# INFERENCES FROM BOSE–EINSTEIN CORRELATIONS IN MULTIPLE PARTICLE PRODUCTION PROCESSES* **

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Data on Bose–Einstein correlations yield information about the interaction regions in multiple particle production processes. The conclusions are model dependent. Several popular models are briefly presented, compared and discussed.

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## 1. Introduction

An important reason for studying Bose–Einstein correlations (BEC) in multiple particle production processes is that they yield information about the interaction regions *i.e.* about the regions, where the hadrons are produced. There is much to learn about such regions. Let us consider two examples. In high energy $e^+e^-$ annihilations the first stages of the process can be described perturbatively. The two primary leptons merge into a heavy photon, or a $Z^0$, this particle decays into a $q\overline{q}$ pair, the quark and the antiquark radiate gluons, the gluons split into $q\overline{q}$ pairs or into gluons, and so on. Then, however, something much more complicated happens: the swarm of parton gets converted into hadrons. This is a nonperturbative process, known as hadronization, which is poorly understood. One would like to know the size and shape of the region where the hadrons are produced, the

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time interval between the first annihilation and the end of the hadronization process, as well as the time interval between the beginning and the end of hadronization. As a more complicated example let us consider central heavy ion collisions at high energy. In this case the initial color fields are so strong that perturbative methods may be unreliable. It is usually assumed that the initial partons, whatever their production mechanism, rapidly thermalize. From that time on they can be considered as a fluid (a kind of liquid or a kind of gas), which expands according to the laws of hydrodynamics or according to some simplified version of the Boltzmann equation. Finally the hadrons are produced in a hadronization process which may be different from that in $e^+e^-$ annihilations. Here, besides the hadronization, one is interested in the flows of the fluid, in its equation of state, and its phase transitions if any. The study of Bose–Einstein correlations among the final hadrons supplies tentative answers to all such questions. The problem is, however, how reliable these answers are? Till a few years ago the mood was optimistic (*cf. e.g.* [1]). The recent results from RHIC, and also some results from LEP, however, have been so unexpected and so puzzling (references can be traced *e.g.* from the recent review [2]) that perhaps some important ideas are still missing. In the present paper we review some of the most popular ways of getting from the experimental data to the physical conclusions. It will be seen that much remains to be clarified there.

The key object in the transition from the experimental data to the physical conclusions is usually a single particle density matrix $\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)$. There are approaches where this is not the case, *e.g.* the string model (*cf.* [3] and references quoted there), but most models can be formulated in terms of this matrix [4]. In this section we will present its relation to the data. The discussion is grossly simplified (*cf. e.g.* [1]), but it contains the main ideas. The relation between the density matrix and the features of the interaction region is the main subject of this paper and is discussed in the following sections. Let us stress that the density matrix $\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)$ is an auxiliary construct, which does not have to coincide with the actual single particle density matrix for the mesons of a given type in the final state.

The diagonal elements of the density matrix are related, as usual, to the single particle momentum distribution

$$\frac{dN}{d^3p} \sim \rho(\boldsymbol{p},\boldsymbol{p})\,. \tag{1}$$

The tilde in most models means equality up to a constant factor. Only in the GGLP model it is a more complicated operation. $d^3p$ may denote the infinitesimal volume in momentum space, or the covariant infinitesimal volume *i.e.* the volume in momentum space divided by the corresponding energy $E(\boldsymbol{p})$. The two-particle distribution for identical bosons is

$$\frac{dN}{d^3p_1d^3p_2} \sim \rho(\boldsymbol{p}_1,\boldsymbol{p}_1)\rho(\boldsymbol{p}_2,\boldsymbol{p}_2) + |\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)|^2 \,. \tag{2}$$

For uncorrelated, distinguishable particles the second term on the right-hand side would be absent. It results from the symmetrization of the product of the single particle density matrices of the two particles and thus reflects the BEC. Usually one considers the correlation function obtained by dividing the two-particle distribution by the product of the corresponding single particle distributions

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) \sim 1 + \frac{|\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)|^2}{\rho(\boldsymbol{p}_1,\boldsymbol{p}_1)\rho(\boldsymbol{p}_2,\boldsymbol{p}_2)} \,. \tag{3}$$

In practice the procedure is usually much more complicated than presented here, but the strategy is as described: the data are used to get the correlation function which is simply related to the single particle density matrix.

## 2. GGLP approach

The first model of BEC in multiple particle production was proposed by the Goldhabers Lee and Pais [5] (GGLP). As a realistic description of BEC in multiple particle production processes this model is outdated, but it is still the best introduction to the subject. GGLP introduced a density $\rho(\boldsymbol{x})$ of particle sources. They considered identical pions and for definiteness also we will call the identical bosons pions, though the analysis can be applied to any identical bosons (or even to identical fermions, when the sign between the two terms in formula (3) is changed from plus to minus). GGLP made the following two assumptions: firstly, that all the pions are produced simultaneously and instantaneously; secondly, that the production is completely incoherent — the pions produced in two different space point do not interfere. Under these assumptions the single particle density operator is

$$\hat{\rho} = \int d^3x|\boldsymbol{x}\rangle\rho(\boldsymbol{x})\langle\boldsymbol{x}| \,. \tag{4}$$

The corresponding density matrix reads

$$\rho(\boldsymbol{p}_1,\boldsymbol{p}_2) = \langle\boldsymbol{p}_1|\hat{\rho}|\boldsymbol{p}_2\rangle \sim \int d^3x \mathrm{e}^{-i\boldsymbol{q}\boldsymbol{x}}\rho(\boldsymbol{x}) \,, \tag{5}$$

where $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$. If this were the density matrix which can be obtained from formula (3), it would be a very nice result. The density of sources $\rho(\boldsymbol{x})$ could then be obtained unambiguously just by inverting the Fourier transform. *E.g.* for

$$\rho(\boldsymbol{p}_1,\boldsymbol{p}_2) \sim \mathrm{e}^{-\frac{1}{2}R^2\boldsymbol{q}^2} \,, \tag{6}$$

where $R^2 > 0$ is a constant, one would obtain

$$\rho(\boldsymbol{x}) \sim \mathrm{e}^{-\boldsymbol{x}^2/2R^2}\,. \tag{7}$$

Unfortunately, the density matrix (5) is untenable as a density matrix proportional to the true single particle density matrix, because it gives a single particle momentum distribution constant in all momentum space. GGLP introduced, therefore, a projection on the states allowed by energy-momentum conservation, *i.e.* they used the density matrix as calculated here to find the momentum distribution for all the particles present in the final state, multiplied it by the delta function of energy-momentum conservation and integrated over the (covariant) momentum space of all the particles except two identical pions for the two particle distribution and over all the momenta except one for the single particle distribution. They got results in qualitative agreement with experiment. The method, however, was cumbersome — no one did calculations for more than six particles in the final state — and was in violent disagreement with experiment at the quantitative level [6].

## 3. Kopylov and Podgoretskii model

Very significant progress was obtained by Kopylov and Podgoretskii (*cf.* [7] and references contained there). In their model the density operator in the Schrödinger picture is

$$\hat{\rho} = \int d^4x_{\mathrm{s}} \mathrm{e}^{iH_0(t_{\mathrm{s}}-t)}|\psi_{\mathrm{s}}\rangle\rho(x_{\mathrm{s}})\langle\psi_{\mathrm{s}}|\mathrm{e}^{-H_0(t_{\mathrm{s}}-t)}\,, \tag{8}$$

where

$$\langle\boldsymbol{x}|\psi_{\mathrm{s}}\rangle = \psi(\boldsymbol{x}-\boldsymbol{x}_{\mathrm{s}})\,; \tag{9}$$

or equivalently

$$\langle\boldsymbol{p}|\psi_{\mathrm{s}}\rangle \sim \int d^3x \mathrm{e}^{-i\boldsymbol{p}\boldsymbol{x}}\psi(\boldsymbol{x}-\boldsymbol{x}_{\mathrm{s}}) = \mathrm{e}^{-i\boldsymbol{p}\boldsymbol{x}_{\mathrm{s}}}A(\boldsymbol{p})\,, \tag{10}$$

where

$$A(\boldsymbol{p}) = \int d^3x \mathrm{e}^{-i\boldsymbol{p}\boldsymbol{x}}\psi(\boldsymbol{x})\,. \tag{11}$$

In these formulae $x_{\mathrm{s}}$ is a point in space-time, as well as a label which defines unambiguously a source. It should be related to the position of the corresponding source in space-time, but the actual relation is a matter of choice. *E.g.* it could be the point where the source got created, or its mean position in space-time. Each source is labeled by a space-time point, but the particle produced by a source is not localized in a point. Its wave function in space is $\psi(\boldsymbol{x}-\boldsymbol{x}_{\rm s})$ with an additional time dependent phase which is zero at $t=t_{\rm s}$. All these functions are related by shifts in space-time. The wave function in momentum space is proportional to $A(\boldsymbol{p})$, thus all the sources yields particles with the same momentum distribution. $H_0$ is the free particle Hamiltonian and we are interested in times $t$ larger than the latest time $t_{\rm s}$, thus the particle evolves freely.

The corresponding density matrix,

$$\rho(\boldsymbol{p}_1,\boldsymbol{p}_2) \sim A(\boldsymbol{p}_1)A^*(\boldsymbol{p}_2)\int d^4x_x {\rm e}^{iqs}\rho(x_{\rm s})\,, \tag{12}$$

yields the single particle distribution,

$$\frac{dN}{d^3p} \sim |A(\boldsymbol{p})|^2\,, \tag{13}$$

which can be made to agree with any experimental distribution by a suitable choice of $A(\boldsymbol{p})$. Thus, there is no obvious need to introduce the energy-momentum conservation constraint. Kopylov and Podgoretskii assumed that for final states with not too few particles, in the interesting momentum region the energy-momentum conservation does not affect significantly the one- and two-body distributions, and that consequently their matrix (12) can be assumed equal, up to a normalizing factor, to the actual single particle density matrix. This assumption makes it easy to get results for high multiplicity exclusive channels and for high energy inclusive processes. Both were beyond the reach of the GGLP method. Besides the satisfactory formula for the single particle distribution given above, the model gives for the correlation function

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = 1 + \int d^4x_{\rm s}{\rm e}^{iqx_{\rm s}}\rho(x_{\rm s})\,. \tag{14}$$

Here the factors $A(\boldsymbol{p})$ cancel and the result is as for point sources. Note, however, that because of the four-fold integration this relation cannot be inverted to give $\rho(x_{\rm s})$, when the correlation function is known. Moreover, a priori the correlation function could depend on the vector $\boldsymbol{q}$ and on the vector $\boldsymbol{K}=(\boldsymbol{p}_1+\boldsymbol{p}_2)/2$, while the right hand side depends only on the four-vector $q$. The reason is that the factor $A(\boldsymbol{p})$ is the same for every source and cancels. Physically this means that there is no correlation between the momentum of the pion and the position of the source. Since such correlation follow from almost every model and since experimentally it is not true, that the correlation function depends only on $q$ (*cf. e.g.* [1] and references quoted there) this is a serious weakness of the model.

## 4. Yano and Koonin method

Another attempt to go beyond the GGLP approximations is due to Yano and Koonin [8]. In our notation their key formula is

$$|\rho(\boldsymbol{p}_1,\boldsymbol{p}_2)|^2 = \mathrm{Re}\int d^4x_1 D(x_1,\boldsymbol{p}_1)\mathrm{e}^{iqx_1}\int d^4x_2 D(x_2,\boldsymbol{p}_2)\mathrm{e}^{-iqx_2}\,. \tag{15}$$

Putting

$$D(x,\boldsymbol{p}) = \delta(t)\rho(\boldsymbol{x}) \tag{16}$$

one recovers the GGLP model. Putting

$$D(x,\boldsymbol{p}) = |A(\boldsymbol{p})|^2\rho(x)\,, \tag{17}$$

which corresponds to the assumptions made in [8], one reproduces the results of Kopylov and Podgoretskii. It is not clear, whether this approach can be given sense in the framework of quantum mechanics, when there are position–momentum correlations, *i.e.* when the $D$ function does not factorize into a momentum dependent and a space dependent factor.

## 5. Covariant current formalism

The covariant current formalism (*cf.* [9] and references given there) can be derived from the model of Kopylov and Podgoretskii introducing the following two generalizations. The label characterizing the source is changed from $x_\mathrm{s}$ to $x_\mathrm{s}, p_\mathrm{s}$. Thus, the source is characterized by its position in space-time and by its four momentum. Correspondingly, the density of sources $\rho(x_\mathrm{s})$ gets replaced by $\rho(x_\mathrm{s},p_\mathrm{s})$. Such labels are not subject to the Heisenberg uncertainty principle. *E.g.* a one-dimensional harmonic oscillator may have $\langle x\rangle = 0$ and $\langle p\rangle = 0$. The universal, momentum dependent function $A(\boldsymbol{p})$ gets replaced by the source dependent $j(\frac{p_\mathrm{s}p}{m_\mathrm{s}})$, where $m_\mathrm{s}$ is the mass and $p_\mathrm{s}$ the momentum of the source. In the Kopylov Podgoretskii model all the sources had the same momentum distribution in some overall reference frame, *e.g.* in the center of mass frame of the collision. In the covariant current formalism each source has the same momentum distribution, when considered *in its rest frame*. Another way of formulating this model is to replace the incoherent sources by incoherent wave packets. Then $p_\mathrm{s}$ and $x_\mathrm{s}$ characterize the wave packet, and since the waves in the packet describe pions, it is natural to put $m_\mathrm{s} = m_\pi$. In this model, assuming a fixed mass for all the sources,

$$\rho(p_1,p_2) = \int d^4x_\mathrm{s}\int d^3p_\mathrm{s}\rho(x_\mathrm{s},p_\mathrm{s})\mathrm{e}^{iqs_\mathrm{s}} j\left(\frac{p_\mathrm{s}p_1}{m_\mathrm{s}}\right) j^*\left(\frac{p_\mathrm{s}p_2}{m_\mathrm{s}}\right)\,. \tag{18}$$

From the point of view of model builders, a nice feature of this approach is that one can assume a classical motion of the source, *e.g.* $\boldsymbol{x}_{\rm s} = \boldsymbol{x}_{\rm s}(t_{\rm s})$ and $\boldsymbol{p}_{\rm s} = \boldsymbol{p}_{\rm s}(t_{\rm s})$, and still have a formula which is consistent with quantum mechanics. In the covariant current formalism position–momentum correlations are naturally included.

## 6. Emission function method

The method of emission functions is very popular nowadays (*cf. e.g.* [1]). The emission function is built by analogy with the Wigner function [10, 11]. The Wigner function $W$ is related to the single particle density matrix in the momentum representation by the formula

$$\rho(\boldsymbol{p}_1, \boldsymbol{p}_2) = \int d^3X \mathrm{e}^{-i\boldsymbol{q}\boldsymbol{X}} W(\boldsymbol{X}, \boldsymbol{K}) , \tag{19}$$

where $\boldsymbol{X} = (\boldsymbol{x}_1 + \boldsymbol{x}_2)/2$, and as usual $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$ and $\boldsymbol{K} = (\boldsymbol{p}_1 + \boldsymbol{p}_2)/2$. The Wigner function is well-defined as a Fourier transform of the density matrix in momentum representation. On the other hand, as a function of $\boldsymbol{X}$ and $\boldsymbol{K}$ it may play the role of a phase space distribution. When studying the interactions regions, it is important to combine information about the space and the momentum distributions. Kopylov, Podgoretskii, and followers proposed to use the position of the source and the momentum of the particle. These two vectors can be measured simultaneously and their joint distribution gives an idea about the phase space distribution of particles. The Wigner function gives another ersatz phase space distribution. The trouble is, however, that the Wigner function refers to a given moment of time, while the hadrons are produced during a time interval. The emission function $S$ is supposed to improve on that and is related to density matrix by the formula

$$\rho(\boldsymbol{p}_1, \boldsymbol{p}_2) = \int d^4X \mathrm{e}^{iqX} S(X, K) . \tag{20}$$

The usual strategy is to find from some classical or quasi-classical argument a phase space distribution as function of time, and then to interpret it as the emission function. Once this is done, one can, by a four-fold integration, obtain the density matrix and compare it with experiment. This comparison may be used to fix the free parameters of the model.

Let us note some difficulties of this approach. Given a density matrix there is an infinity of very different emission functions, which can reproduce it. For instance perfect agreement is obtained for

$$S(X, K) = \delta(X_0) W(\boldsymbol{X}, \boldsymbol{K}) . \tag{21}$$

This, however, corresponds to simultaneous and instant production of all the hadrons, which is not a very plausible scenario. When particle production is an incoherent sum of production amplitudes at various moments of time, the emission function $S(\boldsymbol{X}, t, K)$ can be related to the Wigner function of the particles produced at time $t$. In the general case, however, the relation of the emission function to a Wigner function is hardly visible [12]. Thus, the formula can be used to eliminate wrong models rather than to prove that a model is implied by the data. This is not necessarily bad. Eliminating the unacceptable values of the parameters of a model, one learns which are the good ones. Nevertheless, one must always keep in mind that a completely different model can give equally good, or better results. One should also keep in mind that $\boldsymbol{X}$ and $\boldsymbol{K}$ are not really position and momentum, but only half sums of the corresponding arguments of the density matrix. When the contributions of the sources are strongly smeared in coordinate space or in momentum space, the difference may be very significant.

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