DISCREET HEAVY PHYSICS*

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In this talk I review the conditions under which heavy physics *virtual* effects are naturally suppressed without requiring a large scale for new physics.

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1. The prophecies

Though there are no clear indications of a deficiency in the Standard Model, it is generally believed that this theory does not represent the most fundamental description of nature. This belief is supported by a variety of arguments; for example, *(i)* the unexplained origin and enormous range of Yukawa couplings (within the pure Standard Model with the addition of right-handed neutrinos they range over almost 12 orders of magnitude [1]); *(ii)* the unexplained origin of the gauge group and the rationale for its particular structure; *(iii)* possible stability and triviality problems [2] within the scalar sector suggest that there is an upper scale Λ beyond which the Standard Model must be modified (though the theoretical constraints on this scale depend on the details of the scalar sector and, in particular, are sensitive to the masses of the physical scalars); *(iv)* the unexplained origin of discrete symmetries, such as lepton and baryon number conservation, that are respected within the Standard Model (up to non-perturbative effects, [3]), or almost conserved, such as CP. In addition *(v)* the Standard Model has a relatively large number of unknown constants which is perceived as undesirable

The above arguments suggest that the Standard Model is in fact an effective theory, obtained from a more fundamental one in the limit where

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all external particles have energies below ~ 1 TeV. This idea has led to an intense study of the possible physics underlying the Standard Model . These investigations can be divided into two groups: *(i)* those that are based on specific models [4]; and *(ii)* those using an effective Lagrangian to describe new physics at low energies [5].

The second possibility attempts to constrain the heavy physics using the existing experimental data and is relatively model-independent. It does, however, assume that the underlying physics respects the Standard Model local symmetry and that underlying physics becomes manifest at a scale Λ which is significantly above the Fermi scale: $\Lambda^2 G_{\rm F} \gg 1$ [5]. In addition the underlying dynamics might be strongly or weakly coupled.

In this talk I will consider the second possibility and attempt to describe the constraints on the heavy physics that can reconcile the absence of any deviation from the Standard Model predictions with the possibility of a relatively small value of $\Lambda < O(10 \text{ TeV})$. I will assume that the heavy physics is weakly coupled and it decouples [6].

2. Effective Lagrangians and small effects

The decoupling assumption implies that all observable heavy-physics effects vanish as $\Lambda \to \infty$ and this suggests one simple way of suppressing the effects generated by the heavy particles: Λ is very large. This sometimes puts the heavy physics out of LHC's reach. For example, the experimental constraints on the $e_{\rm L} e_{\rm L} u_{\rm L} u_{\rm L}$ 4-Fermi interaction generated by a heavy vector-boson exchange of mass Λ, leads to $\Lambda > 25$ TeV [1].

There is, however, a second option, assuming first that Λ is large enough to avoid direct particle production, and second, that all leading virtual effects are absent. The first condition leads to limit on Λ derived form the absence of direct observation of the heavy particles and correspond to $\Lambda \gtrsim$ CM collider energy. The second condition requires appropriate particle content and symmetries to insure the absence of the leading graphs containing *virtual* heavy particles. In this talk I will consider this possibility.

2.1. Hierarchy of virtual effects

Adopting the above assumptions it follows that at low energies (small compared to Λ) all virtual effects generated by the heavy physics can be reproduced by an effective Lagrangian that consists of a linear combination of an infinite series of local effective operators involving only the Standard Model fields, and which respect all the local symmetries of the Standard Model [5]. The coefficients of these operators are calculable if the heavy physics action is known; if this is not the case these coefficients are left free and parameterize *any* type of new physics satisfying the above conditions.

The weak-coupling requirement insures that all the anomalous dimensions will be small and so one can classify the operators according to their naive mass dimension: an operator $\mathcal{O}$ of dimension n will appear with a coefficient f/Λ^{n-4} in the effective Lagrangian. The constant f is given by a product of coupling constants when $\mathcal{O}$ is tree-level-generated, while for operators that are generated via loops, f receives an additional suppression factor $\sim 1/(4\pi)^2$. Leading heavy physics effects are then produced by the lowest dimensional tree-level generated operators.

For example, operators of dimension 6 will generate percentile corrections of order $1/(G_{\rm F}\Lambda^2)$ to Standard Model processes if they are generated at tree-level, or of order $1/(16\pi^2 G_{\rm F}\Lambda^2)$ if loop generated. In this last case the heavy physics corrections are of the order of the Standard Model radiative corrections suppressed by an additional factor of $1/(G_{\rm F}\Lambda^2)$. For $\Lambda > 1$ TeV the observation of the effects of loop-generated operators requires a precision of $\sim 0.02\%$.

The above arguments single out tree-level generated operators as being phenomenologically interesting. These operators can be characterized as follows. Imagine a vertex of in the full theory that has h_n heavy particle legs and ℓ_k light legs. If a graph has V_n vertices of this type, I heavy internal lines and no light internal lines, then it will contain no loops provided $\sum V_n = I + 1$; also, since there are no heavy external lines, $\sum h_n V_n = 2I$. Then

$$\sum (h_n - 2) V_n = -2 < 0\,, \tag{1}$$

from which it follows that all tree-level generated operators are generated by graphs containing a vertex with $h_n = 1$.

3. Eliminating tree-level generated operators

In the following I will denote heavy fermions, scalars and gauge bosons by Ψ, Φ and X, respectively, while ψ, ϕ, and A will represent their light counterparts.

Using the previous characterization it is easy to see that all tree-level generated operators will contain one of the following vertices

$$\psi\psi\Phi\,, \quad \phi\phi\Phi\,, \quad \phi\phi\phi\Phi\,, \quad \psi\phi\Psi\,, \quad \psi\psi X\,, \quad \psi A\Psi\,, \quad \phi\phi X\,. \tag{2}$$

The vertices $AA\Phi$ $\;AAX$ $\;\phi AX$ $\;AAAX$ $\;\phi\phi AX$ are not included in this list since they are necessarily absent: AAX, $AAAX \propto f_{\rm light,light,heavy} = 0$ since the unbroken generators form an algebra; $AA\Phi \propto T_{\rm light} V$, where V denotes an $O(\Lambda)$ vacuum expectation value, and these vanish since the light generators remain unbroken; $\phi AX \propto \phi T_{\rm light} T_{\rm heavy} V = 0$ since the light scalars are orthogonal to the broken directions; and $\phi\phi AX$ is absent whenever $\phi\phi X$ is, due to the structure of the kinetic-energy terms. A particular consequence of these observations is that *all* triple vector–boson vertices are loop-generated [7] [1].

For example, the dimension 5 operator [8] $\left(\bar{\ell}\tilde{\phi}\right)\left(\phi^\dagger \ell^c\right)$ (where ℓ denotes a left-handed lepton doublet and ϕ the Standard Model scalar doublet and the superscript c denotes the charge conjugate field) is generated by the exchange of a scalar iso-triplet or a fermion iso-triplet or iso-singlet:

Similarly, the operator $(\bar{\nu}\nu^c)\left(\phi^\dagger\phi\right)$ (where ν denotes a right-handed neutrino singlet) is generated by the exchange of a scalar iso-singlet or a fermion iso-triplet. Aside from these operators there is only one more dimension 5 operator, namely, $\bar{\nu}\sigma_{\alpha\beta}\nu^c B^{\alpha\beta}$ where $B^{\alpha\beta}$ denotes the U(1) field strength.

Operators of dimension 6 are much more numerous: there are 82 of them (for 1 family) [9], of which 45 are tree-level generated operators [7]. These take the generic forms

$$\phi^6\,, \qquad D^2\phi^2\,, \qquad \psi^2\phi^3\,, \qquad D\psi^2\phi^2\,, \qquad \left(\bar{\psi}\psi\right)^2\,, \qquad \left(\bar{\psi}\gamma\psi\right)^2\,. \tag{3}$$

Note that there are no tree-level generated operators of dimension 6 containing only vectors and fermions. For example the operator

$$\left(\bar{\ell}\sigma^I\gamma^\mu D^\nu\ell\right)W^I_{\mu\nu} = \left(\bar{\ell}\sigma^I\gamma^\mu\partial^\nu\ell\right)\left(\partial\mu W^I_\nu - \partial\mu W^I_\nu\right) + \cdots\,.$$

where W denotes the SU(2) gauge field, cannot be generated at tree level since its first term contains 3 external legs, and any graph with zero loops and three external lines has no internal lines. Because of this the vertex $\Psi\psi A$ is irrelevant when considering only operators of dimension ≤ 6. The graphs associated with the surviving operators [7] are given in Fig. 1

[1] In addition it is worth remarking that no vertices of dimension >4 in the heavy theory are included since they would be suppressed by inverse powers of a scale $\Lambda' \gg \Lambda$.

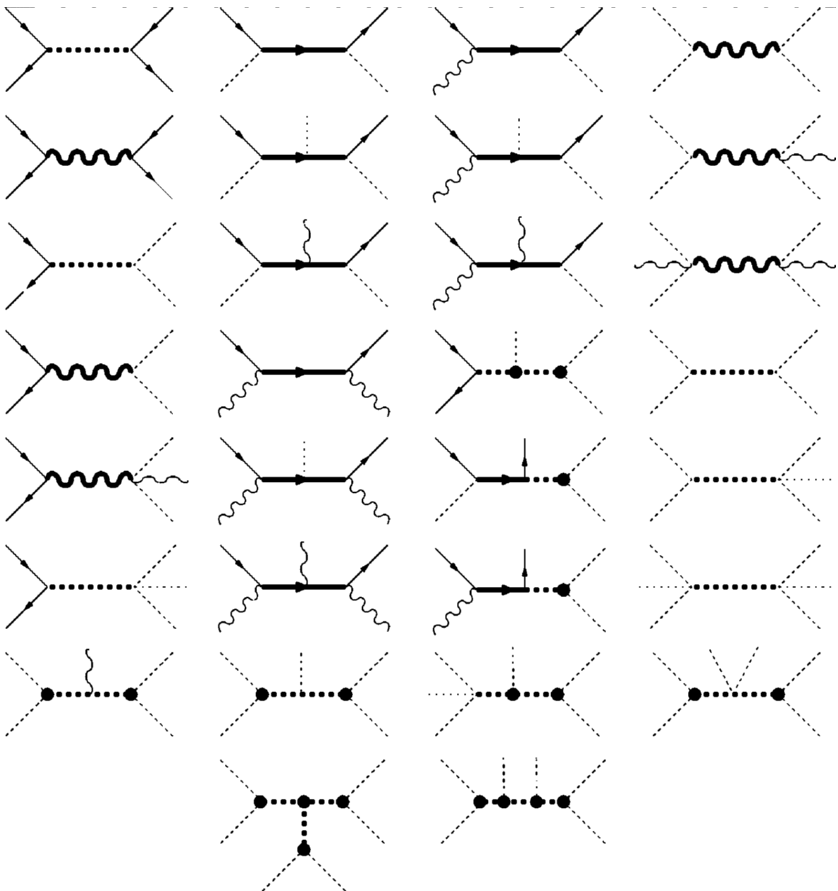

Fig. 1. Graphs responsible for the dimension 6 tree-level generated operators.

4. Eliminating tree-level generated operators

In this section I will describe a simple set of conditions that guarantee the absence of tree-level generated operators and, therefore of all leading new physics virtual effects.

4.1. Discreet symmetry

Since all tree-level generated operators operators contain at least one vertex with a single heavy leg one can eliminate them by requiring that the Lagrangian of the full theory be invariant under a Z_2 symmetry where all heavy fields are odd and all light fields are even. This is what occurs in the MSSM [10] where all s-fermions and bosinos are odd while the Standard Model (including both scalar doublets) are even [11].

4.2. Space-time symmetry

An alternative way of eliminating all tree-level generated operators is realized in models with universal extra dimensions [12]. In these theories space is assumed to be $4+d$-dimensional, and compactified into $\mathbb{R}^4 \times \mathcal{M}$, where $\mathcal{M}$ compact of size $\sim R$; all fields are assumed to propagate in all the $4+d$ dimensions. In a mode expansion a generic field χ can be expanded in terms of the spherical harmonics for $\mathcal{M}$, namely,

$$\chi = \sum_n \chi_n(x) q_n(y) \,, \qquad \nabla^2_{\mathcal{M}} q_n = c_n q_n \,, \quad q_0 = 1 \,,$$

where x and y denote, respectively, the coordinates of $\mathbb{R}^4$ and $\mathcal{M}$, and where the eigenvalues $c_n \sim 1/R^2$ for $n \neq 0$. In this case only χ_0 is light while $\chi_{n\neq 0}$ have masses $\sim 1/R$.

Because of this expansion any vertex containing a single heavy field will be proportional to one of the q_n with $n \neq 0$. Its contribution to the action then vanishes because

$$\int\limits_{\mathcal{M}} q_{n\neq 0} = 0$$

(which can be interpreted as momentum conservation along $\mathcal{M}$ when this manifold has translation symmetry).

4.3. Gauge symmetry

It is also possible to eliminate all tree-level generated operators in a gauge theory by appropriately choosing the particle content and couplings. Here I will only consider a toy model where *(i)* all scalars s are assumed to get a vacuum expectation value $\langle s \rangle = O(\Lambda)$; *(ii)* all fermion and vector masses are generated through spontaneous symmetry breaking; and *(iii)* all physical scalars are are heavy.

With these assumptions and using generic properties of spontaneously broken gauge theories (such as the fact that the unbroken generators close into an algebra, and that they annihilate the vacuum expectation values), it is easy to see that all undesirable vertices are eliminated except $\Psi\psi A$ and $\psi\psi X$. The first, however is irrelevant for dimension 6 operators, as mentioned earlier; the second is more problematic and can be eliminated only though appropriate choice of fermion representations. For example if the left-handed fermions (in the underlying theory) carry the adjoint representation, while right-handed fermions and scalars carry the fundamental representation ad the full gauge group is $\mathrm{SU}(N)$ (broken to $\mathrm{SU}(N-1)$), then all vertices of the form $\psi\psi X$ are disallowed. This type of model will not contain tree-level generated operators, but it is rather trivial having only massless fermions and gauge bosons in the light sector.

4.4. Standard Model gauge symmetry

The vertices we need to eliminate, $\psi^2\Phi$, $\phi^2\Phi$, $\phi^3\Phi$, $\psi\phi\Psi$, $\psi^2 X$, $\psi A\Psi$, $\phi^2 X$ can be used to determine the $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ representations that the heavy fields would carry:

heavy particle	weak isospin	|hypercharge|
X, Φ	0, 1	$n/3,\ 0 \le n \le 5$
Φ	1/2, 3/2	1/2, 3/2
Ψ_{L}	0, 1	$n/3,\ 0 \le n \le 3$
Ψ_{R}	1/2	1/6, 1/2

If we now use this to *forbid* such representations, the vertices would not occur and there would be no tree-level generated operators.

Note however, that these conditions forbid the presence of heavy $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ singlet vector-bosons. In this case the underlying group must be of rank 2 (ignoring color), so either the Standard Model gauge group is the group for the full theory, or else the underlying group is SU(3) (though this last possibility suffers from serious problems — *e.g.* anomalies)

5. Comments and conclusions

There are sensible models which would exhibit no significant deviations from the Standard Model through radiative corrections. If such models are realized in nature it is quite possible for the LHC to uncover new physics without any premonition from LEP or the Tevatron. The above arguments indicate that, at least partly, the phenomenological success of the MSSM and the universal-extra-dimensions theories due to the absence of tree-level generated operators in these theories.

In the absence of tree-level generated operators the the relationship between the experimental limits and the physical value of Λ would change. For example, a limit $\Lambda > M_{\mathrm{exp}}$ obtained for a tree-level generated operator, becomes $\Lambda > M_{\mathrm{exp}}/(4\pi)$ if *all* tree-level generated operators are absent. In particular the limit derived for the scale at which $(\bar\psi\gamma^\mu\psi)^2$ is generated drops to $\Lambda > 360$ GeV, with similar results for non-Standard Model Z couplings.

Finally, it is worth noting that the elimination of tree-level generated operators can be done selectively in sectors distinguished by some global symmetries. For example one can require that all baryon and lepton number conserving tree-level generated operators be absent, while allowing lepton number violating tree-level generated operators (whose effects are small presumably because they are generated at a scale Λ that is very large).

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