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THE PUZZLE OF NEUTRINO MASSES*

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Realistic see-saw models of neutrino masses and mixing are briefly reviewed. It is shown that a rather small hierarchy of the mass scales governing neutrino oscillations can be obtained in models assuming a large hierarchy of the Dirac masses. The low energy mass spectrum of the active neutrinos is sensitive to unitary transformations for the right-handed neutrinos. Predictions for the mass spectrum of active neutrinos are discussed.

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1. The hierarchies of neutrino masses

It is well known that the mass spectrum of fundamental fermions belongs to the most challenging problems of theoretical particle physics. The amazingly successful Standard Model does not offer any reasonable answer to the questions about the number of families and the origin of the highly hierarchical mass structure of quarks and leptons. Non-zero masses of neutrinos make this problem even more surprising and difficult. The combined information on neutrino oscillations and the upper mass limit from tritium decays show that the heaviest active neutrino state is nine to eleven orders of magnitude lighter than the other fermions in the third family, *i.e.* the top and bottom quarks, and the τ lepton. By active neutrinos we understand these experimentally observed neutrinos, ν_e, ν_μ and ν_τ which interact electroweakly. So, in addition to the old question about the ratio of the top quark mass to the electron mass which is of order 10^5 we should explain even more striking observation that the mass ratios within a single family can be much larger. A very attractive and popular answer is the see-saw mechanism [1]. If this idea is true than the masses of neutrinos are of the

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Majorana type in contrast to the masses of all other fundamental fermions which are of the Dirac type. Moreover, the masses of active neutrinos are low energy composite objects rather than fundamental mass parameters of the underlying theory. These more fundamental parameters are the Dirac masses describing couplings between left-handed and right-handed neutrinos, and the Majorana masses of the right-handed neutrinos. The smallness of the active neutrino masses is a consequence of a huge mass scale of the Majorana masses for the right-handed neutrinos. This new mass scale is not restricted by local gauge invariance of the Standard Model because the right-handed neutrinos are singlets of its gauge group $\mathrm{SU}_3 \times \mathrm{SU}_2 \times \mathrm{U}_1$. The effective mass operator for the active neutrinos is a 3×3 symmetric matrix of the form

$$\mathcal{N} = -{m_{\mathrm{D}}}^{\mathrm{T}} M_{\mathrm{R}}^{-1} m_{\mathrm{D}}\,, \tag{1}$$

where m_{D} is a $n_{\mathrm{R}} \times 3$ complex matrix of Dirac masses and M_{R}^{-1} denotes a complex symmetric matrix which is the inverse of the Majorana mass matrix for the right-handed neutrinos. Majorana masses are not allowed for particles with non-zero electric charge. So, the masses of charged leptons and quarks are all of the Dirac type and they all exhibit a clear hierarchy: $m_e \ll m_\mu \ll m_\tau$, $m_u \ll m_c \ll m_t$, and $m_d \ll m_s \ll m_b$. It seems quite natural to assume that this hierarchical structure is a common feature for the Dirac masses of all fundamental fermions, so the Dirac masses of the neutrinos, *i.e.* the eigenvalues of m_{D} should be also hierarchical:

$$m_1 \ll m_2 \ll m_3\,. \tag{2}$$

In the simplest case, assuming $n_{\mathrm{R}} = 3$ and degenerate mass spectrum of M_{R},

$$M_{\mathrm{R}} = M\mathbf{1} \tag{3}$$

a very small mixing and a very strong hierarchy is obtained for the masses of the three active neutrinos:

$$\mu_1 = \frac{m_1^2}{M}\,, \qquad \mu_2 = \frac{m_2^2}{M}\,, \qquad \mu_3 = \frac{m_3^2}{M}\,. \tag{4}$$

Both these predictions are evidently wrong. First of all the data on oscillations of atmospheric neutrinos from the famous SuperKamiokande experiment strongly support maximal mixing between ν_μ and ν_τ. Furthermore, the data on solar neutrinos (Homestake, GALLEX + GNO, SAGE, SuperKamiokande, and SNO) as well as on reactor antineutrinos $\bar{\nu_e}$ (KAMLAND) show a large, although not maximal amount of mixing for the electron neutrino. A recent fit by SNO [2] gives $\theta_{\mathrm{sol}} = 32.5^{+1.7}_{-1.6}$ degrees, *i.e.*

$$\tan^2\theta_{\mathrm{sol}} = 0.41 \pm 0.05\,. \tag{5}$$

The absolute values of the active neutrino masses are not known yet. From the Mainz–Troitsk experiment on tritium decays an upper limit of about 2 eV has been derived on the electron neutrino mass. The oscillations of atmospheric neutrinos are governed by the mass scale

$$\Delta m^2_{\rm atm} \approx \mu_3^2 - \mu_2^2 \approx 2.5 \times 10^{-3}{\rm eV}^2\,. \tag{6}$$

The best fit to the oscillations of solar and reactor neutrinos gives [2]:

$$\Delta m^2_{\rm sol} = \mu_2^2 - \mu_1^2 \approx 7.1 \times 10^{-5}{\rm eV}^2\,. \tag{7}$$

It follows that the upper limit on the mass ratio

$$\frac{\mu_3}{\mu_2} \approx \frac{\sqrt{\Delta m^2_{\rm atm}}}{\sqrt{\Delta m^2_{\rm sol}}} \approx 6 \tag{8}$$

is much smaller than the corresponding mass ratios for the charged leptons $m_\tau/m_\mu \approx 17$, the up type quarks $m_t/m_c \sim 100$, and the down type quarks $m_b/m_s \sim 30$. Of course this is in a drastic disagreement with Eqs. (2) and (4) which suggest a much larger value of μ_3/μ_2. For example in many grand unified models a reasonable assumption is $\mu_3/\mu_2 \sim m_t/m_c$ leading to $\mu_3/\mu_2 \sim 10^4$.

2. Possible solutions

There are many ways to modify the assumptions leading to the disastrous mass spectrum given in Eq. (4) and described in the previous section. In the following a few ideas are described which have been discussed in the literature of the subject:

- $n_{\rm R} \neq 3$. If the number of the right-handed neutrinos is not equal three then $m_{\rm D}$ is not a square matrix and it does not have eigenvalues. The mass hierarchy (3) does not exist from the very beginning for the Dirac masses of the neutrinos. Evidently, $n_{\rm R} = 2$ is the simplest and most attractive possibility. An interesting and in principle testable prediction of $n_{\rm R} = 2$ models is the mass spectrum which consists of one strictly massless and two massive active neutrinos.

- $m_3 \sim m_2$. For three right-handed neutrinos the mass spectrum of the active neutrinos given in Eq. (4) can be consistent with the existing data on neutrino oscillations if for some reason the Dirac masses of neutrinos are not hierarchical and the eigenvalues m_3 and m_2 are of the same order. Dynamical schemes have been proposed based on E_6 grand unification as well as other ideas leading to such models, see *e.g.* [4].

- $M_1 \ll M_2 \ll M_3$. Another possibility for $n_{\mathrm{R}} = 3$ is that the eigenvalues of M_{R} are highly hierarchical and this results in a large reduction of the mass hierarchy for the active neutrinos. A realistic realization of this idea is the model proposed in Ref. [3]. The assumed ratios of the eigenvalues of the Majorana mass matrix M_{R} are:

$$M_1 \ : \ M_2 \ : \ M_3 \ \sim \ \epsilon^6 \ : \ \epsilon^4 \ : \ 1$$

  with $\epsilon \sim 0.1$, so in particular for the second and third generations the hierarchy is much stronger than for other fundamental fermions. However, such a situation can be understood in some grand unified models. Moreover, the hierarchy of the right-handed Majorana masses can be weakened in some models as shown in the following.

- m_{D} and M_{R} diagonal in different reference frames. The strong hierarchy predicted in Eq. (4) can be reduced in a dramatic way by unitary transformations connecting the two frames in the flavor space for which the matrices m_{D} and M_{R} are diagonal. It has been shown [5, 6] that even for a degenerate mass spectrum of M_{R}, *cf.* Eq. (3), and hierarchical Dirac masses as in Eq. (2) a realistic model can be built for the masses and mixing of the active neutrinos. These two frames are correlated in a highly non-trivial way which may reflect some underlying discrete symmetry. In the following section the model of Refs. [5, 6] is briefly described.

3. Hidden hierarchy

It has been shown in [5, 6] that a rather small hierarchy of the observed low energy masses of the active neutrinos can follow from a large hierarchy at the more fundamental level of the Dirac masses. This can happen even in the case when the Majorana masses of the right-handed neutrinos are all equal. Therefore large hierarchies in the Dirac masses of all other fundamental fermions, *i.e.* up quarks, down quarks and charged leptons, can be accompanied by a large hierarchy of the Dirac masses for the neutrinos. This large hierarchy is drastically modified by a symmetric unitary operator R related to unitary transformations of the right-handed neutrinos. In some sense the underlying hierarchy of the Dirac masses is hidden and only mildly reflected in the effective low energy masses. This phenomenon is due to the see-saw mechanism and the algebraic structure of the low energy effective mass operator $\mathcal{N}$ describing the masses of the active neutrinos.

Let us assume that there are three right-handed neutrinos and their Majorana masses are equal to M, which is huge. We choose the reference frame in the flavor space such that M_{R}, the Majorana mass matrix of the right-handed neutrinos is diagonal and has the form (3). The form of the matrix M_{R} depends on the reference frame in a non-trivial way. In particular it need not be proportional to the unit matrix in the frame where m_{D}, the matrix of the Dirac neutrino masses is diagonal and exhibits a strong hierarchy (2). So in our reference frame m_{D} can be an arbitrary complex matrix of the form

$$m_{\mathrm{D}} = U_{\mathrm{R}}\, m^{(\nu)}\, U_{\mathrm{L}}\,, \tag{9}$$

where the matrices U_{L} and U_{R} are unitary and $m^{(\nu)}$ is diagonal:

$$m^{(\nu)} = \mathrm{diag}(m_1, m_2, m_3)\,. \tag{10}$$

The mass spectrum of the active neutrinos is dictated by an effective mass operator $\mathcal{N}$

$$\mathcal{N} = -{m_{\mathrm{D}}}^{\mathrm{T}} M_{\mathrm{R}}^{-1} m_{\mathrm{D}} = -\frac{1}{M} U_{\mathrm{L}}^{\mathrm{T}} m^{(\nu)\mathrm{T}} U_{\mathrm{R}}^{\mathrm{T}} U_{\mathrm{R}} m^{(\nu)} U_{\mathrm{L}}\,. \tag{11}$$

The algebraic structure of $\mathcal{N}$ implies that the resulting mass spectrum is extremely sensitive to the form of the following matrix R,

$$R = U_{\mathrm{R}}^{\mathrm{T}} U_{\mathrm{R}}\,, \tag{12}$$

which is symmetric and unitary. It is remarkable that to some extent the condition (2) fixes the reference frame in the flavor space. Unitary transformations of the right-handed fields are in general complex and cannot be gauged away even if M_{R} is proportional to the unit matrix. The matrix R can drastically reduce the hierarchy of the mass spectrum for the active neutrinos. So, R is observable, in principle at least, if a large hierarchy of the Dirac masses is the common feature of all quarks and leptons. In this sense R is a physical object which is imprinted in low energy physical quantities, namely the masses of the active neutrinos. Unlike the quark sector with its Cabibbo–Kobayashi–Maskawa mixing matrix [7] the lepton sector has therefore two important matrices in the flavor space. One is the lepton mixing matrix $\mathrm{U}_{\mathrm{MNS}}$ [8] which affects the form of the weak charged current. Another is the matrix R defined in Eq. (12). R affects the form of U_{MNS}. Moreover, it is also reflected in the low energy neutrino mass spectrum.

It has been shown in [5, 6] that a realistic model can be built for

$$R = \begin{pmatrix} 0 & 0 & \exp i\phi_1 \\ 0 & \exp i\phi_2 & 0 \\ \exp i\phi_1 & 0 & 0 \end{pmatrix}. \tag{13}$$

In this model a small absolute value of the element U_{e3} of the Maki–Nakagawa–Sakata lepton mixing matrix [8] can be easily explained. Furthermore the model predicts the mass spectrum of the active neutrino mass eigenstates:

$$\mu_1 = \mu r \exp(-t)\,, \quad \mu_2 = \mu r \exp t\,, \quad \mu_3 = \mu\,, \tag{14}$$

with $t = -\ln\tan\theta_{\rm sol}$ and $r = m_1 m_3/m_2^2$. It follows that for the two lighter neutrino mass eigenstates

$$\frac{\mu_1}{\mu_2} = \tan^2\theta_{\rm sol}\ . \tag{15}$$

The above formulae for the neutrino masses can be expressed in terms of measurable quantities:

$$\mu_1 \ = \ \sqrt{\Delta m^2_{\rm sol}}\ \tan^2\theta_{\rm sol}\ /\ \sqrt{1-\tan^4\theta_{\rm sol}}\ , \tag{16}$$

$$\mu_2 \ = \ \frac{\sqrt{\Delta m^2_{\rm sol}}}{\sqrt{1-\tan^4\theta_{\rm sol}}}\,, \tag{17}$$

$$\mu_3 \ \approx \ \sqrt{\Delta m^2_{\rm atm}}\,. \tag{18}$$

Then using the central values for $\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm sol}$ and $\tan^2\theta_{\rm sol}$, *cf.* Eqs. (5)–(7), one obtains

$$\mu_1 = 3.8 \text{ meV}\,, \quad \mu_2 = 9.2 \text{ meV}\,, \quad \text{and} \quad \mu_3 = 50 \text{ meV}\,.$$

This prediction can be tested for neutrinos from very distant astronomical sources. For the ratio r of the Dirac neutrino masses one obtains

$$r = \frac{m_1 m_3}{m_2^2} = \frac{\sqrt{\mu_1\mu_2}}{\mu_3} \approx 0.116\,. \tag{19}$$

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