Vol. 34 (2003) *ACTA PHYSICA POLONICA B* No 11

HIGGS MASS PREDICTION IN FINITE UNIFIED THEORIES*

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(Received November 6, 2003)

Finite Unified Theories (FUTs) are $N=1$ supersymmetric Grand Unified Theories, which can be made all-loop finite, both in the dimensionless (gauge and Yukawa couplings) and dimensionful (soft supersymmetry breaking terms) sectors. This remarkable property provides a drastic reduction in the number of free parameters, which in turn leads to an accurate prediction of the top quark mass in the dimensionless sector, and predictions for the Higgs boson mass and the s-spectrum in the dimensionful sector. Here we examine the predictions of two FUTs taking into account the various theoretical and experimental constraints as well as their restricted parameter space. For the first we present the results of a detailed scanning concerning the Higgs mass prediction, while for the second we present a representative prediction of its spectrum.

PACS numbers: 11.10.Hi, 11.30.Pb, 12.10.Kt, 12.60.Jv

1. Introduction

Finite Unified Theories are $N=1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite even to all-loop orders, including the soft supersymmetry breaking sector. The method to construct GUTs with reduced independent parameters [2, 7] consists of searching for renormalisation group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. Of particular interest is the

* Presented at the XXVII International Conference of Theoretical Physics, "Matter to the Deepest", Ustroń, Poland, September 15–21, 2003.

possibility to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [8, 9]. In order to achieve the latter it is enough to study the uniqueness of the solutions to the one-loop finiteness conditions [8,9]. The constructed *finite unified* $N=1$ supersymmetric SU(5) GUTs using the above tools, predicted correctly from the dimensionless sector (Gauge–Yukawa unification), among others, the top quark mass [1]. The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories [3, 11], which involves parameters of dimension one and two. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the s-spectrum. The purpose of the present article is to start an exhaustive search of the latter predictions.

2. Reduction of couplings and finiteness in $N=1$ SUSY gauge theories

A RGI relation among couplings $g_i, \Phi(g_1, \cdots, g_N) = 0$, has to satisfy the partial differential equation $\mu\, d\Phi/d\mu = \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i = 0$, where β_i is the β-function of g_i. There exist $(N-1)$ independent Φ's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations (REs) [7], $\beta_g\,(dg_i/dg) = \beta_i$, $i = 1, \cdots, N$, where g and β_g are the primary coupling and its β-function. Using all the $(N-1)\,\Phi$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, whose uniqueness can be investigated at the one-loop level.

Finiteness can be understood by considering a chiral, anomaly free, $N=1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ , \tag{1}$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, *i.e.*

$$\sum_i \ell(R_i) = 3C_2(G)\,,\ \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\ , \tag{2}$$

where $l(R_i)$ is the Dynkin index of R_i, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G. There exists a theorem [8] which guarantees the vanishing of the β-functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling, in the sense described above. Alternatively, similar results can be obtained [9, 10] using an analysis of the all-loop NSVZ gauge beta-function [19].

The above described method of reducing the dimensionless couplings has been extended [3, 11] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. More recently a very interesting progress has been made [10-17] concerning the renormalisation properties of the SSB parameters based conceptually and technically on the work of Ref. [15]. In this work the powerful supergraph method [16] for studying supersymmetric theories has been applied to the softly broken ones by using the "spurion" external space-time independent superfields [17]. In the latter method a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire "vacuum expectation values". Based on this method the relations among the soft term renormalisation and that of an unbroken supersymmetric theory have been derived. In particular the β-functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of Refs. [10–17] in solving the set of coupled differential equations so as to be able to express all parameters in a RGI way, was to transform the partial differential operators involved to total derivative operators [12]. This is indeed possible to be done on the RGI surface which is defined by the solution of the reduction equations. In addition it was found that RGI SSB scalar masses in Gauge–Yukawa unified models satisfy a universal sum rule at one-loop [4]. This result was generalised to two-loops for finite theories, and then to all-loops for general Gauge–Yukawa and finite unified theories [13].

In order to obtain a feeling of some of the above results, consider the superpotential given by (1) along with the Lagrangian for SSB terms

$$\begin{aligned} -\mathcal{L}_{\rm SB} \;=\; & \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j \\ & + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}, \end{aligned} \qquad (3)$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since only finite theories are considered here, it is assumed that the gauge group is a simple group and the one-loop β-function of the gauge coupling g vanishes. It is also assumed that the reduction equations admit power series solutions of the form $C^{ijk} =$

$g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n}$. According to the finiteness theorem [8], the theory is then finite to all-orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by $h^{ijk} = -MC^{ijk} + \ldots = -M\rho^{ijk}_{(0)}\, g + O(g^5)$ [18]. To obtain the two-loop sum rule for soft scalar masses, it is assumed that the lowest order coefficients $\rho^{ijk}_{(0)}$ and also $(m^2)^i_j$ satisfy the diagonality relations $\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i$ for all p and q, and $(m^2)^i_j = m^2_j\delta^i_j$, respectively. Then the following soft scalar-mass sum rule is found [5]

$$\frac{(\ m_i^2 + m_j^2 + m_k^2\ )}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(1)} + O(g^4) \tag{4}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction

$$\Delta^{(1)} = -2\sum_l \left[(m^2_l/MM^\dagger) - (1/3)\right]\ T(R_l)\,, \tag{5}$$

which vanishes for the universal choice, *i.e.* when all the soft scalar masses are the same at the unification point.

3. Finite unified theories

In this section we examine two concrete SU(5) finite models, where the reduction of couplings in the dimensionless and dimensionful sector has been achieved. A predictive Gauge–Yukawa unified SU(5) model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, *i.e.*, $\gamma_i^{(1)\,j} \propto \delta^j_i$.
2. Three fermion generations, in the irreps $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1, 2, 3)$, which obviously should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of Ref. [1], which will be labelled **A**, and a slight variation of this model (labelled **B**) , which can also be obtained from the class of the models suggested by Kazakov *et al.* [12] with a modification to suppress non-diagonal anomalous dimensions[1].

[1] An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [14], where several realistic examples are given. These extensions are not considered here.

The superpotential which describes the two models takes the form [1,5]

$$
\begin{aligned}
W &= \sum_{i=1}^{3}\left[\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_iH_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\,\overline{H}_i\right] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3H_4 \\
&\quad + g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4 + g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4 + \sum_{a=1}^{4}g_a^f\,H_a\,\mathbf{24}\,\overline{H}_a + \frac{g^\lambda}{3}\,(\mathbf{24})^3\ ,
\end{aligned}
\tag{6}
$$

where H_a and $\overline{H}_a$ $(a = 1,\ldots,4)$ stand for the Higgs quintets and anti-quintets.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$
\begin{aligned}
(g_1^u)^2 &= \left\{\frac{8}{5},\frac{8}{5}\right\}g^2\ ,\ (g_1^d)^2 = \left\{\frac{6}{5},\frac{6}{5}\right\}g^2\ ,\ (g_2^u)^2 = (g_3^u)^2 = \left\{\frac{8}{5},\frac{4}{5}\right\}g^2\ , \\
(g_2^d)^2 &= (g_3^d)^2 = \left\{\frac{6}{5},\frac{3}{5}\right\}g^2\ ,\ (g_{23}^u)^2 = \left\{0,\frac{4}{5}\right\}g^2\ ,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \left\{0,\frac{3}{5}\right\}g^2\ , \\
(g^\lambda)^2 &= \frac{15}{7}g^2\ ,\ (g_2^f)^2 = (g_3^f)^2 = \left\{0,\frac{1}{2}\right\}g^2\ ,\ (g_1^f)^2 = 0\ ,\ (g_4^f)^2 = \{1,0\}g^2\ .
\end{aligned}
\tag{7}
$$

According to the theorem of Ref. [8] these models are finite to all orders. After the reduction of couplings the symmetry of W is enhanced [1,5].

The main difference of the models $\mathbf{A}$ and $\mathbf{B}$ is that three pairs of Higgs quintets and anti-quintets couple to the $\mathbf{24}$ for $\mathbf{B}$ so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of SU(5).

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale [5]:

$$
m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \quad \text{for} \quad \mathbf{A}\ , \tag{8}
$$

$$
\begin{aligned}
m_{H_u}^2 + 2m_{\mathbf{10}}^2 &= M^2\ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}\ , \\
m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \quad \text{for} \quad \mathbf{B},
\end{aligned}
\tag{9}
$$

where we use as free parameters $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model $\mathbf{A}$, and $m_{\mathbf{10}}$ for $\mathbf{B}$, in addition to M.

4. Predictions of low energy parameters

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalisation property at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (7), the $h = -MC$ relation, and the soft scalar-mass sum rule (4) at $M_{\rm GUT}$, as applied in the various models. So we examine the evolution of these parameters according to their renormalisation group equations at two-loop for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s so that below M_s the SM is the correct effective theory.

The predictions for the top quark mass M_t are ~ 183 and ~ 174 GeV in models **A** and **B**, respectively. Comparing these predictions with the most recent experimental value $M_t = (174.3 \pm 5.1)$ GeV [23], and recalling that the theoretical values for M_t may suffer from a correction of less than $\sim 4\%$ [6], we see that they are consistent with the experimental data. In addition the value of $\tan\beta$ is obtained as $\tan\beta = 54$ and 48 for models **A** and **B**, respectively.

In the SSB sector, besides the constraints imposed by finiteness there are further restrictions imposed by phenomenology. In the case where all the soft scalar masses are universal at the unification scale, there is no region of $M_{\rm s} = M$ below 0 (few TeV) in which $m^2_{\tilde{\tau}} > m^2_{\chi}$ is satisfied (where $m_{\tilde{\tau}}$ is the $\tilde{\tau}$ mass, and m_χ the lightest neutralino mass, which is the lightest supersymmetric particle). But once the universality condition is relaxed this problem can be solved naturally (provided the sum rule). More specifically, using the sum rule (4) and imposing the conditions a) successful radiative electroweak symmetry breaking, b) $m_{\tilde{\tau}^2} > 0$ and c) $m_{\tilde{\tau}^2} > m_{\chi^2}$, a comfortable parameter space for both models (although model **B** requires large $M \sim 1$ TeV) is found.

As a final constraint, we also calculate BR($b \to s\gamma$) [20]. We do not take into account any constraints coming from the muon anomalous magnetic moment $(g-2)$ in this work. In the graphs we show the results for **FUTA** for different values of M for m_h (including the large corrections due to $\tan\beta$), m_{χ^0}, and m_A, for the case when $\mu < 0$ and that the LSP is a neutralino (χ^0). The results for $\mu > 0$ are slightly different: the spectrum starts around 500 GeV. The main difference, though, is in the value of the running bottom mass $m_{\rm bot}(m_{\rm bot})$. In the $\mu < 0$ case, $m_{\rm bot} \sim 3.5 - 4.0$ GeV just below the experimental value $m_{\rm bot-exp} \sim 4.0 - 4.5$ GeV [23]. In the $\mu > 0$, $m_{\rm bot} \sim 4.75 - 5.3$ GeV, *i.e.* above the experimental value.

The prediction for the Higgs mass for the models is

$$m_h = 112 - 132 \ \ {\rm GeV}\,, \tag{10}$$

where the uncertainty comes from variations of the gaugino mass M and the soft scalar masses, and from finite (*i.e.* not logarithmically divergent) corrections in changing renormalisation scheme. In the analysis we have also included a small variation, due to threshold corrections at the GUT scale, of $1-2\%$ of the FUT boundary conditions. This small variation does not give a noticeable effect in the results at low energies. From Fig. 1 we already see that the requirement $m_h > 113.5$ GeV [23] excludes the possibility of $M = 200$ GeV for FUTA.

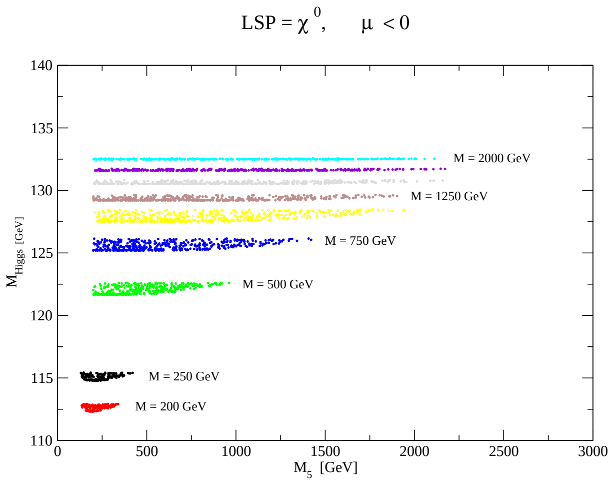

Fig. 1. m_h as function of $m_{\mathbf{5}}$ for different values of M for model **FUTA**, for $\mu < 0$.

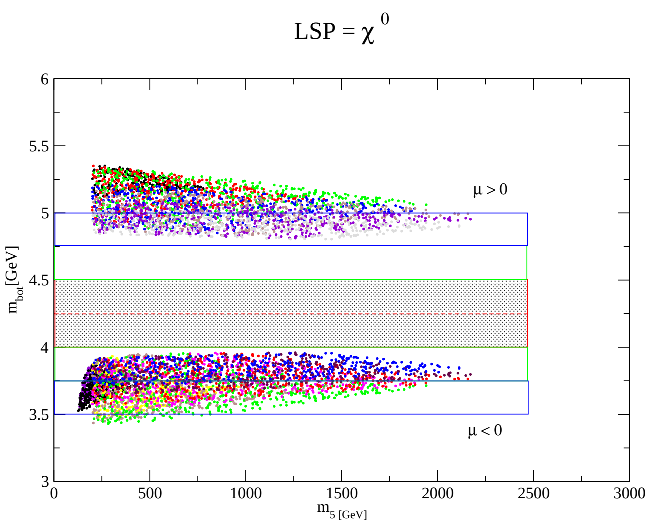

Fig. 2. $m_{\rm bot}(m_{\rm bot})$ as function of $m_{\mathbf{5}}$ for different values of M for model **FUTA**, for $\mu < 0$ and $\mu > 0$.

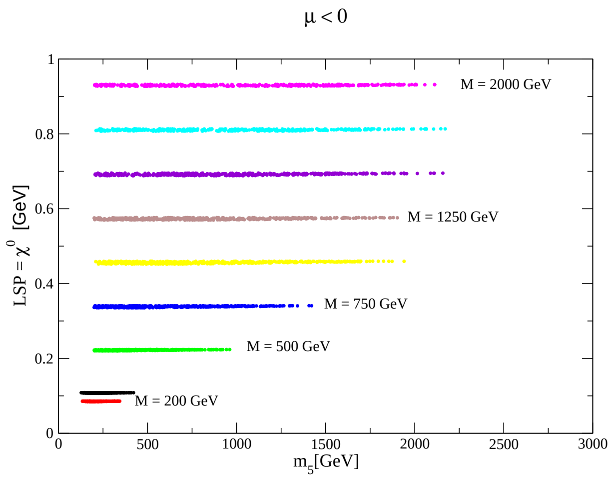

Fig. 3. M_{χ^0} as function of $m_{\mathbf{5}}$ for different values of M for model **FUTA**, for $\mu < 0$.

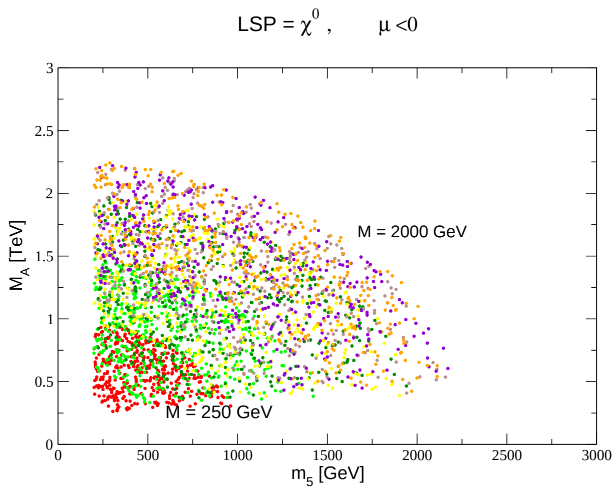

Fig. 4. M_A as function of $m_{\mathbf{5}}$ for different values of M for model **FUTA**, for $\mu < 0$.

A more detailed numerical analysis, where the results of our program and of known programs like FeynHiggs and Suspect are combined, is currently in progress [21].

In Tables I and II we present representative examples of the values obtained for the sparticle spectra in each of the models. The value of the lightest Higgs physical mass m_h has already the one-loop radiative corrections included, evaluated at the appropriate scale [22].

TABLE I

A representative example of the bottom (running) and top (pole) masses, plus the supersymmetric spectrum for Model FUTA, with $m_5 = 697$ GeV, $m_{10} = 806$ GeV, $M_{\rm susy} = 1681$ GeV, $\mu < 0$. All masses in the Table are in GeV.

$M_{\rm top}$	183	$m_{\rm bot}$	3.9
$\tan\beta =$	54.4	$\alpha_{\rm s}$	.118
m_{χ_1}	452	$m_{\tilde{\tau}_2}$	916
m_{χ_2}	843	$m_{\tilde{\nu}_3}$	883
m_{χ_3}	850	μ	-1494
m_{χ_4}	1500	B	3543
$m_{\chi_1^\pm}$	843	m_A	555
$m_{\chi_2^\pm}$	1500	$m_{H^\pm}$	560
$m_{\tilde{t}_1}$	1578	m_H	555
$m_{\tilde{t}_2}$	1776	m_h	127.5
$m_{\tilde{b}_1}$	1580	M_1	452
$m_{\tilde{b}_2}$	1766	M_2	846
$m_{\tilde{\tau}_1}$	654	M_3	2210

TABLE II

A representative example of the bottom (running) and top (pole) masses, plus the supersymmetric spectrum for Model FUTB, with $m_{10} = 945$ GeV, $M_{\rm susy} = 2278$ GeV, $\mu < 0$. All masses in the Table are in GeV.

$M_{\rm top}$	173	$m_{\rm bot}$	4.2
$\tan\beta =$	48	$\alpha_{\rm s}$	.116
m_{χ_1}	669	$m_{\tilde{\tau}_2}$	970
m_{χ_2}	912	$m_{\tilde{\nu}_3}$	916
m_{χ_3}	1289	μ	-1900
m_{χ_4}	1909	B	4010
$m_{\chi_1^\pm}$	1289	m_A	1106
$m_{\chi_2^\pm}$	909	$m_{H^\pm}$	1109
$m_{\tilde{t}_1}$	2236	m_H	1106
$m_{\tilde{t}_2}$	2519	m_h	123.5
$m_{\tilde{b}_1}$	2163	M_1	700
$m_{\tilde{b}_2}$	2501	M_2	1293
$m_{\tilde{\tau}_1}$	766	M_3	3256

It is a pleasure to thank the Organising Committee for the very warm hospitality offered to one of us (G.Z.). We acknowledge many useful discussions with A. Djouadi and S. Heinemeyer. Supported by the projects PAPIIT-IN116206 and partially by the RTN contract HPRN-CT-2000-00148, the Greek–German Bilateral Programme IKYDA-2001 and by the NTUA Programme for Fundamental Research THALES.

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