MODELS OF MAXIMAL ATMOSPHERIC NEUTRINO MIXING*

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We discuss two types of neutrino mass matrices which both give $\theta_{23} = 45^\circ$, *i.e.*, a maximal atmospheric mixing angle. We review three models, based on the seesaw mechanism and on simple extensions of the scalar sector of the Standard Model, where those mass matrices are obtained from symmetries.

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1. Motivation and introduction

In the last years great progress has been made in the measurements and understanding of the solar and atmospheric neutrino fluxes, and the oscillation solutions [1] for the solar and atmospheric neutrino deficits have been established — for recent reviews see, for instance, Ref. [2]. At the same time, the amazing precision of the Solar Standard Model [3], a necessary ingredient for the evaluation of the solar neutrino data, has also become evident. The measurement of the total active ^{8}B neutrino flux with enhanced neutral-current sensitivity (salt phase) by the SNO Collaboration [4] has

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further corroborated this picture for the solar neutrinos, and it has considerably reduced the allowed region in the solar oscillation parameters — for analyses including the SNO result see Refs. [5–10]. The present knowledge of the neutrino mixing angles can be summarized in the following way. For the atmospheric mixing angle the SuperKamiokande Collaboration [11] has obtained the bound $\sin^2 2\theta_{\rm atm} > 0.9$ at 90% CL, which corresponds to $\theta_{\rm atm} = 45^\circ \pm 9^\circ$. The allowed range for the solar mixing angle can be read off from the regions allowed at 90% CL in the above-mentioned papers, and is estimated as $\theta_\odot \sim 33^\circ\,{}^{+4^\circ}_{-3^\circ}$. Furthermore, from the new SNO data it follows that $\theta_\odot \neq 45^\circ$ at the $5\,\sigma$ level [4]. A further interesting development is that solar neutrino data have become numerically important in a three-neutrino analysis of the mixing angle θ_{13} [7, 9]; at the $3\,\sigma$ level Ref. [7] has obtained $\sin^2\theta_{13} < 0.054$. In addition, it has been established that, in the oscillation solution for the solar deficit, matter effects [12] play a decisive role [6].

In the course of time, the best-fit value of the atmospheric mixing has always remained stable at 45°. Therefore, we believe that the above results provide a motivation to search for models which have *maximal atmospheric neutrino mixing enforced by a symmetry* and *large but non-maximal solar neutrino mixing.*

In the following, we will only discuss models of massive Majorana neutrinos with a mass term

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\,\nu_{\rm L}^T C^{-1} \mathcal{M}_\nu \nu_{\rm L} + \text{h.c.} \tag{1}$$

We consider the following two mass matrices:

$$M1: \quad \mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \quad \text{with} \quad x,\, y,\, z,\, w \in \mathbb{C}\,, \tag{2}$$

$$M2: \quad \mathcal{M}_\nu = \begin{pmatrix} a & r & r^* \\ r & s & b \\ r^* & b & s^* \end{pmatrix} \quad \text{with} \quad r,\, s \in \mathbb{C}\,,\, a,\, b \in \mathbb{R}\,. \tag{3}$$

These mass matrices are defined in the basis where the charged-lepton mass matrix is diagonal. Phenomenological discussions of the matrix $M1$ can be found in many papers — see, *e.g.*, Ref. [13] — whereas $M2$ was recently found by Babu, Ma, and Valle in the context of models based on the group A_4 [14, 15]. We will see in the following that $M1$ and $M2$ have maximal atmospheric neutrino mixing. For an attempt to obtain $\theta_{\rm atm} = 45^\circ$ based on the group S_3 see Ref. [16].

The subject of the talk is the following:

(i) Discussion of the phenomenology of $M1$, $M2$;

(ii) Review of the models of Refs. [17,18] and of Ref. [19], which produce $M1$ and $M2$, respectively, by symmetries.

2. Phenomenology of the mass matrices $M1$, $M2$

The matrices $M1$, $M2$ can be algebraically characterized in a very simple way. Defining a unitary matrix

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{4}$$

the relations

$$M1: \quad S\mathcal{M}_\nu S = \mathcal{M}_\nu \,, \tag{5}$$
$$M2: \quad S\mathcal{M}_\nu S = \mathcal{M}_\nu^* \tag{6}$$

can be conceived as defining $M1$ and $M2$, respectively.

The Majorana mass matrix $\mathcal{M}_\nu$ is diagonalized by

$$V^T \mathcal{M}_\nu V = \mathrm{diag}\,(m_1, m_2, m_3)\,, \tag{7}$$

where the real and non-negative neutrino masses have been denoted by m_j ($j = 1, 2, 3$). The matrix V is decomposed as

$$V = \mathrm{e}^{i\hat{\alpha}} U_{23} U_{13} U_{12}\, \mathrm{diag}\left(1, \mathrm{e}^{i\beta_1}, \mathrm{e}^{i\beta_2}\right). \tag{8}$$

The diagonal phase matrix $\mathrm{e}^{i\hat{\alpha}} = \mathrm{diag}\left(\mathrm{e}^{i\alpha_1}, \mathrm{e}^{i\alpha_2}, \mathrm{e}^{i\alpha_3}\right)$ contains unphysical phases which can be absorbed into the charged-lepton fields. The unitary matrices U_{23}, U_{13}, U_{12} are given by

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \tag{9}$$

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{i\delta} & 0 & c_{13} \end{pmatrix}, \tag{10}$$

$$U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

respectively. Here, the notation $c_{12} \equiv \cos\theta_{12}$, *etc.* is used. The phases β_1, β_2 are the so-called Majorana phases (only $2\beta_1$ and $2\beta_2$ are physical). The neutrino mixing matrix $U = U_{23}U_{13}U_{12}$ contains the CP-violating phase δ, which is, in principle, accessible in neutrino oscillations. Our convention for the mixing matrix U is the same as the convention for the CKM matrix used in the Review of Particle Properties [20] (RPP convention). Note that $\theta_{12} \equiv \theta_\odot$ and $\theta_{23} \equiv \theta_{\mathrm{atm}}$.

2.1. Phenomenology of $M1$

Starting with the evident eigenvector relation

$$\begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = (z-w) \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \tag{12}$$

it is easy to check that the mixing matrix in the RPP convention is given by

$$U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \\ \sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{13}$$

Thus we obtain the following results for the neutrino mixing angles:

$$M1: \quad \theta_{13} = 0^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{12} \equiv \theta \text{ arbitrary.} \tag{14}$$

Furthermore, Eq. (12) gives $m_3 = |z-w|$. The neutrino masses in the case of the mass matrix $M1$ are free, *i.e.*, no relations among themselves or with the mixing angles are obtained. The parameter $\sin^2 2\theta_{\mathrm{atm}} = 4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right)$, which is probed in atmospheric and long-baseline experiments, is exactly equal to 1.

On the other hand, if one wishes to use the form (13) of the mixing matrix U as input and work back to $\mathcal{M}_\nu$, it is easy to see that a mass matrix of the form $M1$ ensues [13].

2.2. Phenomenology of $M2$

With relation (6) and the physical requirement of a non-degenerate three-neutrino mass spectrum one can show that the matrix V of Eq. (7) must fulfill the condition [19]

$$SV^* = VX\,, \tag{15}$$

where X is a diagonal phase matrix. From this equation it follows immediately that

$$|U_{\mu j}| = |U_{\tau j}| \;\; \forall j = 1,2,3\,. \tag{16}$$

Equation (16) was originally proposed by Harrison and Scott [21].

Before we proceed further, we note that the sets of matrices of type $M1$ and $M2$ have a non-vanishing overlap; *e.g.*, if a matrix of type $M2$ is real, then it is automatically of type $M1$ also. It has been shown in Ref. [19] that for matrices of type $M2$ one has

$$\sin\theta_{13} = 0 \Leftrightarrow r^2 s^* \in \mathbb{R}\,. \tag{17}$$

One direction of this equivalence is easy to demonstrate, namely if $r^2 s^* \in \mathbb{R}$ then by rephasing one obtains a matrix of type $M1$ from a matrix of type $M2$. Thus, in the following we will always assume that $r^2 s^* \notin \mathbb{R}$ for matrices of type $M2$, in order to genuinely distinguish them from matrices of type $M1$.

Then for the matrix $M2$ one has the following results [14, 15, 19]:

$$\theta_{23} = 45^\circ,\ \mathrm{e}^{i\delta} = \pm i,\ \mathrm{e}^{i\beta_{1,2}} = 1 \text{ or } i\,. \tag{18}$$

The first two results follow readily from relation (16) and the parameterization

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{19}$$

of the mixing matrix. Furthermore, we now have

$$\sin^2 2\theta_{\mathrm{atm}} = 4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2\right) = 1 - s_{13}^4\,;$$

for practical purposes this quantity is equal to 1, due to the smallness of s_{13}^4.

3. The seesaw mechanism with soft breaking of the family lepton numbers

Now we consider the lepton sector of the Standard Model (SM) with an arbitrary number n_H of Higgs doublets ϕ_j, supplemented by three right-handed neutrino singlets ν_{R}, and allow for lepton number violation. Thus we consider the Lagrangian

$$\begin{aligned} \mathcal{L} \;=\; \cdots - &\left[\sum_j \left(\bar{\ell}_{\mathrm{R}}\phi_j^\dagger \Gamma_j + \bar{\nu}_{\mathrm{R}}\tilde{\phi}_j^\dagger \Delta_j\right) D_{\mathrm{L}} + \mathrm{h.c.}\right] \\ &+ \left(\frac{1}{2}\,\nu_{\mathrm{R}}^T C^{-1} M_{\mathrm{R}}^* \nu_{\mathrm{R}} + \mathrm{h.c.}\right). \end{aligned} \tag{20}$$

The charged-lepton singlets are denoted by ℓ_{R} and the lepton doublets by D_{L}. The dots indicate the gauge part of $\mathcal{L}$. The mass matrix M_{R} of the right-handed neutrino singlets is symmetric. The mass matrix of the charged leptons and the so-called Dirac mass matrix in the neutrino sector are given by

$$M_\ell = \frac{1}{\sqrt{2}} \sum_j v_j^* \mathit{\Gamma}_j \,, \quad M_\mathrm{D} = \frac{1}{\sqrt{2}} \sum_j v_j \mathit{\Delta}_j \,, \tag{21}$$

respectively, with the vacuum expectation values (VEVs) $\left\langle \phi_j^0 \right\rangle_0 = v_j/\sqrt{2}$. The total Majorana mass matrix for left-handed neutrino fields is obtained as

$$\mathcal{M}_{\mathrm{D}+M} = \begin{pmatrix} 0 & M_\mathrm{D}^T \\ M_\mathrm{D} & M_\mathrm{R} \end{pmatrix} \quad \text{for} \quad \begin{pmatrix} \nu_\mathrm{L} \\ C(\bar{\nu}_\mathrm{R})^T \end{pmatrix}. \tag{22}$$

With the assumption $m_\mathrm{D} \ll m_\mathrm{R}$, where m_D and m_R are the scales of M_D and M_R, respectively, the seesaw mechanism [22] is obtained where

$$\mathcal{M}_\nu = -M_\mathrm{D}^T M_\mathrm{R}^{-1} M_\mathrm{D} \tag{23}$$

for the three light neutrinos.

Diagonalization of M_ℓ proceeds via $(U_\mathrm{R}^\ell)^\dagger M_\ell U_\mathrm{L}^\ell = \hat{m}_\ell$ with two unitary matrices $U_\mathrm{R,L}^\ell$. Then the neutrino mixing matrix is given by $U_M = (U_\mathrm{L}^\ell)^\dagger V$, where V is defined in Eq. (7). Thus with the seesaw mechanism there are three sources for neutrino mixing: M_ℓ, M_D, and M_R. We may choose as a typical neutrino mass $m_\nu \sim \sqrt{\Delta m^2_\mathrm{atm}} \sim 0.05$ eV where Δm^2_atm is the atmospheric mass-squared difference. Then, if we adopt as a reasonable guess $m_\mathrm{D} \sim m_{\mu,\tau}$, the right-handed scale is typically in the range $m_\mathrm{R} \sim 10^8 \div 10^{11}$ GeV. One could also use $m_\mathrm{D} \sim$ electroweak scale, then $m_\mathrm{R} \sim 10^{15}$ GeV could be identified with the GUT scale.

For the rest of this report we reduce the three sources of neutrino mixing to one, namely to M_R. This means that we choose *diagonal* coupling matrices $\mathit{\Gamma}_j$ and $\mathit{\Delta}_j$. This is a well-defined renormalizable theory: diagonal Yukawa couplings are guaranteed by conservation of the family lepton numbers L_α ($\alpha = e, \mu, \tau$) which are *softly* broken by the Majorana mass term of the ν_R in the Lagrangian of Eq. (20) [17, 23]. We may summarize the properties of such a theory of the seesaw mechanism in the following way:

(i) Soft L_α breaking by the ν_R mass terms occurs at the *high* scale m_R;

(ii) With diagonal Yukawa couplings, the matrices M_ℓ, M_D are diagonal as well;

(iii) M_R is the only source of neutrino mixing;

(iv) For $n_H > 1$, in the limit $m_\mathrm{R} \to \infty$, there is a non-decoupling in the scalar sector, stemming from the neutral-scalar–charged-lepton vertices, in the following sense [23]:

— Amplitudes of, *e.g.*, $\mu \to e\gamma$ and $Z \to \mathrm{e}^- \mu^+$ scale with $1/m_\mathrm{R}^2$ for large m_R;

— The amplitude of, *e.g.*, $\mu \to 3e$ approaches a constant in that limit and is not suppressed by m_R; rather, it is suppressed by a product of four Yukawa couplings and, possibly, the branching ratio of this process is within future experimental reach.

The models developed in Refs. [17–19], which will be reviewed in the following, are all of this type.

4. Models for obtaining mass matrix $M1$

4.1. The $\mathbb{Z}_2$ model

According to the previous section, the $\mathbb{Z}_2$ model of Ref. [17] contains the SM multiplets supplemented by three right-handed heavy neutrino singlets ν_R; moreover, it has three Higgs doublets ϕ_j. The symmetries are the following:

- $\mathrm{U}(1)_{L_\alpha}$ $(\alpha = e, \mu, \tau)$ associated with the family lepton numbers L_α;
- $\mathbb{Z}_2^{(\mathrm{tr})}: \quad D_{\mu\mathrm{L}} \leftrightarrow D_{\tau\mathrm{L}}\,,\ \mu_\mathrm{R} \leftrightarrow \tau_\mathrm{R}\,, \nu_{\mu\mathrm{R}} \leftrightarrow \nu_{\tau\mathrm{R}}\,,\ \phi_3 \to -\phi_3\,; \qquad (24)$
- $\mathbb{Z}_2^{(\mathrm{aux})}: \quad \nu_{e\mathrm{R}},\ \nu_{\mu\mathrm{R}},\ \nu_{\tau\mathrm{R}},\ \phi_1,\ e_\mathrm{R}$ change sign. $\qquad (25)$

The symmetry $\mathbb{Z}_2^{(\mathrm{tr})}$ transposes the muon and tau family and is spontaneously broken by the VEV of ϕ_3. The symmetry $\mathbb{Z}_2^{(\mathrm{aux})}$, spontaneously broken by the VEV of ϕ_1, is an auxiliary $\mathbb{Z}_2$ which prevents — at the tree level — $\mathbb{Z}_2^{(\mathrm{tr})}$ breaking in the neutrino sector. The above symmetries determine the Yukawa Lagrangian as

$$
\begin{aligned}
\mathcal{L}_\mathrm{Y} \ = \ & -y_1 \bar{D}_{e\mathrm{L}} \nu_{e\mathrm{R}} \tilde{\phi}_1 - y_2 \left(\bar{D}_{\mu\mathrm{L}} \nu_{\mu\mathrm{R}} + \bar{D}_{\tau\mathrm{L}} \nu_{\tau\mathrm{R}} \right) \tilde{\phi}_1 \\
& - y_3 \bar{D}_{e\mathrm{L}} e_\mathrm{R} \phi_1 - y_4 \left(\bar{D}_{\mu\mathrm{L}} \mu_\mathrm{R} + \bar{D}_{\tau\mathrm{L}} \tau_\mathrm{R} \right) \phi_2 \\
& - y_5 \left(\bar{D}_{\mu\mathrm{L}} \mu_\mathrm{R} - \bar{D}_{\tau\mathrm{L}} \tau_\mathrm{R} \right) \phi_3 + \mathrm{h.c.}
\end{aligned} \tag{26}
$$

The mass matrix M_R is S-invariant, *i.e.*, $S M_\mathrm{R} S = M_\mathrm{R}$. Moreover, from Eq. (26) we read off that $M_\mathrm{D} = \mathrm{diag}\,(c, d, d)$, *i.e.*, M_D is S-invariant as well. Consequently, $\mathcal{M}_\nu$ given by Eq. (23) is S-invariant and due to Eq. (5) has the form $M1$.

The family lepton number groups $\mathrm{U}(1)_{L_\mu}$ and $\mathrm{U}(1)_{L_\tau}$ do not commute with $\mathbb{Z}_2^{(\mathrm{tr})}$; thus the basic non-abelian symmetry group in the μ–τ sector is O(2) [24]. Furthermore, it was shown that the $\mathbb{Z}_2$ model can be embedded in an SU(5) Grand Unified Theory [24].

4.2. The D_4 model

This model [18] has the same multiplets as the $\mathbb{Z}_2$ model, but we add two real scalar gauge singlets χ_1 and χ_2. The symmetries of the D_4 model are the following:

- $\mathbb{Z}_2^{(\mathrm{tr})}$: $\ldots,\ \chi_1 \leftrightarrow \chi_2$, where the dots indicate the transformations of Eq. (24);

- $\mathbb{Z}_2^{(\tau)}$: $D_{\tau\mathrm{L}},\ \tau_\mathrm{R},\ \nu_{\tau\mathrm{R}},\ \chi_2$ change sign; (27)

- $\mathbb{Z}_2^{(\mathrm{aux})}$ as in Eq. (25).

The symmetries $\mathbb{Z}_2^{(\mathrm{tr})}$ and $\mathbb{Z}_2^{(\tau)}$ generate the 2-dimensional irreducible representation $\underline{2}$ of the group D_4. Thus the pairs $(D_{\mu\mathrm{L}}, D_{\tau\mathrm{L}})$, $(\mu_\mathrm{R}, \tau_\mathrm{R})$, $(\nu_{\mu\mathrm{R}}, \nu_{\tau\mathrm{R}})$, and (χ_1, χ_2) transform all as $\underline{2}$ under D_4. With the above symmetries we obtain the Yukawa Lagrangian

$$\mathcal{L}'_\mathrm{Y} = \mathcal{L}_\mathrm{Y} + \left[\frac{1}{2}\, y_\chi\, \nu_{e\mathrm{R}}^T C^{-1} \left(\nu_{\mu\mathrm{R}} \chi_1 + \nu_{\tau\mathrm{R}} \chi_2 \right) + \mathrm{h.c.} \right], \tag{28}$$

where $\mathcal{L}_\mathrm{Y}$ is given by Eq. (26). Furthermore, there is a Majorana mass term of the right-handed neutrino singlets

$$\mathcal{L}_M = \frac{1}{2} \left[M^* \nu_{e\mathrm{R}}^T C^{-1} \nu_{e\mathrm{R}} + M'^* \left(\nu_{\mu\mathrm{R}}^T C^{-1} \nu_{\mu\mathrm{R}} + \nu_{\tau\mathrm{R}}^T C^{-1} \nu_{\tau\mathrm{R}} \right) \right] + \mathrm{h.c.} \tag{29}$$

The mass matrix M_R has not only contributions from $\mathcal{L}_M$, but also from the VEVs of the χ_i, which may be parameterized in the following way:

$$\langle \chi_1 \rangle_0 = W \cos\gamma\,, \quad \langle \chi_2 \rangle_0 = W \sin\gamma\,, \tag{30}$$

with $W > 0$. The VEVs of the Higgs doublets represent the electroweak scale via $v^2 \equiv \sum_j |v_j|^2 = (246\ \mathrm{GeV})^2$. According to the seesaw mechanism we assume

$$W \sim |M|\,,\ |M'| \gg v\,. \tag{31}$$

Then, by considering the scalar potential, one can show [18] that $\cos 2\gamma = \mathcal{O}(v^2/W^2)$ or $\gamma = 45^\circ$ up to corrections of order v^2/W^2; such corrections are completely negligible and, therefore, $\langle \chi_1 \rangle_0 = \langle \chi_2 \rangle_0 = W/\sqrt{2}$. These VEVs break D_4 down to $\mathbb{Z}_2^{(\mathrm{tr})}$.

Finally, we arrive at

$$M_\mathrm{D} = \mathrm{diag}\,(c, d, d)\,, \quad M_\mathrm{R} = \begin{pmatrix} M & M_\chi & M_\chi \\ M_\chi & M' & 0 \\ M_\chi & 0 & M' \end{pmatrix}, \tag{32}$$

with $M_\chi = y_\chi^* W/\sqrt{2}$. As in the previous section, $M_{\rm D}$ and $M_{\rm R}$ are both S-invariant and, therefore, $\mathcal{M}_\nu$ is of the form $M1$.

In the D_4 model, the effective mass probed in neutrinoless $\beta\beta$-decay can be expressed by the masses of the light neutrinos, namely $|\langle m \rangle| = m_1 m_2/m_3$. This is a consequence of $(M_{\rm R})_{\mu\tau} = 0$. For a further discussion of $|\langle m \rangle|$ see Ref. [18].

5. Models for obtaining mass matrix $M2$

5.1. A_4 models

Because of its irreducible representations, the group A_4 of the even permutations of four objects is an interesting discrete group for model building [25]. Originally, the mass matrix $M2$ of Eq. (3) was obtained in a supersymmetrized version of the SM with additional fermionic and scalar singlets [14]. Then a model without supersymmetry, where the SM was enlarged by an A_4-triplet of charged scalar singlets of Zee type and heavy gauge singlets $E_{\rm L,R}$, was devised in Ref. [15]. However, we will not pursue this line but concentrate instead on relation (6) which suggests the use of a non-standard CP transformation (for a review of the theory of CP transformations see Ref. [26]).

5.2. The CP model

In this model [19] the multiplets are the same as in the $\mathbb{Z}_2$ model. The symmetries are the following:

- $\mathrm{U}(1)_{L_\alpha}$ $(\alpha = e, \mu, \tau)$;
- The non-standard CP transformation [19, 21]
$$\begin{aligned} &D_{\alpha \rm L} \to i S_{\alpha\beta} \gamma^0 C \bar{D}^T_{\beta \rm L}\,, \quad \nu_{\alpha \rm R} \to i S_{\alpha\beta} \gamma^0 C \bar{\nu}^T_{\beta \rm R}\,, \quad \alpha_{\rm R} \to i S_{\alpha\beta} \gamma^0 C \bar{\beta}^T_{\rm R}\,, \\ &\phi_{1,2} \to \phi^*_{1,2}\,, \quad \phi_3 \to -\phi^*_3\,, \end{aligned} \tag{33}$$
where $\alpha, \beta = e, \mu, \tau$ and S is defined in Eq. (4);
- $\mathbb{Z}_2^{(\rm aux)}$.

With these symmetry operations, we obtain the Yukawa Lagrangian

$$\begin{aligned} \mathcal{L}''_{\rm Y} \;=\; & -y_1 \bar{D}_e \nu_{e\rm R} \tilde{\phi}_1 - \left(y_2 \bar{D}_\mu \nu_{\mu \rm R} + y_2^* \bar{D}_\tau \nu_{\tau \rm R} \right) \tilde{\phi}_1 \\ & -y_3 \bar{D}_e e_{\rm R} \phi_1 - \left(y_4 \bar{D}_\mu \mu_{\rm R} + y_4^* \bar{D}_\tau \tau_{\rm R} \right) \phi_2 \\ & - \left(y_5 \bar{D}_\mu \mu_{\rm R} - y_5^* \bar{D}_\tau \tau_{\rm R} \right) \phi_3 + \text{h.c.} \end{aligned} \tag{34}$$

The coupling constants y_1 and y_3 are real, whereas y_2, y_4, and y_5 are in general complex.

Assuming without loss of generality $v_1 \in \mathbb{R}$, we have $M_\mathrm{D} = \mathrm{diag}\,(c, d, d^*)$ with $c \in \mathbb{R}$, and therefore M_D fulfills $M_\mathrm{D}^* = S M_\mathrm{D} S$. Since, by virtue of Eq. (33), $M_\mathrm{R}^* = S M_\mathrm{R} S$ holds, it follows that $\mathcal{M}_\nu$ fulfills relation (6) and has, therefore, the form $M2$. We note that in the CP model $m_\mu \neq m_\tau$ is a consequence CP violation [19].

6. Summary

In this report we have first discussed the phenomenology of the neutrino mass matrices $M1$ and $M2$ of Eqs. (2) and (3), respectively, Then we have reviewed the $\mathbb{Z}_2$ model of Ref. [17] and the D_4 model of Ref. [18], which both yield the mass matrix $M1$, and the CP model of Ref. [19], which yields mass matrix $M2$. These three models have several features in common:

- The SM is enlarged with three right-handed neutrino singlets, there are three Higgs doublets instead of one, and — only in the case of the D_4 model — there are two real scalar gauge singlets.
- The seesaw mechanism is responsible for the smallness of the neutrino masses.
- Below the seesaw scale, the family lepton numbers L_α are softly broken by the mass term of the right-handed singlets, *i.e.*, M_R is the sole source of neutrino mixing.[1]
- Neutrino mass matrices of form $M1$, $M2$ are obtained by non-abelian horizontal symmetry groups in the case of the $\mathbb{Z}_2$ and D_4 models, and by a non-standard CP transformation which does not commute with $\mathrm{U}(1)_{L_\alpha}$ ($\alpha = \mu, \tau$) in the case of the CP model.
- All three models have a maximal atmospheric neutrino mixing $\theta_{23} = 45^\circ$.
- The models have no predictions for the neutrino mass spectrum, *i.e.*, there are no relations among the masses or between the masses and mixing angles.

Other predictions for the mixing angles are $\sin\theta_{13} = 0$ in the case of matrix $M1$, and $\sin\theta_{13} \neq 0$, $\mathrm{e}^{i\delta} = \pm i$ in the case of matrix $M2$. Finally we note that, looking at the Yukawa Lagrangians of Eqs. (26), (28), and (34),

[1] We want to stress that this means also that the charged-lepton mass matrix is diagonal not by assumption but by virtue of the lepton numbers L_α.

one would expect the "natural relation" $m_\mu \sim m_\tau$, following from the μ–τ interchange symmetry. However, simply by introducing an additional symmetry but no further multiplets one can achieve $m_\mu \ll m_\tau$ in a technically natural way [27].

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