Vol. 34 (2003) *ACTA PHYSICA POLONICA B* No 11

TRIVIALITY AND STABILITY LIMITS ON THE HIGGS BOSON MASS IN EFFECTIVE THEORIES*

Bohdan Grzadkowski

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warsaw, Poland
and
Theory Division, CERN, CH-1211 Genève 23, Switzerland

Jacek Pliszka

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warsaw, Poland
and
Department of Physics, University of California, Riverside, CA 92521-0413, USA

and Jose Wudka

Department of Physics, University of California, Riverside, CA 92521-0413, USA

(Received November 3, 2003)

The impact of new interactions on the triviality and stability Higgs-boson mass bounds has been studied. The interactions have been parametrized in a model-independent way by a set of effective operators of dimension 6. Constraints from electroweak observables at 1-loop level have been included. In the analyzed region of scale of new physics $\Lambda \simeq 2 \div 50$ TeV the classic triviality bound remains unchanged. An extension of the triviality condition that has been introduced leads to strong constraints on the possible models. The stability bound on the Higgs boson mass is substantially modified depending on the scale Λ and strength of coefficients of relevant effective operators.

PACS numbers: 14.80.Bn, 14.80.Cp

* Presented at the XXVII International Conference of Theoretical Physics, "Matter to the Deepest", Ustroń, Poland, September 15–21, 2003.

1. Introduction

Highly successful and experimentally verified Standard Model of the electroweak interactions has still one crucial particle undiscovered — the Higgs boson. It is of great importance to estimate even possible range of the Higgs boson mass for two main reasons. Firstly, the experimental methods that should be used for the discovery depend on the mass of the Higgs boson. Secondly, narrowing the possible ranges of the Higgs boson mass may exclude certain models describing physics beyond the Standard Model. Current direct experimental bound based on the LEP2 data [1] is $m_h > 113.2$ GeV for the Standard Model Higgs boson. In addition there exists the upper limit based from the indirect precision electroweak data [2] $m_h \lesssim 212$ GeV at 95% C.L. Yet for many of the extensions of the Standard Model these limits are much looser. In addition to these experimental limits there exist additional ones based on the theoretical arguments — so called triviality and vacuum stability bounds.

The triviality bound originates from the fact first described in the work by Wilson [3]. It states that in the renormalizable theory containing massive scalar field the strength of the quartic self-interaction term reaches infinity at some scale κ — and the stronger the interaction, the lower the scale at which it happens. As a consequence — the requirement of validity up to given scale bounds the size of the coupling from above what directly translates into the bound for the Higgs boson mass. The only theory valid at all scales is the trivial one — with no quartic self-interaction, which is reflected in the name of this bound. Since in practical terms the couplings have to be at most of the order of one in order for the theory to remain perturbative, we demand that up to some scale the coupling is smaller than some arbitrary value. We call this operational definition of the triviality condition. One should have in mind that violating this condition does not exclude the theory completely — it just means that the theory loses its predictivity, which may disfavors it against other, more predictive theories.

It was noted by Cabibbo [4] that quantum corrections may destabilize the electroweak potential leading to minimum in the wrong place, additional minimum or no minimum at all. With m_h being proportional to the curvature of the potential at the minimum the vacuum is less stable for lower m_h. Thus the requirement for the correct vacuum structure leads to lower bound on the Higgs boson mass.

This talk is based on our work in which we established these bounds in the presence of the general high energy interactions described in terms of the effective Lagrangian. For more detailed description of topics discussed here see the original publication [5].

2. Effective Lagrangian and RGE

The effective Lagrangian method has been widely used in the past [6]. We followed the approach and conventions of Buechmueller and Wyler [7]. In the effective Lagrangian method low-energy effects of the unknown high-energy physics are described in terms of the set of additional, non-renormalizable operators that are added to the original Lagrangian.

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \frac{1}{\Lambda}\mathcal{L}_1 + \frac{1}{\Lambda^2}\mathcal{L}_2 \,. \tag{1}$$

The operators are suppressed by the power of the scale of new physics Λ. As terms contributing to $\mathcal{L}_1$ violate either lepton or baryon number — they are strongly constrained by experiment. Hence we will limit ourselves to 81 independent operators of dimensions 6 constituting $\mathcal{L}_2$

$$\mathcal{L}_2 = \sum_i \alpha_i \mathcal{O}_i \,. \tag{2}$$

Out of these, if just third generation fermions are taken into account, only 16 can be generated at the tree-level by the unknown, high-energy physics described by weakly coupled gauge theory. Among these 16 only 5 listed below contribute directly to effective potential while 11 only through RG-mixing.

- $\mathcal{O}_\phi = \frac{1}{3}|\phi|^6$,
- $\mathcal{O}_{\partial\phi} = \frac{1}{2}\left(\partial|\phi|^2\right)^2$,
- $\mathcal{O}_\phi^{(1)} = |\phi|^2\,|D\phi|^2$,
- $\mathcal{O}_\phi^{(3)} = \left|\phi^\dagger D\phi\right|^2$,
- $\mathcal{O}_{t\phi} = |\phi|^2\left(\bar{q}\tilde{\phi}t + \text{h.c.}\right)$.

In addition to the above operators we have included one of the the remaining 11: $\mathcal{O}_{qt}^{(1)} = \frac{1}{2}\,|\bar{q}t|^2$ in order to estimate the importance of the mixing.

The effective potential obtained after inclusion of these operators has the following form:

$$V_{\rm eff}(\bar\varphi) = -\eta\Lambda^2|\bar\varphi|^2 + \lambda|\bar\varphi|^4 - \frac{\alpha_\phi|\bar\varphi|^6}{3\Lambda^2} + \frac{1}{64\pi^2}\left[\sum_{X=H,G,W,Z,T,\eta\Lambda^2} \xi_X X^2\left(\ln\frac{X}{\kappa^2} - \zeta_X\right)\right],$$

with coefficients $(\xi_X, \zeta_X) = (1, \frac{3}{2}), (3, \frac{3}{2}), (6, \frac{5}{6}), (3, \frac{5}{6}), (-12, \frac{3}{2}), (-4, \frac{3}{2})$ for $H, G, W, Z, T, \eta\Lambda^2$, respectively, where $\eta \equiv \lambda v^2/\Lambda^2$.

$$\begin{aligned}
H &= (6\lambda|\bar\varphi|^2 - \eta\Lambda^2) - \left[(6\lambda|\bar\varphi|^2 - \eta\Lambda^2)(2\alpha_{\partial\phi} + \alpha_\phi^{(1)} + \alpha_\phi^{(3)}) + 5\alpha_\phi|\bar\varphi|^2\right]\frac{|\bar\varphi|^2}{\Lambda^2}\,,\\
G &= (2\lambda|\bar\varphi|^2 - \eta\Lambda^2) - \left[(2\lambda|\bar\varphi|^2 - \eta\Lambda^2)\left(\alpha_\phi^{(1)} + \frac{1}{3}\alpha_\phi^{(3)}\right) + \alpha_\phi|\bar\varphi|^2\right]\frac{|\bar\varphi|^2}{\Lambda^2}\,,\\
W &= \frac{g^2}{2}|\bar\varphi|^2\left(1 + \frac{|\bar\varphi|^2\alpha_\phi^{(1)}}{\Lambda^2}\right)\,, \quad T = f^2|\bar\varphi|^2\left(1 + \frac{2\alpha_{t\phi}|\bar\varphi|^2}{f\Lambda^2}\right),\\
Z &= \frac{g^2 + g'^2}{2}|\bar\varphi|^2\left(1 + \frac{|\bar\varphi|^2(\alpha_\phi^{(1)} + \alpha_\phi^{(3)})}{\Lambda^2}\right).
\end{aligned}$$

Therefore only $\mathcal{O}_\phi = \frac{1}{3}|\phi|^6$ contributes to the effective potential at the tree level and should have the dominant impact on the potential stability.

As both triviality and stability conditions are checked at various energy scales it is crucial to implement the Renormalization Group Equation (RGE) evolution for the couplings used. The full set of the RGE running equations was presented in the earlier papers: [5, 8]. Here we concentrate on those essential for further discussion — for λ and α_ϕ:

$$\begin{aligned}
\frac{d\lambda}{dt} &= 12\lambda^2 - 3f^4 + 6\lambda f^2 - \frac{3}{2}\lambda\left(3g^2 + g'^2\right) + \frac{3}{16}\left(g'^4 + 2g^2g'^2 + 3g^4\right)\\
&\quad + 2\eta\left[2\alpha_\phi + \lambda\left(3\alpha_{\partial\phi} + 4\bar\alpha + \alpha_\phi^{(3)}\right)\right],\\
\frac{d\alpha_\phi}{dt} &= 9\alpha_\phi\left(6\lambda + f^2\right) + 12\lambda^2\left(9\alpha_{\partial\phi} + 6\alpha_\phi^{(1)} + 5\alpha_\phi^{(3)}\right) + 36\alpha_{t\phi}f^3\\
&\quad - \frac{9}{4}\left(3g^2 + g'^2\right)\alpha_\phi - \frac{9}{8}\left[2\alpha_\phi^{(1)}g^4 + \left(\alpha_\phi^{(1)} + \alpha_\phi^{(3)}\right)\left(g^2 + g'^2\right)^2\right],
\end{aligned} \tag{3}$$

where $\bar\alpha = \alpha_{\partial\phi} + 2\alpha_\phi^{(1)} + \alpha_\phi^{(3)}$.

As one can see the dominant terms in the λ evolution are those related to the quartic scalar self-coupling and to the top quark Yukawa coupling, so the impact of the 6-dim operators is expected to be small. On the other hand, for α_ϕ, for small and moderate λ the dominant terms are those proportional to α_ϕ and $\alpha_{t\phi}$. Yet it would be beneficial to check these hypothesis in detail.

3. The results

In this study we have included the basic tests against the precision electroweak observables. Since only large, tree-level generated operators have been included, it was enough to calculate only one-loop electroweak corrections. We limited our study to the observables that are relatively independent and are measured with sufficient precision: m_t, m_W, m_Z, ρ. For each tested point we demanded that combined $\chi^2 < 25$. For m_W and m_Z for which experimental measurements are much more precise than the 1-loop theoretical calculations we used the approximated 1-loop theoretical error set to 1%. Apart from $\Lambda < 5$ TeV region the tested models were fully consistent with the electroweak data at the assumed precision level. For $\Lambda < 5$ TeV the only problematic region was one with large λ but even then the consistency with EW data could be obtained by allowing moderate changes in the coupling constants.

As our test models we chose models with $\alpha_i(\Lambda) = \pm 1$ $(i = 1 \ldots 6)$ giving 64 different combinations of signs and employed the following algorithm:

1. pick set of α_i at $\kappa = \Lambda$,
2. pick $\Lambda = 2, 3, 4, \ldots, 50$ TeV,
3. pick λ ($m_h > 65$ GeV, $\lambda < \frac{\pi}{2}$),
4. set f, η, g, g' to SM values at $\kappa = m_Z$,
5. solve RGE running equations,
6. check if m_t, m_W, m_Z and ρ are consistent with the experiment, if not, try to adjust them several times (step 4) (problems only at $\Lambda < 5$ TeV and large λ),
7. check stability and triviality (specified below),
8. repeat procedure for other values of parameters.

We generalized the standard triviality condition to the following ones:

T1: $\lambda < \frac{\pi}{2}$,

T2: $\forall_i |\alpha_i| < 1.5$,

T3: logical product of the following 3 conditions:

- T3a: $|\eta\alpha_i| < \frac{\lambda}{4}$,
- T3b: $|\frac{\alpha_\phi}{\lambda}| < \frac{3}{4}|\frac{\Lambda}{\kappa}|^2$,
- T3c: $|\eta(4\alpha_{\partial\phi} + 2\alpha_\phi^{(1)} + 2\alpha_\phi^{(3)} + \alpha_\phi/\lambda)| < |\lambda|$.

The condition T1 is the classic triviality condition. The condition T2 is its natural generalization for α_i. The condition T3 demands that corrections from new physics are not too large so the whole procedure remains perturbative. One should remember that the conditions T2 and T3 are not triviality conditions in the strict sense but they are very similar to the operational definition of the triviality condition as they demand that given couplings are small enough for the theory to be consistent. In addition, the borders for all triviality conditions are fuzzy as the limit for the size of the couplings is arbitrary. The results for T1 have been plotted in Fig. 1. There is no significant departure from the SM limit as all 65 (64+the SM) curves are very close to each other.

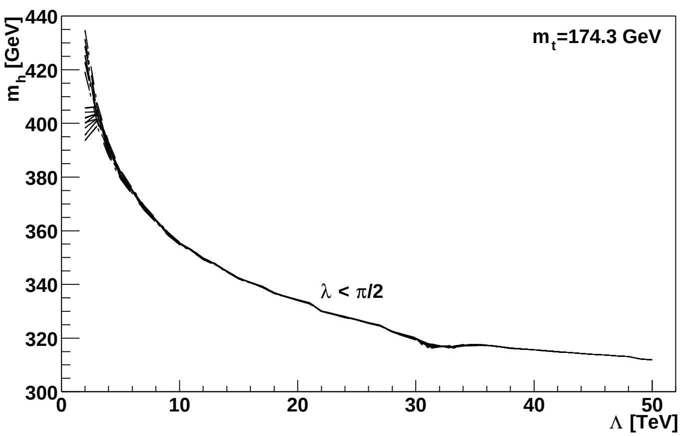

Fig. 1. The upper bound on the Higgs boson mass from the standard triviality condition T1: $\lambda < \pi/2$ (the small-scale structure is due to numerical inaccuracies).

As stability condition we have used the following checks:

S1: For $\bar{\varphi} \leq \frac{3}{4}\Lambda$, $V_{\rm eff}(\bar{\varphi})$ has a unique minimum at $\bar{\varphi} = v_0$ within 20% of the SM tree-level value $v \simeq 246$ GeV,

S2: The potential at $\bar{\varphi} = \frac{3}{4}\Lambda$ lies above its value at the minimum.

The results are plotted in Fig. 2. All 64 curves group into 4 sets of 16 curves. As it was expected, the dominant role is played by α_ϕ. The positive sign destabilizes the potential while the negative sign stabilizes it. In spite of the complicated nature of the analysis performed here, it is worth to trace the way in which $\alpha_{t\phi}$ could influence the lower limit on the Higgs-boson mass. The key point is the fact that $\mathcal{O}_{t\phi}$ modify the relation between the top-quark mass and its Yukawa coupling. Another mechanism of enhancing the contribution from $\mathcal{O}_{t\phi}$ is the very large numerical factor in front of $\alpha_{t\phi}$

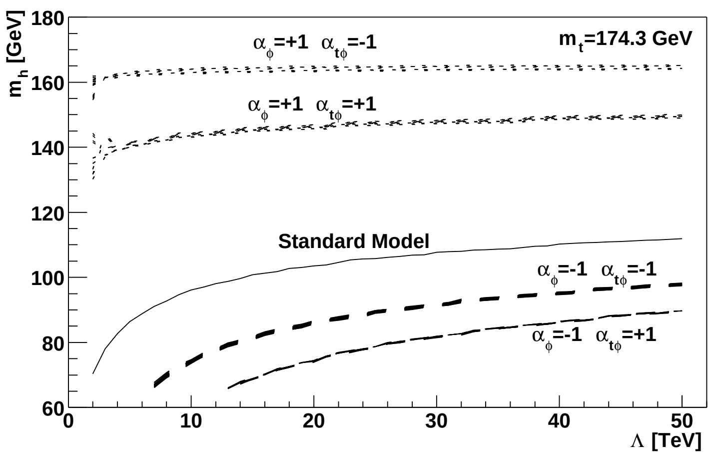

Fig. 2. Lower bounds on the Higgs boson mass (all the S's conditions satisfied), the black curve represents the SM limit, the upper (dotted) curves are for $\alpha_\phi > 0$, and the lower (dashed) ones for $\alpha_\phi < 0$. For each style the higher branches correspond to $\alpha_{t\phi} < 0$ while lower for $\alpha_{t\phi} > 0$.

in the evolution equation of α_ϕ. Because of this a larger $|\alpha_{t\phi}|$ amplifies the evolution of α_ϕ thereby requiring a larger m_h. Both effects combine leading to the dependence on $\alpha_{t\phi}$ illustrated in Fig. 2.

It is worth noticing that $\alpha_\phi > 0$ whenever the effective operator $\mathcal{O}_\phi$ is generated through the tree-level exchange of a heavy scalar isodoublet in the fundamental high-scale theory. Finally for each of the 64 analyzed models we applied all conditions combined. We present here figures (Fig. 3) for 3 representative cases with the α_i signs as written on the figures. The upper and lower borders of the grey-colored regions are triviality T1 and stability bounds as plotted on the previous figures. The black color regions, labeled T2 T3, are such that both conditions are satisfied; medium-grey areas, labeled $\not{T}2$ T3, represent the regions where T3 is satisfied but T2 is not; dark-grey areas, labeled T2 $\not{T}3$, represent the regions where T2 is satisfied but T3 is not; and light-grey areas, labeled $\not{T}2$ $\not{T}3$, represent the regions where neither T2 nor T3 are satisfied. The conditions T2 and T3 seem to be quite strong and exclude a significant part of the parameters space. The violation of T2 is always due to α_ϕ or $\alpha_{t\phi}$. For large (small) m_h condition T2 is always stronger(weaker) than T3 what can be deducted from the definitions of T2 and T3. On all of the plots condition T3 bounds from below as it is expected while T2 bound can be from above or from both sides. We have also found that for $\alpha_{qt} = \alpha_\phi = -1$ and $\alpha_{t\phi} = \alpha_\phi^{(3)} = \alpha_\phi^{(1)} = \alpha_{\partial\phi} = +1$ both T2 and T3 are violated in the whole parameter space.

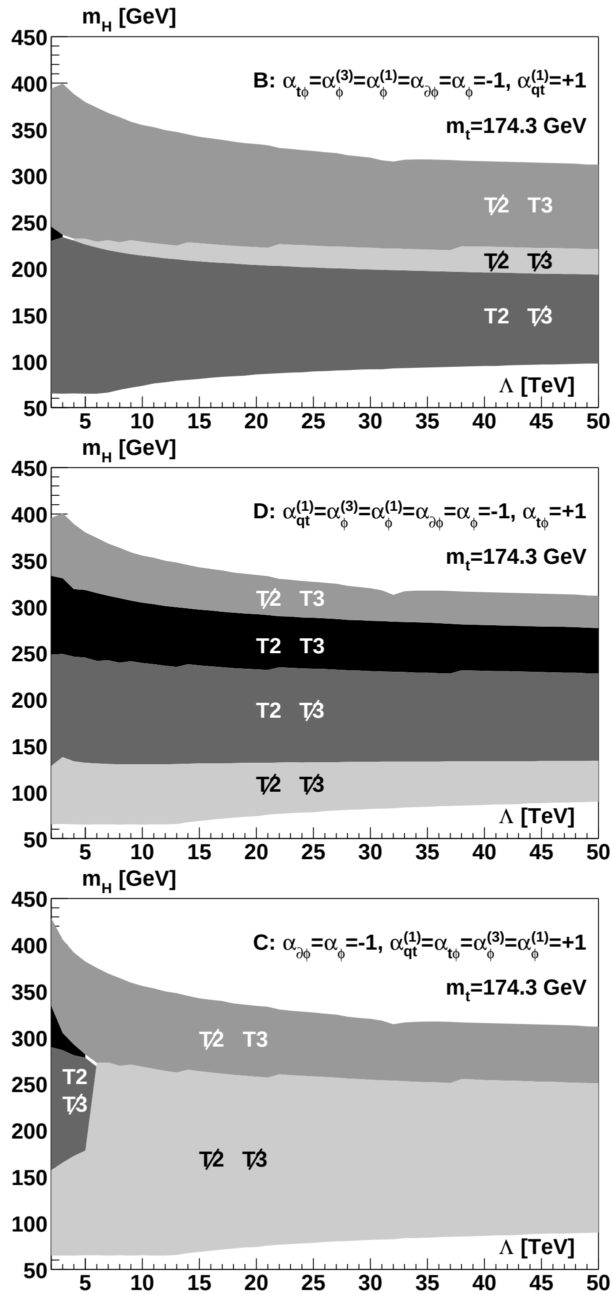

Fig. 3. Exclusion regions for the 3 selected models described in the text.

However, we would like to emphasize again that the bounds are fuzzy and that theory violating T2 and T3 may be still acceptable — nevertheless in such case corrections from new physics are large leading to potential lack of predictivity. If this happens to be the case in Nature, the violation of T2 or/and T3 conditions will have to be thoroughly examined.

4. Conclusions

We have calculated triviality and stability bounds on m_h in the effective Lagrangian approach for scale of new physics in the range: $2\,\text{TeV} \lesssim \Lambda \lesssim 50\,\text{TeV}$. The dominant operators contributing to the effective potential at one-loop level have been included in this analysis. One-loop basic electroweak constraints have been taken into account. We concluded that the classic triviality bound remains unchanged. In contrast, the stability limit is very sensitive to the effective operators: $\mathcal{O}_\phi = \frac{1}{3}|\phi|^6$ and $\mathcal{O}_{t\phi} = |\phi|^2 \left(\bar{q}\tilde{\phi}t + \text{h.c.}\right)$. We have developed generalized triviality-like conditions and we have shown how strongly they constrain the parameter space.

This work is supported in part by the Polish State Committee for Scientific Research (KBN) under grant 5 P03B 121 20 and funds provided by the U.S. Department of Energy under grant No. DE-FG03-94ER40837. BG and JP are indebted to U.C. Riverside for the warm hospitality extended to them while this work was performed.

REFERENCES

[1] T. Junk, The LEP Higgs Working Group, at LEP Fest October 10th 2000, `http://lephiggs.web.cern.ch/LEPHIGGS/talks/index.html`

[2] E. Tournefier, The LEP Electroweak Working Group, talk presented at the 36th Rencontres De Moriond On Electroweak Interactions And Unified Theories, 2001, Les Arcs, France, `hep-ex/0105091`.

[3] K.G. Wilson, *Phys. Rev.* **B4**, 3184 (1971).

[4] N. Cabibbo *et al.*, *Nucl. Phys.* **B158**, 295 (1979); for a review see M. Sher, *Phys. Rep.* **179**, 273 (1989) and references therein.

[5] B. Grzadkowski, J. Pliszka, J. Wudka, `hep-ph/0307338`, to appear in *Phys. Rev.* **D**.

[6] S. Weinberg, `hep-th/9702027`; H. Georgi, *Ann. Rev. Nucl. Part. Sci.* **43**, 209 (1993).

[7] W. Buchmüeller, D. Wyler, *Nucl. Phys.* **B268**, 621 (1986).

[8] B. Grzadkowski, J. Wudka, *Phys. Rev. Lett.* **88**, 041802 (2002).