NEUTRINO MIXING AND LEPTON FLAVOUR VIOLATION IN SUSY–GUT MODELS*

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In supersymmetric (SUSY) models the misalignment between fermion and sfermion families introduces unsuppressed flavour-changing processes. Even if the mass parameters are chosen to give no flavour violation, family dependent radiative corrections make this adjustment not stable. In particular, due to the observed large neutrino mixings and potentially large neutrino Yukawa couplings, sizable lepton flavour violation (LFV) is expected. After introducing the basic concepts, the framework and the main assumptions, we report on a recent study of rare leptonic decays in a class of SUSY–GUT models with three quasi-degenerate neutrinos. We show that LFV effects are likely visible in forthcoming experiments.

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1. Lepton flavour violation

In the standard model (SM) neutrinos are massless and hence individual lepton flavour is conserved. This is in contrast with the quark sector, where the Cabibbo–Kobayashi–Maskawa (CKM) mixing allows for (one-loop, GIM suppressed) flavour changing neutral currents. In particular, the process $b \to s\gamma$ is predicted at a rate in agreement with experiment.

The observation of neutrino oscillations has opened for the first time the gate to physics beyond the SM. It requires the introduction of right-handed neutrino isosinglets and Dirac mass terms analogous to the ones in the quark sector. Majorana mass terms for the singlets are also possible (although not indispensable), and they could be used to explain the lightness of the neutrino masses via the seesaw mechanism. In either case, LFV processes

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like $\mu \to e\gamma$, $Z \to \mu e$, $\mu \to 3e$ are possible at a tiny rate in this so-called νSM. Thus

$$\nu\text{SM}: \ \text{BR}(\ell_i \to \ell_j \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|M_{\nu_{ik}}^* M_{\nu_{kj}}|^2}{M_W^8} \lesssim 10^{-45} \ , \tag{1}$$

where the neutrino mass matrix $\boldsymbol{M_\nu} = \boldsymbol{U}^* \text{diag}(m_1, m_2, m_3)\boldsymbol{U}^\dagger$ contains off-diagonal entries due to the misalignment between the charged-lepton and neutrino mass matrices encoded in the Maki–Nakagawa–Sakata (MNS) mixing matrix $\boldsymbol{U}$. Present experimental constrains $M_{\nu_{ij}} \lesssim 1$ eV are used.

SUSY extensions of the SM introduce new sources of flavour non-conservation: the misalignment of lepton and slepton mass matrices causes unsuppressed LFV. The dominant contribution in the minimal supersymmetric standard model (MSSM) is

$$\text{MSSM}: \ \text{BR}(\ell_i \to \ell_j \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|m_{\tilde{L}_{ij}}^2|^2}{m_S^8} \tan^2 \beta \lesssim (\delta_{LL}^{ij})^2 \tan^2 \beta \times 10^{-5} \ , \tag{2}$$

where the off-diagonal (in the basis of charged-lepton mass eigenstates) slepton mass insertions $m_{\tilde{L}_{ij}}^2 \sim m_S^2 \delta_{L_{ij}}$ are normalized to a typical SUSY mass scale $m_S \gtrsim 10^2$ GeV. Even for low $\tan\beta$, the experimental limits require a very fine alignment, in particular for the first two generations: $\text{BR}(\mu \to e\gamma) \lesssim 10^{-11} \Rightarrow \delta_{LL}^{12} \lesssim 10^{-3}$. This is generally formulated as the SUSY *flavour problem*: how to explain such an alignment when fermion and sfermion masses have a different origin; the former come from (SUSY-preserving) Yukawa interactions (Y_{ij}) and the latter from a SUSY-breaking mechanism (m_{ij}^2).

The most economical solution is that $\boldsymbol{m^2 \propto 1}$. This can be the case in SUGRA scenarios, where SUSY is broken in a hidden sector only connected via gravitational interactions with the SM. Gravitational interactions could generate identical masses for the three sfermion masses at the Planck scale. But the renormalization-group (RG) corrections from the Planck down to the electroweak scale would then introduce off-diagonal entries due to the different Yukawa interactions of the three slepton families. The impact of this effect on LFV decays is the subject of our analysis [1], with special emphasis on the off-diagonal slepton mass terms generated from the Planck to the GUT scale, usually neglected in the literature.

2. Seesaw and neutrino Yukawa couplings

The seesaw mechanism requires the addition of one gauge singlet (N_i) to each SM doublet (ν_i), and needs that Majorana masses ($\boldsymbol{M}$) of the singlets

are much larger than Dirac mass terms ($\boldsymbol{M_D}$). As a consequence one gets, apart from heavy neutrinos with masses of order $\boldsymbol{M}$, light neutrinos with masses $\boldsymbol{M_\nu} = \boldsymbol{M_D^T} \boldsymbol{M^{-1}} \boldsymbol{M_D}$, where $\boldsymbol{M_D} = \langle H_2 \rangle \boldsymbol{Y_\nu}$ is obtained via Yukawa interactions after the electroweak symmetry breaking when the hypercharge $+1/2$ Higgs field (H_2 in the MSSM) gets a *vev* $\langle H_2 \rangle = v_2 = v \sin\beta$. Notice that a mass of the order of 1 eV can be generated by a coupling $Y_\nu \sim 1$ with $M \sim 10^{14}$ GeV or by $Y_\nu \sim 10^{-2}$ with $M \sim 10^{10}$ GeV.

Neutrino oscillations give us a partial information [3] on neutrino masses (the squared mass differences Δm_{21}^2 and Δm_{32}^2) and the neutrino mixing matrix $\boldsymbol{U}$ ($\theta_{12}, \theta_{23}, \theta_{13}$). In the basis where the charged-lepton Yukawa matrix $\boldsymbol{Y_e}$ is diagonal, $\boldsymbol{U}$ relates the weak eigenstates ν_α ($\alpha = e, \mu, \tau$) and the mass eigenstates ν_i ($i = 1, 2, 3$): $\nu_\alpha = U_{\alpha i} \nu_i$.

The neutrino Yukawa matrix can be expressed [4] in a basis where the Majorana mass matrix is diagonal ($\boldsymbol{D_M}$) in terms of the Majorana masses, the diagonal light neutrino mass matrix ($\boldsymbol{D_m}$) and the neutrino mixings up to an unknown complex orthogonal matrix $\boldsymbol{R}$,

$$v_2 \boldsymbol{Y}_\nu = \sqrt{\boldsymbol{D_M}} \boldsymbol{R} \sqrt{\boldsymbol{D_m}} \boldsymbol{U}^\dagger . \tag{3}$$

Therefore, it depends on six masses, six angles and six phases. Only a few of these parameters are accessible to current experiments. To simplify the analysis we adopt the following *conservative* assumptions (they will lead to a minimal LFV): (1) We concentrate on the case of quasi-degenerate (QD) neutrinos ($\boldsymbol{D_m} \approx m_\nu \boldsymbol{1}$), that *naturally* implies $\boldsymbol{D_M} = M\boldsymbol{1}$. (2) We assume that $\boldsymbol{R}$ is real, that implies that $v_2^2 \boldsymbol{Y}_\nu^\dagger \boldsymbol{Y}_\nu = M \boldsymbol{U} \boldsymbol{D_m} \boldsymbol{U}^\dagger$ (relevant for LFV) is independent of $\boldsymbol{R}$. It must be noticed that a complex $\boldsymbol{R}$ is necessary for leptogenesis and enhances LFV by several orders of magnitude [5]. (3) Finally we assume that CP is conserved and the light neutrinos have all the same CP parities ($\boldsymbol{U}$ is real and the eigenvalues of $\boldsymbol{M_\nu}$ have all the same sign). For the light neutrino mass scale we take $m_\nu \approx 0.2$ eV, compatible with $m_\nu < 2.2$ eV (^{3}H $\beta-$decay), $m_\nu < 0.7/3$ eV (WMAP) and $m_\nu < (0.35 - 1.05)$ eV ($\beta\beta_{0\nu}$) [6].

3. Scenarios for quasi-degenerate neutrinos

High-energy Yukawa couplings are constrained by masses and mixings at the electroweak scale through the RGE. The neutrino Yukawa couplings do not run below the Majorana mass scale M, where they are matched to the light neutrino mass matrix $\boldsymbol{M_\nu} = v_2^2 \boldsymbol{Y}_\nu^T \boldsymbol{Y}_\nu / M$. The evolution of neutrino mixings embodied in this effective mass matrix is controlled by charged-lepton Yukawa couplings. Between the electroweak and the SUSY scale (m_S) the running is negligible but above m_S the down-type Yukawa couplings are enhanced if $\tan\beta \gg 1$ (in particular $Y_\tau = m_\tau / (v \cos\beta) \to 1$)

and as a consequence mixings get magnified for QD neutrinos [7]. Therefore we distinguish two scenarios at high-energy compatible with the mass differences and bimaximal mixings observed at low energy, irrespective of the Majorana scale: (a) Bimaximal mixings for low $\tan\beta$ ($Y_\tau \ll 1$): almost no running; (b) CKM-like mixings for high $\tan\beta$ ($Y_\tau \sim 1$): from unification of quark and neutrino mixings at high scale one gets two large mixings θ_{12} and θ_{23}. Concerning θ_{13}, even if it is taken zero at a high scale it never vanishes at low energy, but becomes between 5×10^{-5} and 0.1 depending on $\tan\beta$.

4. Radiative corrections to slepton masses

SUSY-breaking masses receive quantum corrections from gaugino (flavour blind) and Yukawa (flavour dependent) interactions. The sleptons (since all leptons are much lighter) get their mass from SUSY-breaking terms:

$$m_{\tilde\ell}^2 \sim \begin{pmatrix} m_{\tilde L}^2 & m_{LR}^{2\dagger} \\ m_{LR}^2 & m_{\tilde e_R}^2 \end{pmatrix} , \quad m_{\tilde\nu}^2 \sim m_{\tilde L}^2 . \tag{4}$$

Sneutrino singlets are very heavy (their masses are at least of order M) and decouple. We neglect the scalar trilinears ($A_e = A_\nu = 0$) which in practice suppresses the left-right mixing, $m_{LR}^2 \sim 0$. The slepton masses, taken flavour universal at tree level, $m_{\tilde L}^2 = m_{\tilde e_R}^2 = m_0^2 \mathbf{1}$, are changed by quantum corrections, developing in particular flavour-changing entries

$$m_{\tilde L_{ij}}^2 \sim m_S^2 \delta_{LL}^{ij} , \quad m_{\tilde e_{R\,ij}}^2 \sim m_S^2 \delta_{RR}^{ij} . \tag{5}$$

In the next subsections the size of the radiative corrections is presented in two cases: universal slepton masses at the GUT scale M_X, where gauge couplings unify in the MSSM, and universality at the Planck scale $M_{\rm P} = M_{\rm Planck}/\sqrt{8\pi}$, assuming the simplest scenario with SU(5) gauge symmetry between both scales. The numerical results are obtained using the complete one-loop RG equations (see [1] and references therein for more details). Analytical approximations are given below just to understand the main effects. We define the two scenarios for high-scale neutrino mixing described above as (a) $\tan\beta = 50$ ($Y_\tau \sim 1$) and (b) $\tan\beta = 3$ ($Y_\tau \ll 1$) and consider the following cases relevant for LFV: (1) $M = 10^{14}$ GeV ($Y_\nu \sim 1$) and (2) $M = 5 \times 10^{11}$ GeV ($Y_\nu \ll 1$).

4.1. Universality at the GUT scale

In the MSSM with right-handed neutrinos the trilinear couplings of the superpotential in the leptonic sector are

$$\mathcal{W} = Y_e^{ij} E_i^c H_1 L_j + Y_\nu^{ij} N_i^c H_2 L_j , \tag{6}$$

where L_i, E_i^c and N_i^c stand for the three families of lepton doublets, charged singlets and neutrino singlets, respectively. H_1 and H_2 are the MSSM Higgs-doublet superfields.

We take diagonal charged-lepton Yukawa couplings $\boldsymbol{Y_e}$ at M_X and include all the lepton mixing in $\boldsymbol{Y_\nu}$. From universal SUSY-breaking masses at the GUT scale, the RG corrections introduce two relevant effects. First, the running down to the Majorana scale M generates off-diagonal terms in the slepton-doublet mass matrix:

$$m^2_{\tilde{L}_{ij}} \approx -\frac{3}{8\pi^2} m_0^2 (\boldsymbol{Y_\nu^\dagger Y_\nu})_{ij} \log \frac{M_X}{M} \,. \tag{7}$$

The slepton-singlet mass matrix remains diagonal, $m^2_{\tilde{e}_{R\,ij}} \approx 0$. Second, the running also generates off-diagonal terms in $\boldsymbol{Y_e}$. The low energy slepton mass matrices must be rediagonalized accordingly. Since the rotations in the space of the three lepton doublets $\theta^e_{ij} \approx Y_e^{ji}/Y_\tau$ $(ij = 12, 13, 23)$ are

$$\theta^e_{ij} \approx -\frac{1}{16\pi^2} (\boldsymbol{Y_\nu^\dagger Y_\nu})_{ij} \log \frac{M_X}{M} \,, \tag{8}$$

and the third family separation (the only relevant) is

$$m^2_{\tilde{L}_{33}} - m^2_{\tilde{L}_{ii}} \approx -\frac{3}{8\pi^2} m_0^2 Y_\tau^2 \log \frac{M_X}{M_Z} \,, \tag{9}$$

one gets a correction $\Delta m^2_{\tilde{L}_{i3}} \approx (m^2_{\tilde{L}_{33}} - m^2_{\tilde{L}_{ii}})\, \theta^e_{i3}$ to the terms $m^2_{\tilde{L}_{i3}}$ in (7), which is of the order of $Y_\tau^2/(16\pi^2) \log(M_X/M_Z) \approx 10\%$ for high $\tan\beta$.

To quantify the lepton-slepton misalignment we show the symmetric matrix $\boldsymbol{m^2_{\tilde{L}}}$ at the electroweak scale generated by the given values of the scalar and gaugino masses m_0 and $m_{1/2}$ at M_X. Only scenarios (*a1*) and (*b1*) are displayed, the ones where the effects are larger according to (7). The δ^{ij}_{LL} can be read directly:

$$[m_0 = m_{1/2} = 300 \text{ GeV at } M_X]$$

$$(a1) \quad \boldsymbol{m^2_{\tilde{L}}} = (353 \text{ GeV})^2 \begin{pmatrix} 1 & -10^{-4} & -2 \times 10^{-5} \\ * & 0.999 & -5 \times 10^{-4} \\ * & * & 0.793 \end{pmatrix}, \tag{10}$$

$$(b1) \quad \boldsymbol{m^2_{\tilde{L}}} = (352 \text{ GeV})^2 \begin{pmatrix} 1 & -5 \times 10^{-5} & 5 \times 10^{-5} \\ * & 0.997 & -3 \times 10^{-3} \\ * & * & 0.996 \end{pmatrix}. \tag{11}$$

4.2. Universality at the Planck scale

In SUSY-SU(5) each generation of quark doublets, up-quark singlets and charged-lepton singlets can be accommodated in the same $\mathbf{10}$ irrep (Ψ_i) of the group, whereas lepton doublets and down-quark singlets would be in the $\overline{\mathbf{5}}$ (Φ_i). We also need gauge singlets (N_i^c) to generate neutrino masses, and a vectorlike $\mathbf{5} + \overline{\mathbf{5}}$ (H_2 and H_1) to include the two standard Higgs doublets. Other vectorlike fermions or the Higgs representations needed to break the GUT symmetry are not essential in our calculation. Including just the three fermion families the trilinear terms in the superpotential read

$$\mathcal{W}_{\mathrm{SU}(5)} = \frac{1}{4} Y_u^{ij} \, \Psi_i \Psi_j H_2 + \sqrt{2} \, Y_{d/e}^{ij} \, \Psi_i \Phi_j H_1 + Y_\nu^{ij} \, N_i^c \Phi_j H_2 \; . \tag{12}$$

At M_X we do the matching of Yukawa couplings in the following way. $\boldsymbol{Y_d}$ and $\boldsymbol{Y_e}$ are diagonalized. All the quark mixing included in $\boldsymbol{Y_u}$ and all the lepton mixing in $\boldsymbol{Y_\nu}$. The matrix $\boldsymbol{Y_u}$ (symmetrized through a rotation of the up quark singlets) is then matched to the analogous matrix above M_X. We do *not* assume tau-bottom unification. $\boldsymbol{Y_{d/e}}$ is matched to $\boldsymbol{Y_e}$, what is justified [1] for both the small and the large values of $\tan\beta$ considered. The matching of SUSY-breaking of the slepton doublet scalar masses is $\boldsymbol{m_{\tilde{L}}^2 = m_{\tilde{\Phi}}^2}$ in the studied scenarios and charged-slepton singlets $\boldsymbol{m_{\tilde{e}_R}^2 = m_{\tilde{\Psi}}^2}$.

The running introduces three main effects on the flavour structure of the model, two of them add to the corrections described for universal masses at M_X, whereas the third one is new. At M_X there appear new off-diagonal terms in the slepton-doublet mass matrix:

$$m_{\tilde{\Phi} \, ij}^2 \approx -\frac{3}{8\pi^2} m_0^2 (\boldsymbol{Y_\nu^\dagger Y_\nu})_{ij} \log \frac{M_{\mathrm{P}}}{M_X} \; . \tag{13}$$

The second effect is a large mass separation of the third slepton family produced by tau corrections:

$$m_{\tilde{\Phi} \, 33}^2 - m_{\tilde{\Phi} \, ii}^2 \approx -\frac{3}{2\pi^2} m_0^2 \, Y_\tau^2 \, \log \frac{M_{\mathrm{P}}}{M_X} \; . \tag{14}$$

This mass splitting will introduce off-diagonal mass terms once the charged-lepton Yukawa matrix is rediagonalized at low energies. The final (new) effect has to do with the slepton-singlet mass matrix. In the minimal SU(5) model this matrix coincides with the one for up squarks, so it will be affected by large top quark radiative corrections. At M_X there will be off-diagonal terms of order

$$m_{\tilde{\Psi} \, ij}^2 \approx -\frac{9}{8\pi^2} m_0^2 (\boldsymbol{Y_u^\dagger Y_u})_{ij} \log \frac{M_{\mathrm{P}}}{M_X} \; , \tag{15}$$

where $\boldsymbol{Y_u}$ contains the whole CKM rotation (we take $\boldsymbol{Y_{d/e}}$ diagonal at M_X). This effect was first considered in [8] and produces off-diagonal terms (δ_{RR}^{ij}) in $\boldsymbol{m_{\tilde{e}_R}^2}$, giving the dominant contribution to LFV for all scenarios except for the first two generations in cases (*a1*) and (*b1*) which are dominated by the term $m_{\tilde{L}_{12}}^2$.

To illustrate the relevance of the corrections between M_{P} and M_X we give $\boldsymbol{m_{\tilde{L}}^2}$ at M_Z and compare it with the matrices obtained for universal scalar masses at M_X. We take values of m_0 and $m_{1/2}$ at M_{P}, which give at M_X similar values to the ones used in Eqs. (10,11):

$$[m_0 = 300 \text{ GeV}, \ m_{1/2} = 275 \text{ GeV at } M_{\mathrm{P}}]$$

$$(a1) \quad \boldsymbol{m_{\tilde{L}}^2} = (353 \text{ GeV})^2 \begin{pmatrix} 1 & -4 \times 10^{-4} & -7 \times 10^{-5} \\ * & 0.997 & -2 \times 10^{-3} \\ * & * & 0.567 \end{pmatrix}, \quad (16)$$

$$(b1) \quad \boldsymbol{m_{\tilde{L}}^2} = (349 \text{ GeV})^2 \begin{pmatrix} 1 & -2 \times 10^{-4} & 2 \times 10^{-4} \\ * & 0.990 & -10^{-2} \\ * & * & 0.989 \end{pmatrix}. \quad (17)$$

Finally, contrary to the usual claim, we find that it is not justified to take universal SUSY-breaking masses at the GUT scale even in the no-scale models (scalar masses taken zero at Planck scale and radiatively generated), since off-diagonal terms in the scalar sector induced by RGE between M_{P} and M_X are only reduced only by an 80% compared to the ordinary case (Fig. 1).
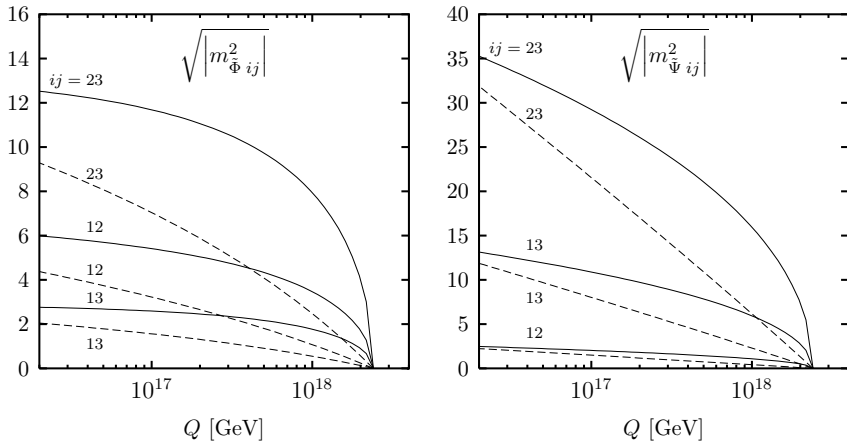

Fig. 1. Off-diagonal terms [GeV] generated in the running of $\boldsymbol{m_{\tilde{\Phi}}^2}$ and $\boldsymbol{m_{\tilde{\Psi}}^2}$ from M_{P} to M_X. Solid lines correspond to no-scale models ($m_0 = 0$ at M_{P}).

5. Predictions for LFV decays

Let us show next how the misalignment between lepton and slepton families translates into the rate of flavour-violating decays. We concentrate on the (best constrained) rare lepton decays. The present (future) experimental limits are [9]

$$\mathrm{BR}(\mu \to e\gamma) \; < \; 1.2 \times 10^{-11} \quad (10^{-14}) \; , \tag{18}$$

$$\mathrm{BR}(\tau \to e\gamma) \; < \; 2.7 \times 10^{-6} \; , \tag{19}$$

$$\mathrm{BR}(\tau \to \mu\gamma) \; < \; 6 \times 10^{-7} \qquad (10^{-9}) \; . \tag{20}$$

A detailed description of all the diagrammatics involved in these processes can be found in [10].

The results for the four cases (*a1, a2, b1, b2*) assuming universality at M_X and choosing $\mu > 0$ are summarized in Fig. 2. For each case we plot the branching ratios of $\mu \to e\gamma$ (solid), $\tau \to e\gamma$ (dashes) and $\tau \to \mu\gamma$ (dots). In all the cases the three branching ratios are dominated by diagrams with
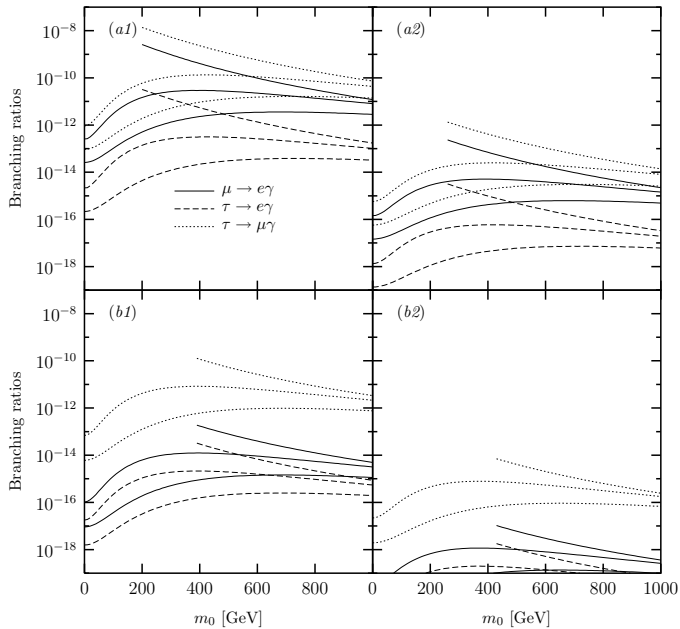

Fig. 2. Branching ratios of $\ell \to \ell'\gamma$ for $m_{1/2} = 100, 300, 500$ GeV and different values of the scalar mass parameter m_0 at M_X. Cases (*a*) and (*b*) correspond to $\tan\beta = 50, 3$, whereas cases (*1*) and (*2*) correspond to $M = 10^{14}, 5 \times 10^{11}$ GeV, respectively. Lower values of m_0 for $m_{1/2} = 100$ GeV give slepton masses excluded by present bounds.

exchange of charginos and sneutrinos [10]. The angle θ_{13} is taken zero at M_X for scenarios with low $\tan\beta$. The rates of $\mu \to e\gamma$ and $\tau \to e\gamma$ in those cases would be scaled by a factor $\approx (\theta_{13}/10^{-2})^2$ for values of this angle close to the experimental bound $\theta_{13} \approx 0.2$. Only the process $\mu \to e\gamma$ and only for the scenario (*a1*) is constrained by present bounds and would be discovered or ruled out in the near future.

The corresponding results assuming universality at $M_{\rm P}$ are shown in Fig. 3. It is remarkable the big enhancement of all processes in scenarios (*a2*) and (*b2*) and of $\tau \to e\gamma$ and $\tau \to \mu\gamma$ in (*a1*) and (*b1*), due to the dominant contribution of the δ_{RR} insertions radiatively generated by top-quark corrections. The rate of $\mu \to e\gamma$ is controlled by δ_{LL}^{12} for (*a1*) and (*b1*). The latter is obtained with $\theta_{13} = 0$ at M_X and would be scaled, as before, by a factor $\approx (\theta_{13}/10^{-2})^2$ for larger values of this angle. Comparing Figs. 2 and 3 we find that corrections from the Planck to the GUT scale can increase BR($\tau \to \mu\gamma$) in four orders of magnitude (case *a2*) or BR($\mu \to e\gamma$) in two orders of magnitude (case *b2*).
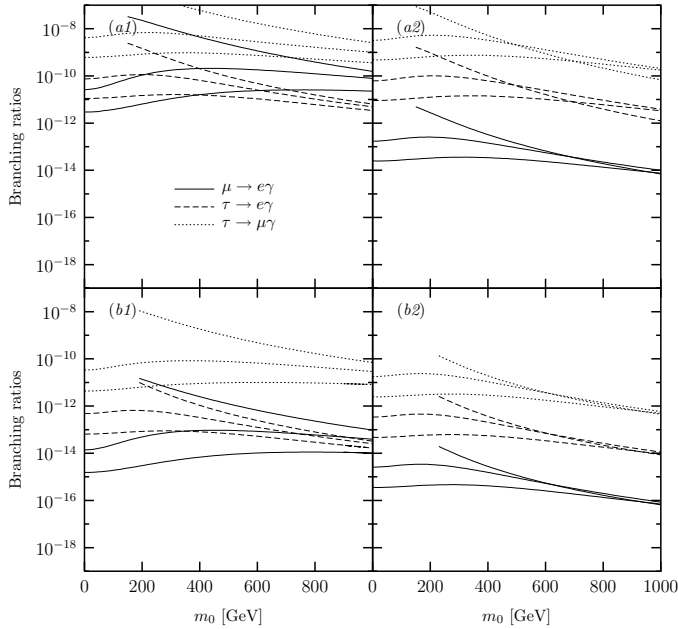

Fig. 3. The same as in Fig. 2 but assuming universality at $M_{\rm P}$.

To conclude, in models with a gravity-mediated origin of the SUSY-breaking parameters the lepton flavour problem may be avoided at the tree level. However, radiative corrections from tau and neutrino couplings introduce flavour violation at rates that should be observable in near future experiments.

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