)\,. \qquad (8)$$

h_v is the HQET field operator with velocity v ($v^2 = 1$) describing the b quark, satisfying

$$P_+\;h_v = h_v,\quad P_-\;h_v = 0\,,$$

where $P_\pm = (1 \pm \hat{v})/2$ are the projectors over the components with positive and negative energies, respectively. In the B meson, the b quark exchanges momenta of order $\Lambda_{\rm QCD}$ with the light degrees of freedom (valence quark), so it has momenta of the form

$$p_b = m_B v + k' \tag{9}$$

and

$$|k'| \sim \Lambda_{\rm QCD} \ll m_B \,.$$

By using Eq. (8) we obtain

$$T_{\mu\nu} = -i \int d^4x \; {\rm e}^{iQ\cdot x} \langle B(v)| T \; \bar{h}_v(x) \Gamma_\mu \; q(x) \; \bar{q}(0) \Gamma_\nu^\dagger \; h_v(0) |B(v)\rangle \,, \tag{10}$$

where

$$Q \equiv m_B v - q \,.$$

By using the Wick theorem we single out the only contraction that is relevant for B decay:

$$T_{\mu\nu} = \int d^4x \; {\rm e}^{iQx} \langle B| \bar{h}_v(x) \Gamma_\mu S(x|0) \Gamma_\nu^\dagger h_v(0) |B\rangle \,, \tag{11}$$

where $S(x|0)$ is the light quark propagator. We can express the Fourier transform of the light quark propagator in a compact notation as

$$\tilde{S}(Q+iD) = \frac{1}{i\hat{D} + \hat{Q} + i\eta} = \frac{i\hat{D} + \hat{Q}}{Q^2 + 2iD\cdot Q - D^2 + g/2 \; \sigma_{\mu\nu} G^{\mu\nu} + i\eta} \,, \tag{12}$$

where $\sigma_{\mu\nu} \equiv i/2[\gamma_\mu, \gamma_\nu]$, $G_{\mu\nu} \equiv -i/g[D_\mu, D_\nu]$ and $D_\mu \equiv \partial_\mu - igA_\mu$. We will consider the light quark being in the LEET approximation [1]. The term $2iQ\cdot D$ and Q^2 are of the same order in the end-point region. At lowest order:

$$\tilde{S}(Q+iD) \simeq \frac{i\hat{Q}}{Q^2 + 2iD\cdot Q + i\eta} \,.$$

By setting $Q_\mu = Q\cdot v \; n_\mu - k_\mu$, with n being a light-like vector, $n \equiv (1;0,0,-1)$, we write:

$$\tilde{S}(Q+iD) = \frac{1}{2v\cdot Q} \; \frac{\hat{Q}}{iD_+ - k_+ + i\eta} \,, \tag{13}$$

where

$$k_+ \equiv -\frac{Q^2}{2v\cdot Q}$$

and $D_+ \equiv n \cdot D$. We can simplify the tensor structure of $T_{\mu\nu}$ by using the identity

$$\bar{h}_v \Gamma h_v = \frac{1}{2}\mathrm{Tr}(\Gamma P_+)\ \bar{h}_v h_v - \frac{1}{2}\mathrm{Tr}(\gamma_\mu \gamma_5 P_+ \Gamma P_+)\, \bar{h}_v \gamma^\mu \gamma_5 h_v \tag{14}$$

which is valid for any Γ. The matrix element of the axial vector current between the B meson states vanishes by parity invariance and we have:

$$T_{\mu\nu} = s_{\mu\nu}\, \frac{1}{2v \cdot Q}\, F(k_+)\,, \tag{15}$$

where [2]

$$F(k_+) \equiv \langle B(v) \mid h_v^\dagger(0) \frac{1}{iD_+ - k_+ + i\eta} h_v(0) \mid B(v) \rangle$$

and

$$s_{\mu\nu} \equiv \frac{1}{2}\,\mathrm{Tr}(\Gamma_\mu^\dagger \hat{Q} \Gamma_\nu P_+)$$

contains the leading spin effects. By using the formula

$$\frac{1}{iD_+ - k_+ + i\eta} = \mathrm{P}\frac{1}{iD_+ - k_+} - i\pi\delta(iD_+ - k_+) \tag{16}$$

and taking the imaginary part of $T_{\mu\nu}$, we obtain (see Eq. (6)):

$$W_{\mu\nu} = \frac{1}{2v \cdot Q}\, s_{\mu\nu}\, f(k_+)\,, \tag{17}$$

where $f(k_+)$ is the shape function defined as the forward matrix element of the nonlocal operator

$$h_v^\dagger(0)\delta(k_+ - iD_+)h_v(0) \tag{18}$$

on a single heavy hadron state with velocity v, that is:

$$f(k_+) \equiv \langle B(v) \mid h_v^\dagger(0)\delta(k_+ - iD_+)h_v(0) \mid B(v) \rangle\,.$$

The shape function $f(k_+)$ is introduced to take care of non perturbative effects. In order to be meaningful, an effective theory must have the same infrared behavior of the original, high energy theory. By computing the shape function in the double logarithmic approximation with a hard cut-off, we find indeed the same double logarithm of k_+ as in QCD. In other words, the leading IR singularity, $\log^2 k_+$, cancels in the matching constant (coefficient function), implying the factorization of infrared physics into the shape function.

However, repeating the same calculation with dimensional regularization, it appears an additional factor of two in the term proportional to $\log^2 k_+$, implying that the double logarithm of k_+ does not cancel in the matching constant. In order to obtain a meaningful matching constant one can choose to use a non minimal subtraction [2] or to modify the effective theory by adding collinear terms [3].

3. Matching

A shape function $f(k_+)^{\rm QCD}$ and a light-cone function $F(k_+)^{\rm QCD}$ can also be defined in full QCD by means of the relations [2, 4]:

$$T_{\mu\nu}^{\rm QCD} \equiv (s_{\mu\nu} + \Delta s_{\mu\nu}) \frac{1}{2v\cdot Q} F(k_+)^{\rm QCD}$$

and

$$W_{\mu\nu}^{\rm QCD} \equiv \left(s_{\mu\nu} + \Delta s'_{\mu\nu}\right) \frac{1}{2v\cdot Q} f(k_+)^{\rm QCD},$$

where $\Delta s_{\mu\nu}$ and $\Delta s'_{\mu\nu}$ are defined as the part of the spin structure not proportional to $s_{\mu\nu}$. The tensors $\Delta s_{\mu\nu}$ and $\Delta s'_{\mu\nu}$ represent residual spin effects not described by the effective theory, which do not contribute to the Double-Logarithmic Approximation (DLA); throughout the paper, we will consider DLA one-loop corrections.

The tensor $T_{\mu\nu}^{\rm QCD}$ can be computed using the formula

$$T_{\mu\nu}^{\rm QCD} = \frac{-i}{2}\,{\rm Tr}\left[P_+ \mathcal{M}_{\mu\nu}\right] ,$$

where $\mathcal{M}_{\mu\nu}$ is the Feynman amplitude for the forward scattering

$$b(v) + \gamma^*(q) \to b(v) + \gamma^*(q)$$

with the external spinors and the photon polarizations amputated. In DLA the forward tensor can be written as

$$T_{\mu\nu}^{\rm QCD} = s_{\mu\nu} \frac{1}{2v\cdot Q} F(k_+)^{\rm QCD}, \tag{19}$$

and an analogous formula holds for $W_{\mu\nu}^{\rm QCD}$.

Both $F(k_+)$ and $f(k_+)$ receive perturbative QCD corrections which determine their evolution through a renormalization group equation [5,6]. The starting point of the evolution (boundary value) is determined by matching the effective field theory onto full QCD. The matching constant (or coefficient function) is defined through the relation

$$f(k_+)^{\rm QCD} = Z\; f(k_+)\,, \tag{20}$$

where $f(k_+)$ is computed in the effective theory. We can calculate the matching constant in an easier way by exploiting the relation

$$F(k_+)^{\rm QCD} = Z\, F(k_+)\,. \tag{21}$$

Up to one loop:

$$F(k_+)^{\rm QCD} \equiv \frac{1}{-k_+ + i0}\left[1 + a\, C\right]\,, \tag{22}$$

where $a \equiv \alpha_{\rm S}\, C_{\rm F}/\pi$ and C is the leading contribution in DLA. In the Feynman gauge, the leading term comes from the vertex correction diagram and we have

$$C \equiv \; -i\, v\cdot Q \int \frac{d^4 l}{\pi^2}\, \frac{1}{(l+Q)^2 + i0}\; \frac{1}{v\cdot l + l^2/2m + i0}\; \frac{1}{l^2 + i0}\,, \tag{23}$$

where m is the b quark mass. We have set the light quark mass equal to zero [7].

The one-loop contribution is [2, 6]

$$F(k_+)^{\rm QCD} = \frac{1}{-k_+ + i\eta}\left(-\frac{1}{2}\right)\; a\; \log^2\left(\frac{2\, m}{k_+ - i\eta}\right). \tag{24}$$

The hadronic tensor relevant to the decay is obtained by taking the imaginary part. This transforms the products in convolutions, which are converted again into ordinary products by the well-known Mellin transform [8].

Let us now pass to the effective field theory. We introduce the following regularization: we set a hard cut-off Λ (HC) on the spatial loop momenta

$$|\overrightarrow{l}| < \Lambda\,,$$

while leaving the loop energy l_0 to vary on the entire real axis

$$-\infty < l_0 < +\infty\,.$$

The one loop contribution is [2]

$$F(k_+) = \frac{1}{-k_+ + i\eta}\left(-\frac{1}{2}\right)\; a\; \log^2\left(\frac{2\, \Lambda}{k_+ - i\eta}\right)\,. \tag{25}$$

Inserting expressions (25) and (24) into Eq. (21), we find

$$\begin{aligned} Z \;&=\; 1 - \frac{1}{2} a\left[\log^2\left(\frac{2\, m}{k_+ - i\eta}\right) - \log^2\left(\frac{2\, \Lambda}{k_+ - i\eta}\right)\right] \\ &=\; 1 - a \log\left(\frac{m}{\Lambda}\right)\log\left(\frac{2\sqrt{m\,\Lambda}}{k_+ - i\eta}\right)\,. \end{aligned} \tag{26}$$

The double logarithm of k_+, *i.e.* the leading infrared singularity, cancels in the difference of the integrals, at any value of Λ, implying the factorization of infrared physics into the effective theory light-cone function. In particular, with $\Lambda = m$, $Z = 1$.

Let us now consider a different regularization, *i.e.* dimensional regularization. The bare light-cone function at fixed ε is given by [2,6]

$$\begin{aligned} F(k_+) &= \frac{1}{-k_+ + i\eta}\left(-\frac{a}{2}\right)\frac{\Gamma(1+\varepsilon)\Gamma(1+2\varepsilon)\Gamma(1-2\varepsilon)}{\varepsilon^2}\left(\frac{\mu}{k_+ - i\eta}\right)^{2\varepsilon} \\ &= \frac{1}{-k_+ + i\eta}a\left[-\frac{1}{2\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{\mu}{k_+ - i\eta}\right) - \log^2\left(\frac{\mu}{k_+ - i\eta}\right)\right], \end{aligned} \quad (27)$$

where $\varepsilon \equiv 2 - D/2$, D is the space-time dimension and μ is the regularization scale. We immediately observe that the finite term containing the $\log^2 k_+$ has an additional factor two with respect to the HC result (Eq. (25)) or the QCD result (Eq. (24)). Computing the coefficient function according to Eq. (21) we have

$$Z = 1 + a\left[\frac{1}{2\varepsilon^2} + \frac{1}{\varepsilon}\log\left(\frac{\mu}{k_+ - i\eta}\right) - \frac{1}{2}\log^2\left(\frac{2m}{k_+ - i\eta}\right) + \log^2\left(\frac{\mu}{k_+ - i\eta}\right)\right].$$

The matching constant contains a double logarithm of k_+ that does not cancel whatever is the choice of the matching scale.

In dimensional regularization a consistent matching requires to go to a non-minimal scheme [2] or to modify the effective theory by adding collinear terms [3]. The conclusions enlighten the different interplay of HC and dimensional regularization with the effective theories. The HC regularization cuts off the same collinear degrees of freedom that are not described in the effective field theory we have been using; therefore the matching is consistent with the infrared QCD behavior. On the other side, the dimensional regularization includes additional collinear modes, that need to be introduced also in the effective theory, in order to provide a correct matching.

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