THE RESUMMED HIGGS BOSON TRANSVERSE MOMENTUM DISTRIBUTION AT THE LHC *

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We apply QCD resummation techniques to study the transverse momentum distribution of Higgs bosons produced via gluon–gluon fusion at the LHC. In particular we focus on the joint resummation formalism which resums both threshold and transverse momentum corrections simultaneously. A comparison of results obtained in the joint and the standard recoil resummation frameworks is presented.

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1. Introduction

The Large Hadron Collider (LHC) is widely expected to be the Higgs boson discovery machine. In particular, the dominant channel for the production of a light Higgs particle at the LHC is gluon fusion $gg \to HX$ [1]. The search strategies for the Higgs boson rely deeply on the knowledge of production characteristics, the transverse momentum (Q_{T}) of the produced boson being one of the most important quantities. In this talk we describe

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an application of the joint resummation formalism [2] to calculate transverse momentum distribution of Higgs bosons produced through the gluon fusion mechanism at the LHC.

It is a general feature of perturbative calculations in QCD that close to a phase space boundary partonic hard-scattering cross sections acquire large logarithmic corrections. These corrections are related to soft and collinear gluon emission and arise from cancellations between virtual and real contributions at each order in perturbation theory. The threshold and recoil corrections are the two notable examples often discussed in this context. The threshold corrections of the form $\alpha_{\rm s}^n \ln^{2n-1}(1-z)/(1-z)$ become large when the partonic c.m. energy approaches the invariant mass Q of the produced boson, $z = Q^2/\hat{s} \to 1$. The recoil corrections, in turn, are of the form $\alpha_{\rm s}^n \ln^{2n-1}(Q^2/Q_{\rm T}^2)$ and grow large if the transverse momentum carried by the produced boson is very small, $Q_{\rm T} \ll Q$. Thus, sufficiently close to the phase-space boundary, *i.e.* in the limit of soft and/or collinear radiation, fixed-order perturbation theory is bound to fail. A proper treatment of higher-order corrections in this limit requires resummation of logarithmic corrections to all orders.

In the Standard Model the leading $\mathcal{O}(\alpha_{\rm s}^2)$ process for Higgs boson production via gluon–gluon fusion proceeds through a heavy quark loop, with the top quark loop providing the most significant contribution. In the limit of large top mass, $m_t \to \infty$, the Higgs coupling to gluons through the top loop can be described by the effective ggH vertex [3]. This simplifying approximation was shown to be valid up to a few percent accuracy in the case of NLO calculations [4]. The fixed-order predictions for the total production rate are currently known at the NNL (next-to-next-to-leading) order. Although not as large as the NLO corrections [3], the NNLO corrections were found to be substantial, increasing the NLO predictions by around 30% [5]. Moreover, it was shown that the prevailing contribution to these corrections corresponds to the soft and collinear gluon emission [6, 7], thus reinforcing the need for a careful treatment of logarithmic corrections to all orders.

The resummation techniques are well established both in the threshold [8, 9] and in the recoil [10, 11] case for the Drell–Yan production-type processes. The Drell–Yan mechanism and the mechanism for Higgs boson production through gluon–gluon fusion are similar. This makes it possible to apply (after implementing necessary changes accounting for gluons, instead of quarks, in the initial state) the already developed resummation methods to Higgs boson production. The resummed predictions were obtained in [4, 12] for threshold resummation and in [13–16] for recoil resummation. A joint, simultaneous treatment of the threshold and recoil corrections was first introduced in [2, 17]. It relies on a novel refactorization of short-distance and long-distance physics at fixed transverse momentum and energy [2]. Similarly to standard threshold and recoil resummation, exponentiation of logarithmic corrections occurs in the impact parameter b space [11], Fourier-conjugate to transverse momentum Q_{T} space as well as in the Mellin-N moment space [8, 9], conjugate to z space. The resulting expression respects energy and transverse momentum conservation. A full phenomenological analysis of Z boson production at the Tevatron in the framework of joint resummation can be found in [18], whereas Higgs boson production at the LHC was studied in [19].

2. The jointly resummed cross section

The general expression for the jointly resummed cross section [2], applied to Higgs boson production via gluon fusion, reads [19]

$$\frac{d\sigma_{AB}^{\mathrm{res}}}{dQ^2\, d^2\vec{Q}_{\mathrm{T}}} = \pi\tau\sigma_0^h \delta(Q^2 - m_h^2)\, H(\alpha_{\mathrm{s}}(Q^2)) \int\limits_{C_N} \frac{dN}{2\pi i}\, \tau^{-N} \int \frac{d^2 b}{(2\pi)^2}\, \mathrm{e}^{i\vec{Q}_{\mathrm{T}}\cdot\vec{b}}$$
$$\times \mathcal{C}_{g/A}(Q, b, N, \mu, \mu_{\mathrm{F}})\ \exp\left[\, E_{gg}^{\mathrm{PT}}(N, b, Q, \mu)\right]\ \mathcal{C}_{g/B}(Q, b, N, \mu, \mu_{\mathrm{F}})\ , \qquad (1)$$

where $\tau = Q^2/S$, m_h is the mass of the Higgs boson, and $\pi\tau\sigma_0^h\delta(Q^2 - m_h^2)$ denotes the lowest order partonic cross section for the process $gg \to HX$ in the limit of large m_t, with

$$\sigma_0^h = \frac{\sqrt{2} G_{\mathrm{F}} \alpha_{\mathrm{s}}^2(m_h)}{576\pi}\ . \qquad (2)$$

The function H contains the hard virtual part of the perturbative corrections and up to $\mathcal{O}(\alpha_{\mathrm{s}})$ is given by [3, 21]

$$H(\alpha_{\mathrm{s}}) = 1 + \frac{\alpha_{\mathrm{s}}}{2\pi} H^{(1)} = 1 + \frac{\alpha_{\mathrm{s}}}{2\pi}\left(2\pi^2 + 11\right)\ . \qquad (3)$$

Apart from a lower limit of integration, at the next-to-leading-logarithm (NLL) accuracy the form of the Sudakov factor $E_{gg}^{\mathrm{PT}}(N, b, Q, \mu)$ in the jointly resummed expression (1) is the same as for the recoil resummation [18]:

$$E_{gg}^{\mathrm{PT}}(N, b, Q, \mu) = -\int\limits_{Q^2/\chi^2}^{Q^2} \frac{dk_{\mathrm{T}}^2}{k_{\mathrm{T}}^2}\left[A_g(\alpha_{\mathrm{s}}(k_{\mathrm{T}}))\, \ln\left(\frac{Q^2}{k_{\mathrm{T}}^2}\right) + B_g(\alpha_{\mathrm{s}}(k_{\mathrm{T}}))\right]\ . \qquad (4)$$

The functions A and B are perturbative series in α_{s} and their coefficients can be determined by comparing fixed order predictions with an expansion of resummed result [20, 21]. NLL accuracy requires using $A_g^{(1)}$, $B_g^{(1)}$ and $A_g^{(2)}$ in Eq. (4):

$$\begin{aligned} A_g^{(1)} &= C_A \,, \qquad\qquad B_g^{(1)} \;=\; -\frac{1}{6}\left(11C_A - 4T_{\rm R}N_{\rm F}\right)\,, \\ A_g^{(2)} &= \frac{C_A}{2}\left[C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{10}{9}T_{\rm R}N_{\rm F}\right], \end{aligned} \tag{5}$$

where $C_A = 3$, $C_{\rm F} = 4/3$, $T_{\rm R} = 1/2$, and $N_{\rm F}$ is the number of flavours. The higher order (needed at NNLL) coefficient B_g^2 is also known [21]

$$B_g^{(2)} = C_A^2\left(-\frac{4}{3}+\frac{11}{36}\pi^2-\frac{3}{2}\zeta_3\right)+\frac{1}{2}C_{\rm F}T_{\rm R}N_{\rm F}+C_AN_{\rm F}T_{\rm R}\left(\frac{2}{3}-\frac{\pi^2}{9}\right). \tag{6}$$

The quantity $\chi(N,b)$ appearing in the lower limit of integration in (4) is specific to joint resummation. The LL and NLL logarithmic terms in the threshold limit, $N \to \infty$ (at fixed b), and in the recoil limit $b \to \infty$ (at fixed N) are correctly reproduced with the following choice of the form of χ

$$\chi(\bar{N},\bar{b}) = \bar{b} + \frac{\bar{N}}{1+\eta\,\bar{b}/\bar{N}}\,, \tag{7}$$

where η is a constant and we define

$$\bar{N} = N{\rm e}^{\gamma_{\rm E}}\,, \tag{8}$$

$$\bar{b} \equiv \frac{bQ{\rm e}^{\gamma_{\rm E}}}{2}\,, \tag{9}$$

with $\gamma_{\rm E}$ the Euler constant.

The functions $\mathcal{C}(Q,b,N,\mu,\mu_{\rm F})$ in Eq. (1) are given by:

$$\mathcal{C}_{a/H}(Q,b,N,\mu,\mu_{\rm F}) = \sum_{j,k} C_{a/j}\left(N,\alpha_s(\mu)\right)\,\mathcal{E}_{jk}\left(N,Q/\chi,\mu_{\rm F}\right)\,f_{k/H}(N,\mu_{\rm F})\,. \tag{10}$$

The product of parton distribution functions $f_{k/H}$ at scale $\mu_{\rm F}$, and a matrix $\mathcal{E}_{jk}$ can be seen as corresponding to parton densities evaluated at the scale Q/χ. The evolution from the scale $\mu_{\rm F}$ to Q/χ is accurate to NLL in χ and represented by the matrix $\mathcal{E}\left(N,Q/\chi,\mu_{\rm F}\right)$. The origin and the structure of the evolution matrix $\mathcal{E}$ was discussed in detail in Ref. [18, 19]. The coefficients $C_{a/j}(N,\alpha_{\rm s})$ have a structure of a perturbative series in $\alpha_{\rm s}$, and are determined in the same way as for recoil resummation, *i.e.* up to $\mathcal{O}(\alpha_{\rm s})$,

$$C_{g/g}\left(N,\alpha_{\rm s}\right) = 1+\frac{\alpha_{\rm s}}{2\pi}C_{g/g}^{(1)} = 1+\frac{\alpha_{\rm s}}{4\pi}\pi^2\,, \tag{11}$$

$$C_{g/q}\left(N,\alpha_{\rm s}\right) = \frac{\alpha_{\rm s}}{2\pi}C_{g/q}^{(1)} = \frac{\alpha_{\rm s}}{2\pi}C_{\rm F}\frac{1}{N+1} = C_{g/\bar{q}}\left(N,\alpha_{\rm s}\right)\,. \tag{12}$$

The expression (1) is formally accurate to the next-to-leading-logarithm (NLL) level. However, due to the large colour charge of the incoming gluons, the cross section exhibits increased sensitivity to the Sudakov logarithms. It is known for recoil resummation that the NNLL terms have a significant impact on numerical results. Motivated by this finding, we decide to include the NNLL terms containing the $B^{(2)}$ coefficient, in the way consistent with the recoil resummation. In our formalism we also include the $\mathcal{O}(\alpha_{\rm s})$ perturbative expansions for the functions H and C, which formally give NNLL contributions. As discussed in Ref. [22], the NNLL resummed cross sections are resummation scheme dependent, and the choice of the resummation scheme is reflected in the value of the coefficients $B^{(2)}$, $H^{(1)}$ and $C^{(1)}$. We exercise the freedom of the resummation scheme choice by demanding that the function H, calculated with $\alpha_{\rm s}$ taken at the scale Q, collects the hard virtual part of the NLO corrections. The Sudakov factor and the $\mathcal{C}$ coefficients contain then only soft or collinear contributions. The values of $B^{(2)}$ and $C^{(1)}$ listed above are for this particular resummation scheme.

By incorporating full evolution of parton densities the cross section (1) correctly includes also the leading $\alpha_{\rm s}^n \ln^{2n-1}(\bar{N})/N$ collinear non-soft terms to all orders. At the NLO, this can be seen by expanding jointly resummed cross section to $\mathcal{O}(\alpha_{\rm s})$ (here for illustration purposes only in the gg channel), integrated over $Q_{\rm T}$

$$\hat{\sigma}^{gg} = \sigma_g^{(0)} \frac{\alpha_{\rm s}}{2\pi} \Big\{ -4C_A \ln^2 \bar{N} + 8\pi b_0 \ln \bar{N} + 11 + 3\pi^2 \\ -2\ln \bar{N} \Big[\frac{4C_A}{N(N-1)} + \frac{4C_A}{(N+1)(N+2)} - 4C_A S_1(N) + 4\pi b_0 \Big] \Big\}\,, \quad (13)$$

where $S_1(N) = \sum_{j=1}^{N} j^{-1} = \psi(N+1) + \gamma_{\rm E}$, with ψ the digamma function. In the large N limit this gives

$$\hat{\sigma}^{gg} = \sigma_g^{(0)} \frac{\alpha_{\rm s}}{2\pi} \left\{ 4C_A \ln^2 \bar{N} + 4C_A \frac{\ln \bar{N}}{N} + 11 + 3\pi^2 \right\} + \mathcal{O}\left(\frac{\ln \bar{N}}{N^2} \right)\,, \quad (14)$$

which can be compared to the large N limit of the NLO result in the Mellin space

$$\hat{\sigma}^{gg} = \sigma_g^{(0)} \frac{\alpha_{\rm s}}{2\pi} \left\{ 4C_A \ln^2 \bar{N} + 4C_A \frac{\ln \bar{N}}{N} + 11 + 4\pi^2 \right\} + \mathcal{O}\left(\frac{\ln \bar{N}}{N^2} \right) . \quad (15)$$

The agreement between the expanded jointly resummed expression and the exact NLO result down to the $\mathcal{O}(1/N)$ is clear. The mismatch in the constant π^2 term between (14) and (15) originates from the value of the $C_{g/g}^{(1)}$ coefficient and is a NNLL effect. A development of the joint formalism at the NNLL would eliminate this disagreement as well as provide a way to include other NNLL coefficients, most notably $B^{(2)}$.

2.1. Numerical results and discussion

In order to have predictive power, the resummed expression (1) needs to be supplemented by a definition of the inverse Mellin and Fourier transforms from N and b space. In the joint approach, the inverse integrals are both treated as contour integrals in the complex space of N and b. For the integrals to be well defined, the contours must not run into the Landau pole or singularities associated with the form of the function χ. This procedure provides an unambiguous definition of resummed perturbation theory without an introduction of additional dimensional scales and implies a functional form of non-perturbative corrections. We refer the reader to Ref. [19] for a detailed discussion of the parametrization of the contours.

The joint resummation formalism with the inverse transforms defined as contour integrals ensures that predictions can be obtained for any non-zero value of $Q_{\rm T}$. This is not possible in the standard recoil approach without adding some non-perturbative term of the form $-gb^2$ to the Sudakov exponent. However, for the purely technical reasons of numerical stability we also include such a factor in our jointly resummed cross section. The value of the g parameter, $g = 1.67$ GeV^2, is adopted from the study in Ref. [24]. However, we checked that the dependence of the results at small $Q_{\rm T}$ on the value of g is negligible, in agreement with what was found for the case of pure b space resummation. At large $Q_{\rm T}$, where the $\ln(Q_{\rm T}^2/Q^2)$ terms taken into account by resummation lose their importance, it is necessary to match the resummed result with a fixed-order result. Here the jointly resummed result is matched to the $\mathcal{O}(\alpha_{\rm s})$ perturbative result [13], in the way described in [18].

The numerical results for the Higgs boson transverse momentum distribution calculated in the joint resummation framework were obtained assuming $m_h = 125$ GeV, $\mu = \mu_{\rm F} = Q = m_h$ and using CTEQ5M [23] parton distribution functions. The parameter η in the definition of χ (7) is chosen to be $\eta = 1$. We checked that the numerical dependence of the predictions on the value of η is small [19].

Apart from the joint resummation predictions, in Fig. 1 we also show the recoil-only (*i.e.* $\chi = \bar{b}$) resummed result. At small to moderate $Q_{\rm T}$, the b space resummed prediction is slightly higher and broader than the one provided by the joint resummation but the difference is small. Consequently, we conclude that the threshold effects are of modest importance at these values of $Q_{\rm T}$ and the pure recoil resummation is fully applicable there.

The $Q_{\rm T}$-integrated joint cross section, by definition, is expected to return the threshold resummed result. Although it does so formally up to NLL, numerically the integrated joint distribution returns a result which is $\sim 10\%$ lower than the threshold cross section. We find that this suppression is caused by subleading terms included in the joint resummation, more specifically terms $\propto 1/(N-1)$ in the expansion of the joint expression (13). These terms arise from our treatment of evolution in the coefficients $\mathcal{C}$, *cf.* Eq. (10). They are important only in the small N limit and therefore not present in the threshold resummed expression. (However, it is interesting to observe that the NLO cross section in Mellin space contains the same subset of terms $\propto 1/(N-1)$ as the expanded joint expression taken at $\bar{b}=0$, plus an additional subset of terms $\propto 1/(N-1)^2$. The numerical effects of the two subsets cancel almost entirely, leading to a relatively good approximation of the NLO cross section by the threshold resummed result.) The small N limit corresponds to the limit of small $z = Q^2/\hat{s} \ll 1$. Given partonic c.m. energies available at the LHC, the small z terms of the form $\alpha_s^n \ln^{(2n-1)}(z)/z$ may indeed play a significant role for light Higgs production. These terms can be resummed on their own [25], but their full inclusion in the joint formalism alongside the threshold and recoil corrections requires further work.

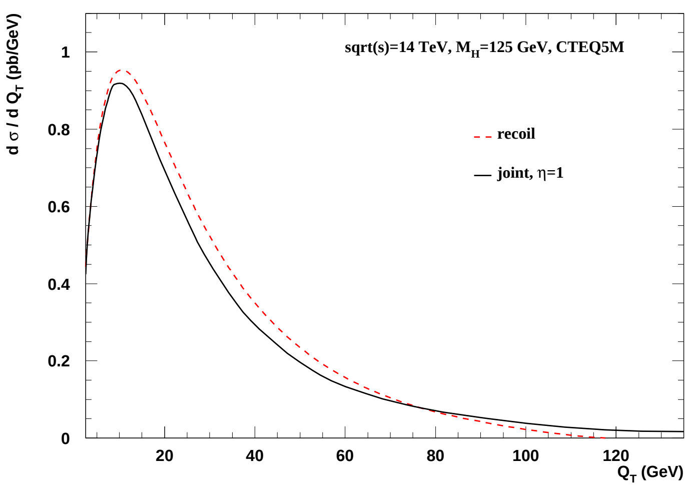

Fig. 1. Transverse momentum distribution for Higgs production at the LHC in the framework of joint resummation and of "pure-Q_{T}" resummation.

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