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QUASI-PARTICLES AND STRONG FIRST ORDER TRANSITION IN HOT QCD *

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Dedicated to the memory of Ian Kogan

The strong first order nature for more than five colours found in simulations of hot QCD leads to a quasi-particle picture valid down to the critical temperature. We review the evidence for magnetic quasi-particles and suggest simulations that put this picture into evidence.

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1. Introduction

Plasmas are ubiquitous in the universe. Most of visible matter is in this state, in which atoms are fully ionized. A plasma we know quite well is the sun. It sustains the thermonuclear power generation, that one tries to imitate in Tokamaks. Large static magnetic fields show up on the sun's surface as sunspots. They tell us that there is no magnetic screening effect. Of course long range static electric fields are absent, due to Debye screening.

What I will concentrate on in these two lectures is the strong first order character of the transition to and on the magnetic screening present in the QCD plasma for a large number of colours. This screening renders the QCD plasma so different from the plasmas we are used to. That the magnetic screening length had to be there was recognized already in the early days [1] of hot QCD. It was based on the simple observation, that the static magnetic sector of QCD is three-dimensional Yang–Mills theory and that this theory contains a mass gap.

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This mass gap is, alas, not computable in terms of the small gauge coupling constant. But it is accessible by lattice simulations. On the basis of these simulations we find that the ratio of the square of the magnetic screening mass and of the string tension is a large number. In terms of length scales this is a small number.

Some time ago a model for the three dimensional Yang–Mills theory was proposed, in which precisely this ratio was supposed to be small. It supposes that the partition function can be approximately computed by thinking of the theory as being a dilute 3D gas of lumps with size equal to the magnetic screening length. The lumps are non-perturbative quantities in terms of the 3D gluons. But they are supposed to be in an adjoint $\mathrm{SU}(N)$ multiplet.

This model has simple consequences for the Wilson loops, which have been verified to 1–2% by lattice simulations. Apart from a group theory factor the model predicts the tension σ of the loop to be simply the product of the magnetic screening l_{M} and the density n_{M} of the lumps:

$$\sigma \sim l_{\mathrm{M}} n_{\mathrm{M}} \,. \tag{1.1}$$

Multiplying this relation with l_{M}^2 gives the desired diluteness $l_{\mathrm{M}}^3 n_{\mathrm{M}}$. This diluteness equals $l_{\mathrm{M}}^2 \sigma$ and we know from lattice simulations this is a small number. Thus we have an *a posteriori* justification for the model.

In the next Section 2 we recall some basic facts and numerology for the Debye screening of electric charges, and how to simulate them on the lattice.

In the Section 3 we formulate the effective actions at high temperature. In the next Section 4 we pursue the same but now for the magnetic screening.

Finally we pass to the subject of magnetic quasi-particles in Section 5.

The last section contains conclusions and prospects.

2. The Debye mass

Let us consider some gas heated at a temperature well above the ionization energy. The electrons and ions are then moving more or less independently from each other.

Since the ions are much heavier than the electrons one can consider them to be a charged background with a density $|e|n$. If you immerse a heavy point charge Q in this medium its Coulomb interaction changes the density $n(\vec{x})$ of the electrons around it. The ions will remain unperturbed. As all of you know this gives rise to screening of the Coulomb law. The argument is purely classical. The Poisson equation in the presence of the ionized electrons with charge e reads:

$$\Delta A_0 = 4\pi e n(\vec{x}) - 4\pi e n_i - 4\pi Q \delta(\vec{x}) \,. \tag{2.1}$$

The variation in the electron density is due to the variation in the energy eA_0 of electrons in the field A_0, and the corresponding Boltzmann factor $\exp eA_0/T$, setting Boltzmann's constant equal 1s. Putting $A_0(\vec{x} = \infty) = 0$ we get $n(\vec{x}) = n \exp eA_0/T$. The Poisson equation becomes in the linear approximation in the energy:

$$\Delta A_0 = 4\pi \frac{e^2 n_i}{T} A_0 - 4\pi e_0 \delta(\vec{x})\,. \tag{2.2}$$

Eq. (2.2) is solved by a Yukawa potential, $A_0 \sim \frac{\exp - m_{\rm D} r}{r}$, with the Debye screening mass $m_{\rm D}$:

$$m_{\rm D}^2 = 4\pi \frac{e^2 n_i}{T}\,. \tag{2.3}$$

The screening length $l_{\rm D}$ is the inverse of the screening mass $m_{\rm D}$. Its raison d'être is statistical, due to the Boltzmann factor. So we expect the screening to involve many electrons. And this is precisely what Eq. (2.3) tells us: in a sphere of radius $l_{\rm D}$ we have $T^{3/2}/(e^3 n^{1/2}) = (T/(e^2/r_a))^{3/2}(r_a^3 n)^{-1/2}$ electrons with r_a the atomic radius. The first dimensionless factor is large because T is larger than the ionization energy e^2/r_a. The second dimensionless factor is large because the number of electrons inside the atomic radius is small in the ionized state .

Typically, for the sun's corona T/e^2 is about10^{2-3} the atomic scale and the number of electrons in the Debye sphere is 10^6. This condition is called the statistical screening or plasma condition.

It is amusing to do the following Gedanken experiment. Suppose we want to compute the electric flux going through some large (with respect to the atomic size) closed loop L with area $A(L)$. Normalize the flux $\Phi = \int_{\rm L} d\vec{S} \cdot \vec{E}$ by the electron charge e and define :

$$V(L) = \exp i2\pi \Phi/e\,. \tag{2.4}$$

Of course, at T below the ionization temperature no flux would be detected by the loop, because there are only neutral atoms moving through the loop. Only at the perimeter of the loop there might be an effect.

Let us now raise the temperature above $T_{\rm ionisation}$. What will happen? Both electrons and ions are screened. For simplicity we will take the ions to have the opposite of one electron charge. Then one electron (ion) on the down side of the loop will contribute $+1/2(-1/2)$ to the flux, and with opposite sign if on the up side of the loop. That is: $V(L)|_{\text{one charge}} = -1$. Of course if we plot $|\Phi/e|$ as function of the distance of the particle to the loop you find an exponential curve with the maximum $1/2$ at zero distance. For the sake of the argument we will replace that curve by a theta function of height $1/2$ and width $2l_{\rm D}$. If one wants to do better one has to deal with infinitesimally thin slabs, and integrate over the thickness. The result is parametrically the same as the one we will derive keeping the simplistic method.

Assuming that all charges move independently, the average of the flux loop $V(L)$ is determined by the probability $P(l)$ that l electrons (ions) are present in the slab of thickness $2l_{\mathrm{D}}$ around the area spanned by the loop. Taking for $P(l)$ the Poisson distribution $\frac{1}{l!}(\bar{l})^l \exp -\bar{l}$- $\bar{l}$ is the average number of electrons (ions) in the slab- we find for thermal average of the loop:

$$\langle V(L)\rangle_T = \sum_l P(l)V(L)_l = \sum_l P(l)(-)^l = \exp -4\bar{l}\,. \tag{2.5}$$

Now $\bar{l} = A(L)2l_{\mathrm{D}}n(T)$, so the electric flux loop obeys an area law $\exp -\rho(T)A(L)$, with a tension $\rho(T) = 8l_{\mathrm{D}}n(T)$. We know from the statistical screening condition that $l_{\mathrm{D}}^2\rho = 8l_{\mathrm{D}}^3 n(T)$ is a large number: hence the electric tension in units of the Debye mass is large.

We leave it as an exercise to the reader to compute the tension in the realistic case, where the ions carry charge $Z \times |e|$. Now one single ion produces a sign $(-)^Z$ and our one-slab approximation breaks down if Z is even: we have to revert to the method where we integrate over infinitesimally thin slabs.

So the behaviour of the loop is very different for the ionised state. It behaves with an area law. In the de-ionised state it records only perimeter effects.

3. Effective field theories at high temperature

With what we have learnt above in mind we turn to gauge theory at high T. Any field theory in equilibrium at non-zero temperature can be formulated as a Euclidean path integral. The time direction in that integral is periodic mod $1/T$ for bosons, and anti-periodic for fermions. For the statistical Gibbs sum one has

$$\mathrm{Tr}_{\mathrm{phys}} \exp -\frac{H}{T} = \int DADq ... \exp -\frac{1}{g^2}S(A,q,...)\,, \tag{3.1}$$

where the gauge potentials A, the quark fields q, and eventually other fields in the Standard Model are integrated over. In the limit that T becomes small with respect to typical particle scales the time direction can be neglected. This is called dimensional reduction [5]. It can be formulated as a systematic approximation scheme using that QCD at high temperature T has a small running coupling $g(T)$. The inverse propagator of a boson is proportional to $(2\pi nT)^2 + \vec{p}^2$. Here n takes on integer values. For a fermion n is replaced by

$n+1/2$. As T becomes large one can integrate all heavy T-modes ("hard" modes) and stay with a three dimensional theory in terms of only the static bosonic modes with $n=0$. For QCD this 3d Lagrangian reads:

$$\begin{aligned}\mathcal{L}_{\mathrm{E}} \;=\; & \mathrm{Tr}(\vec{D}(A)A_0)^2 + m_{\mathrm{E}}^2\mathrm{Tr}A_0^2 + \lambda_{\mathrm{E}}(\mathrm{Tr}(A_0^2))^2 \\ & +\bar{\lambda}_{\mathrm{E}}(\mathrm{Tr}(A_0)^4 - \frac{1}{2}(\mathrm{Tr}A_0^2)^2) + \mathrm{Tr}F_{ij}^2 + \delta\mathcal{L}_{\mathrm{E}}\,. \end{aligned} \tag{3.2}$$

This is called the electrostatic Lagrangian. The last term contains higher powers of A_0 and of the covariant derivative $\vec{D}(A) = \vec{\partial} + ig_{\mathrm{E}}[\vec{A}$. Neglecting it means one neglects $O(g^4)$ in the correlations you compute with the first six terms. The parameters in this Lagrangian are computed from the corresponding n-point functions in one and two loop accuracy in terms of $g_{\mathrm{E}}^2 = g^2(T)T$. Higher loop order adds only to the accuracy if one takes into account $\delta\mathcal{L}_{\mathrm{E}}$. The electrostatic coupling g_{E}^2 is to one loop order in terms of $\Lambda_{\overline{MS}}$ (no flavours):

$$\frac{g_{\mathrm{E}}^2 N}{T} = \frac{24\pi^2}{11\log(\frac{6.742.T}{\Lambda_{\overline{MS}}})}\,. \tag{3.3}$$

The subtraction was chosen to minimize the one loop effects [4].

We have swept one problem under the rug. When integrating the hard modes we have to admit a lower cut-off Λ_{E}, in between the scale T and the electrostatic scale gT. In principle the parameters will depend on this cut-off.

One expects that we have the same picture as before: above the deconfining temperature $T_{\mathrm{c}} \sim 200$ MeV) we have a gas of "ions", the quarks, and of "electrons", the gluons. There is screening as before, as witnessed by the mass parameter $m_{\mathrm{E}}^2 = \frac{N}{3}g^2T^2$ in electrostatic Lagrangian, Eq. (3.2).

This constitutes the Stephan–Boltzmann picture of QCD and interactions between gluons and quarks describe deviations from this free quasi-particle system.

Specific to QCD is that there is not only an electrostatic scale set by the Debye mass. We can integrate out in electrostatic Lagrangian all degrees of freedom corresponding to the Debye scale. That leaves us with the magnetostatic Lagrangian:

$$\mathcal{L}_{\mathrm{M}} = \mathrm{Tr}F_{ij}^2 + \delta\mathcal{L}_{\mathrm{M}} \tag{3.4}$$

with a magnetostatic gauge coupling g_{M}^2 in F_{ij}.

Here an ultra-violet cut-off Λ_{M} is needed. It separates the electrostatic scale gT and the magnetic scale g^2T.

Terms with higher order covariant derivatives are contained in $\delta\mathcal{L}_{\mathrm{M}}$. They are needed when we want an accuracy of $O(g^3)$.

The magnetic action gives a non-perturbative theory. In calculating a Green's function with a typical external momentum p we find as dimensionless parameter $g_{\rm M}^2/p$ which is $O(1)$ if the momentum is the magnetic scale. In particular calculation of the free energy in this theory will give for dimensional reasons $(g_{\rm M}^2)^3$ times a non-perturbative constant. That is the contribution one expects from a 4-loop diagram. As for the Green's function all higher loops are of the same order.

Still we can compute a series in the small coupling $g(T)$. Only the coefficients are non-perturbative.

For asymptotically large temperatures such a picture is indeed accurate. But asymptotic means temperatures about $10^6 T_{\rm c}$, well above the electroweak scale.

To put the calculation of the contributions of order higher than three in perspective and to see how the different scales come in, we recall once more the hierarchy of scales, cut-offs Λ and reduced actions needed to compute the pressure:

$$T \gg \Lambda_{\rm E} \gg gT \gg \Lambda_{\rm M} \gg g^2 T\,.$$

The pressure is normalized by $p_0 = P_{\rm Stefan-Boltzmann}$ and consists of three parts:

$$\frac{p}{p_0} = p_{\rm h} + p_{\rm E} + p_{\rm M}\,.$$

The hard modes are cut-off in the infrared by $\Lambda_{\rm E}$ and equal $p_{\rm h}$. Schematically we get:

$$p_{\rm h} = 1 + g^2 + g^4 \log\frac{T}{\Lambda_{\rm E}} + g^4 + g^6 \log\frac{T}{\Lambda_{\rm E}} + g^6 + \ldots\,.$$

All powers of the coupling are even, since infrared divergencies are cut-off by $\Lambda_{\rm E}$. The short distance scales (larger than T) are absorbed in the running coupling, Eq. (3.3). The cut-off $\Lambda_{\rm E}$ appears only in logarithms. The electric mode contributions are computed with $\mathcal{L}_{\rm E}$ and give $p_{\rm E}$:

$$p_{\rm E} = g^3 + g^4 \log\frac{\Lambda_{\rm E}}{m_{\rm E}} + g^4 + g^5 + g^6 \log\frac{\Lambda_{\rm E}}{m_{\rm E}} + g^6 \log\frac{m_{\rm E}}{\Lambda_{\rm M}} + g^6 + \ldots\,.$$

Note the odd powers in g. They come in because the electric mass gT comes in through propagators from the electrostatic action, Eq. (3.2). For example, the contribution from the scalar potential A_0 gives the first term in $p_{\rm E}$:

$$-\frac{1}{2}(N^2-1)\int \frac{d\vec{l}}{(2\pi)^3}\log(\vec{k}^2 + m_{\rm E}^2) = \frac{\Gamma(-\frac{3}{2})}{16\pi^{\frac{3}{2}}} m_{\rm E}^3\,. \tag{3.5}$$

The dominant cubic term was computed in Eq. (3.5). We can expect logarithms of the two ratios of the three scales $m_{\rm E}$, $\Lambda_{\rm E}$ in the electrostatic action and $\Lambda_{\rm M}$.

Finally the magnetic contribution is computed with $\mathcal{L}_{\rm M}$:

$$p_{\rm M} = g^6 \log \frac{\Lambda_{\rm M}}{g_{\rm M}^2} + g^6 + \dots .$$

We only put in the obvious dependence on the parameters in the electrostatic and magnetostatic actions. There are three comments:

- All terms shown are perturbatively calculable, except the last one in $p_{\rm M}$.
- All perturbatively calculable terms have been computed [15], except for the g^6 terms. In particular the log's are known by now [8].
- All dependence on the cut-offs cancels, as expected.

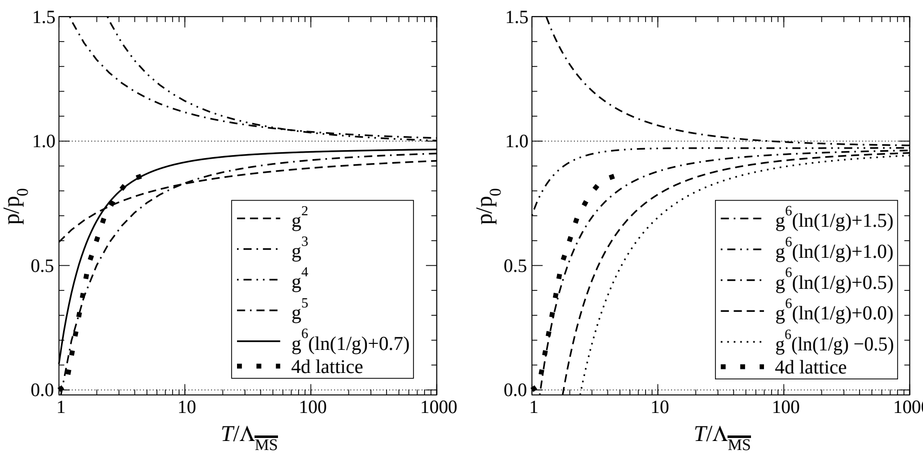

Fig. 1. Left: perturbative results at various orders for pure SU(3) gauge theory, including $\mathcal{O}(g^6)$ for an optimal constant. Right: the dependence of the $\mathcal{O}(g^6)$ result on the (not yet computed) constant, which contains both perturbative and non-perturbative contributions. The 4d lattice results are from [6]. From Ref. [8].

This is dramatically illustrated by Fig. 1. You see on the left the lattice data for the pressure in units of its Stephan–Boltzmann value $\frac{8\pi^2}{45}T^4$ plotted together with the known low order (up to $O(g^6)$) perturbative results. For the asymptotically large temperatures mentioned the quasi-particle picture is indeed accurate as the figure shows. The right panel shows how the prediction can improve, when the known [8] logarithmic contribution to the $O(g^6)$ coefficient is included, together with a guess for the non-perturbative part of the coefficient.

For any reasonable T, say below 2 GeV, the coupling obeys $g_{\rm E}^2(T)/T \leq 2.5$. This is about 30 times bigger then e^2, so we may already surmise that low order perturbation theory will be far from accurate.

3.1. The Debye mass and electric flux loop in QED and QCD

Now we discuss the Debye screening in the QCD plasma.

Let us put a probe charge in the plasma, say a very heavy quark. In Fig. 2 the exchange of a single gluon is shown, together with its multiply

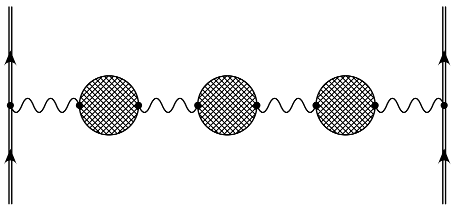

Fig. 2. A single gluon exchanged between two static test charges.

inserted self-energy. Once we compute the self energy Π_{00} of the gluon, shown in Fig. 3, the resumed propagator D_{00} becomes:

$$D_{00}(\vec{p}) = \frac{1}{\vec{p}^2 + \Pi_{00}(\vec{p})}\,. \tag{3.6}$$

It is easy to see that only the hard modes contribute to the self energy.

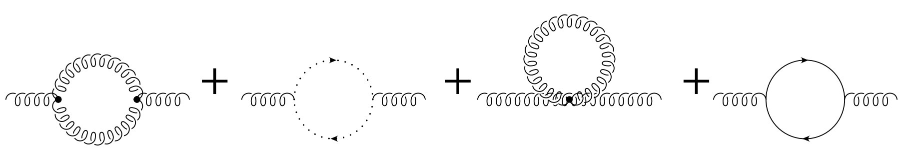

Fig. 3. One-loop self-energy of a gluon.

Hence one finds the one loop result for $m_{\rm E}^2$ in the electrostatic Lagrangian.

$$D_{00}(r) \sim \frac{\exp -m_{\rm E} r}{r}\,. \tag{3.7}$$

To two loop order one finds that already at that order non-perturbative effects contribute.

Hence a definition independent of perturbation theory is called for, and a natural candidate is the correlator between two heavy test charges: its fall-off as a function of distance gives us the screening mass.

The test charge put in the plasma changes the free energy. This change can be expressed in terms of an expectation value of the thermal Wilson line:

$$\exp -\frac{\Delta F_\psi}{T} = \frac{\int DA \frac{1}{N}\mathrm{Tr}\mathcal{P}(A_0)\exp -S(A)}{\int DA \exp -S(A)} \equiv \langle P(A_0)\rangle \,, \tag{3.8}$$

where the thermal Wilson line is given by:

$$P(A_0(\vec{x})) = \frac{1}{N}\mathrm{Tr}\mathcal{P}\exp ig \int\limits_0^{\frac{1}{T}} d\tau A_0(\vec{x},\tau)\,. \tag{3.9}$$

If the test charge is in the fundamental representation then so is A_0.

The path ordering is defined by dividing the interval $[0,1/T]$ into a large number N_τ of bits of length $\Delta\tau = \frac{1}{N_\tau T}$:

$$\mathcal{P}(A_0) = \lim_{N_\tau\to\infty} U(\tau=0,\Delta\tau)U(\tau=\Delta\tau,2\Delta\tau)\ldots U\left(\tau=\frac{1}{T}-\Delta\tau,\frac{1}{T}\right)\,. \tag{3.10}$$

From a formal and from a computational point of view the correlator $\langle P(A_0)P(A_0)^\dagger\rangle$ has two advantages over the calculation presented above using the scalar potential:

- The correlator is gauge invariant.
- The correlator can be evaluated non-perturbatively, *i.e.* on the lattice.

Both are needed for an accurate determination of the screening length in QCD. Perturbation theory is not enough, despite the small coupling $g(T)$ for asymptotically large temperature (*i.e.* well above the electro-weak scale). In the confined phase $\langle P(A_0(r)) < P(A_0)^\dagger(0)\rangle$ obeys an area law $\exp -\sigma\frac{r}{T}$, in the deconfined phase the area law is replaced by the Yakaw potential controlled by the Debye mass.

There is a further advantage: correlators of gauge invariant operators will excite the levels of a fictitious Hamiltonian describing Yang–Mills dynamics in a space with one periodic mod $1/T$ direction and two other infinite directions. The time conjugate to this Hamiltonian is now the direction of the correlation.

Conserved quantum numbers are then, apart from those from the two dimensional rotation group, the usual discrete parities, charge conjugation C, parity P (now in 2D), and a new quantum number, that changes $A_0 \to -A_0$, called R-parity.

Note that the Debye mass defined this way should coincide in one loop order with what we found before: m_{E} in the electrostatic Lagrangian. So it is associated with R-parity -1. Clearly, this corresponds to the imaginary part of the Wilson line. The real part excites $R=+,\ P=C=+$ states. To wit: if the correlator $\langle PP\rangle$ between two like charges were zero, then the difference between correlators of imaginary and real parts would be zero. That would mean, in turn, that the masses controlling their decays would be degenerate. This is not the case. Two like screened charges are compatible on a torus. In the confined phase (no screening) their correlation is indeed zero.

3.2. Z(N) symmetry, universality, and the order of the transition

There is a symmetry due to invariance of the Yang–Mills action under gauge transformations that are not periodic in Euclidean time, but only periodic modulo a center group element $\exp ik\frac{2\pi}{N}I_N$. I_N is the $N\times N$ unit matrix. So with k integer, the determinant is one. What is not invariant is the periodicity of fields in representations with non-zero N-allity, like quark fields. So we will discard them for the moment.

Now the Wilson line, Eq. (3.9), under such a transformation is multiplied by the Z(N) phase factor $\exp ik\frac{2\pi}{N}$. So the probability to find the system with a given value for the Wilson line:

$$E(\widetilde{P})\sim\int DA\delta(\widetilde{P}-\overline{P(A_0)}\ )\exp -S(A) \tag{3.11}$$

has the same value in $\widetilde{P}$ as in $\exp ik\frac{2\pi}{N}\widetilde{P}$, because the measure stays the same, so does the action, only the argument of the delta function will change. This is most useful manifestation of Z(N) symmetry [11].

Note that the Wilson line is a scalar quantity in every point $\vec{x}$.

So it bears resemblance to a Z(N) spin variable $z(\vec{x})$ defined on a three dimensional lattice. If we endow this Z(N) spin system with a nearest neighbour Z(N) invariant action, we have a system that has a transition point where the spin system changes from disordered into ordered behaviour.

There is now the hypothesis [23] that the transition of this spin system and that of the Yang–Mills system are in the same universality class. This is interesting because it relates critical behaviour of a rather simple system to that of our Yang–Mills system.

For $N=2$ and 3 this spin system is unique in that one can write down only one action for this spin system per link:

$$S_{N=2,3}=\beta(z+z^*)\,. \tag{3.12}$$

Here z is a shorthand for the product of the two spin variables at the end points.

Indeed the transition is second (first) order for $N = 2(3)$, and many studies have found that critical behaviour is identical [9].

However for $N = 4$ the spin system is not unique. Let us parametrize the action per link like:

$$S_{N=2,3} = \beta((z + z^*) + xz^2)\,. \tag{3.13}$$

In Fig. 4 [2] the phase diagram of this theory is plotted. Only positive couplings are of interest to us.

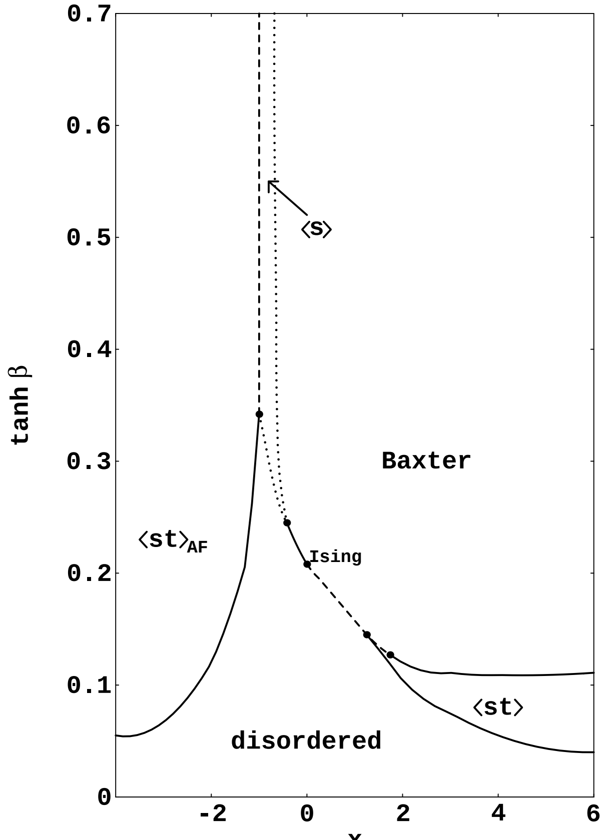

Fig. 4. Schematic phase diagram of three-dimensional Z(4) spin model from Eq. (3.13) on a simple cubic lattice, taken from Ref. [2], where it was extracted from series analysis and Monte Carlo data. Dashed and solid lines indicate first and second-order transitions respectively. Dotted lines indicate cases where the nature of the transition has not been unambiguously determined. The phases are labeled disordered ($\langle z\rangle = \langle z^2\rangle = 0$); Baxter (ferromagnetic with $\langle z\rangle$, $\langle z^2\rangle$ both non-zero); "$\langle st\rangle$" (where $\langle z^2\rangle$ is ferromagnetically ordered but $\langle z\rangle = 0$).

The VEV of the z spin corresponds to the VEV of the Wilson line $P(A_0)$ [12], the VEV of z^2 to that of $P(A_0)^2$). The region in between the two second order transition lines corresponds to the subgroup Z(2) already broken, but not yet Z(4). That would imply two Debye masses (not very natural from the plasma point of view), one corresponding to $P(A_0)$ and still zero in that region. Another corresponding to $P(A_0)^2$ and already non zero in that region.

But Nature has decided differently: in the gauge system the transition is first order [10,7], and, from the phase diagram, that corresponds to both order parameters jumping at the same time. This is what has been confirmed [10] in gauge theory within errors.

Recent data [24] show that the first order transition becomes stronger with increasing N. This is consistent with the idea that quasi-particles govern the behaviour of the plasma from very high T down to just above the critical temperature.

3.3. Electric flux and the spatial 't Hooft loop

The phenomenon of deconfinement involves the breaking of the electric flux tubes, and the appearance of quasi-particles, the gluons. This reflects itself in the change in the force law between test charges, discussed above.

How does it manifest itself in other measurable quantities? A natural candidate is the spatial loop that measures the electric flux, as we discussed in the first section. This loop is formed by a closed magnetic flux line, the 't Hooft loop [13].

What will perspire [3, 17] is that the behaviour of this loop in the deconfined phase is again quite different from that in the confined phase. The quantitative behaviour of the loop at very high T can be computed along the same lines as in Section 2. This is what we will do below.

We start with a definition of the loop as a magnetic flux loop, *i.e.* as a gauge transformation $\exp i\omega_{\rm L}(\vec{x})Y_k$ with a discontinuity $\exp ik\frac{2\pi}{N}$ when going around the loop. Here $Y_k = \mathrm{diag}\ (k,k,...,k,k-N,...k-N)$ with $N-k$ entries k and k entries $k-N$, so that it generates the center group element:

$$\exp\left(i\frac{2\pi}{N}Y_k\right) = \exp ik\frac{2\pi}{N} = z_k. \tag{3.14}$$

$\omega_{\rm L}(\vec{x})$ is half the solid angle defined by the loop.

In the Hilbert space this operator reads:

$$\tilde{V}_k(L) = \exp i\int d\vec{x}\frac{1}{g}\mathrm{Tr}\vec{E}(\vec{x})\cdot\vec{D}\omega_{\rm L}(\vec{x})\frac{Y_k}{2N}. \tag{3.15}$$

A representation which has the same effect in the physical Hilbert space is:

$$V_k(L) = \exp i\frac{2\pi}{g}\int\limits_{S(L)} d\vec{S}\mathrm{Tr}\vec{E}(\vec{x})\frac{Y_k}{N}. \tag{3.16}$$

Using the canonical commutation relations you can check that $V_k(L)$ and $\tilde{V}_k(L)$ have the same effect on physical states. That is, they multiply Wilson loops with the center group factor if the latter intersects with $S(L)$.

So Eq. (3.16) is the dual Stokes version of Eq. (3.15).

In the confined phase the particles are colour neutral so will at most contribute a perimeter law.

Let us now repeat the quasi-particle argument for the area law in the average of the flux loop in the deconfined phase:

$$\langle V_k(L)\rangle = \exp -\rho_k(T)A(L)\,. \tag{3.17}$$

The gluons are in the adjoint representation, so their Y_k charge follows from the differences of the diagonal elements of Y_k. So there are $2k(N-k)$ gluons with charge $\pm gN$, and the remaining gluons have charge 0.

Again, the total flux of a gluon inside the slab of thickness $l_{\mathrm{D}} = m_{\mathrm{E}}^{-1}$ on both sides of the loop as seen by the loop is $\frac{1}{2}gN$. The other half is lost on the loop. So the contribution of a fixed gluon species with non-zero charge is -1. As we suppose the gluons to be independent, the probability distribution for all the species inside the slab will factorize into single species distributions $P(l)$, l the number of gluons of that species inside the slab. Only the $2k(N-k)$ species with non-zero flux will contribute. Because of the factorization:

$$\langle V_k(L)\rangle = \left(\sum_l P(l)(-1)^l\right)^{2k(N-k)} \tag{3.18}$$

and with the Poisson distribution for $P(l)$ we get $\sum_l P(l)(-1)^l = \exp -2\bar{l}$, with $\bar{l} = n(T)l_{\mathrm{D}}A(L)$ the average number of the gluon species in the slab. It follows from Eq. (3.17) that the tension equals:

$$\rho_k = 4l_{\mathrm{D}}n(T)k(N-k)\,. \tag{3.19}$$

It is the dependence on the strength k of the loop, which is typical for the quasi-particle picture. We have checked in perturbation theory that deviations of this behaviour start to develop only to three loop order [18]. Just above the transition we have no reason to trust the loop expansion. If the strong first order transition found at $N \geq 6$ really implies a quasi-particle picture one should simulate the loop just above the transition for its k-dependence. Paraphrasing Ref. [24] such a behaviour just above T_c would suggest that the upper limit to the interface tension $\tau_{c,d}$ will scale like N, in accordance with a strong first order transition! To understand this argument, consider the complex plane with the possible phase of the Wilson line $P = \exp ik\frac{2\pi}{N}$ on the unit circle. In the deconfined phase the effective potential has degenerate minima in the Z(N) vacua. At $T = T_c$ one has another degenerate minimum in $P = 0$, the confined phase, which in coexistence with any of the Z(N) phases, will have the same tension $\tau_{c,d}$ because of the obvious symmetry in the plane. A region of space with $P = 1$ is separated from a region where $P = \exp ik\frac{2\pi}{N}$ by a wall given by ρ_k, for which the tension $\sim N$ from Eq. (3.19). If just above the transition T_c the deconfined phase with $P = 0$ starts to form in between the two Z(N) phases (so-called "wetting") we have $2\tau_{c,d} \leq \rho_k$. If wettting is to be true for all Z(N) interfaces the upper limit follows. If we know the Z(N) spin model, that falls in the same universality class as our gauge theory, we can check these statements quantitatively[1].

4. Magnetic screening mass and spatial Wilson loop

Not only the force law between heavy electric charges like the heavy quark, but also the force between heavy magnetic charges tells us about the medium. The original idea of 't Hooft and Mandelstam was that of a dual superconductor, with the electric Cooper pairs replaced by some form of magnetic condensate. This condensate would be expected to screen the colour-magnetic field.

In Section 2 we constructed an operator $V_k(L)$ creating a magnetic flux of strength $\exp ik\frac{2\pi}{N}$, Eq. (3.15). This loop was space like.

To get the monopole anti-monopole pair at points $(0, r)$ we have the vortex end at 0 and r on the positive z-axis. The vortex is given by a gauge transformation $V_k(\vec{x})$ which is discontinuous modulo a center group element $\exp ik\frac{2\pi}{N}$ when going around the vortex. The vortex is like the Dirac string in QED. It is unobservable by scattering with particles in the adjoint representation, as long as it has center group strength.

$$V_k = \exp i\vec{D}(A)v_k(x,y)\vec{E} \tag{4.1}$$

with $v_k(x,y) = \frac{\arctan(\frac{y}{x})}{N}Y_k$. When encircling the point $x = y = 0$ the gauge transformation $\exp iv_k(x,y)$ picks up a factor $\exp i\frac{2\pi}{N}Y_k = z_k$. This gauge transformation remains, by definition, unchanged along the z-direction and will be denoted by $V_k(r)$. We say that $V_k(r)$ creates a vortex or "Z(N)

[1] It is known that in the N state Potts model $\rho_k = \rho = \tau_{c,d}$ at the first order phase transition.

Dirac string" of length r. That means, a Wilson loop W in the fundamental representation that encircles the vortex will pick up the z_k factor:

$$V_k W V_k^\dagger = z_k W \,. \tag{4.2}$$

Any Wilson loop with non-zero N-ality l will pick up a factor $(z_k)^l$. But Z(N) neutral loops will not sense the Z(N) Dirac string, hence the name.

On the lattice the Hamiltonian operator will have magnetic plaquette operators. These will pick up the z_k factor and it is not hard to see that the Gibbs trace can be worked into a path integral along the usual lines, and on the lattice the latter takes the form:

$$\exp -\frac{F_{\mathrm{M}}(r)}{T} = \frac{\int DA \exp -S_{(k)}(A)}{\int DA \exp -S(A)} \,. \tag{4.3}$$

The action $S_{(k)}$ is the usual lattice action, except for those plaquettes pierced by the Dirac string. Those plaquettes are multiplied by a factor $\exp ik\frac{2\pi}{N}$, as in Fig. 5. This string is repeated at every time slice between $\tau = 0$ and $\tau = 1/T$.

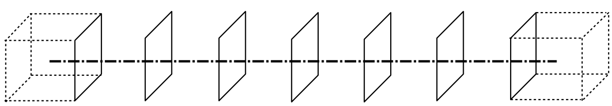

Fig. 5. Monopole antimonopole pair induced by twisting the plaquettes pierced by the Dirac string.

Screening is expected in both confined and deconfined phases:

$$F_{\mathrm{M}}(r) = F_{\mathrm{M}0} - c_{\mathrm{M}} \frac{\exp -m_{\mathrm{M}} r}{r} \,. \tag{4.4}$$

All parameters are function of T. In the cold phase the screening is a consequence of the electric flux confinement. This is natural because the ground state contains a condensate of "magnetic Cooper pairs", according to the dual superconductor analogy. It is a screening mechanism whose details are not understood. We dropped for notational reason the dependence on the strength k of the monopole in the coefficient c_{M}.

The magnetic mass does probably not depend on the strength k of the source, just like the Debye mass does not, as discussed in the previous section.

In the hot phase there are indications from spatial Wilson loop simulations that there is additional thermal screening from magnetic quasi-particles, as discussed in Section 5.

Analogous to the Wilson line correlator we consider the Hamiltonian $\hat{H}$ in the fictitious system of (x, y, τ) variables. We search the operator V_k acting on the Hilbert space of physical states of this Hamiltonian, that reproduces the path integral Eq. (4.3) [2] . So V_k should create a vortex in the (x, y) plane at every time slice τ and the Hamiltonian $\hat{H}$ should propagate every one of these vortices in the z-direction over a distance r. So V_k is the 't Hooft vortex operator discussed around Eq. (4.1):

$$V_k = \exp i \int_{x,y,\tau} \mathrm{Tr}\vec{D}(A) v_k(x, y)\vec{E}. \tag{4.5}$$

with $v_k(x, y) = \arctan\left(\frac{y}{x}\right) \frac{1}{N} Y_k$.

Both under parity (remember: only $y \to -y$!) and charge conjugation the vortex V_k transforms into $V_k^1 \dagger$. Its spin J equals 0, despite the appearance of the rotated singularity line. On physical states the location of the singularity does not matter. Hence the operator $\mathrm{Im}V_k$ excites spin zero states with $P = C = -1$. The magnetic screening mass should correspond to the self-energy of a magnetic gluon, just like the correlator of the thermal Wilson line had to correspond to the self energy of a temporal gluon, So we choose the negative charge conjugation component $\mathrm{Im}V_k$. The magnetic screening mass distinguishes itself from the electric screening mass by the opposite parity. This will prove important!

Perturbation theory is not reliable and we need lattice simulations [22]. Up to now these simulations are four dimensional and have limited accuracy. They need to be repeated, also in dimensionally reduced form. Once they reproduce the mass levels of the fictitious Hamiltonian with sufficient accuracy, we can use them with more confidence for determining the tension of the space like 't Hooft loops.

At high T where reduced calculations are valid we expect to find the mass level of the 0^{--} of the reduced Hamiltonian. From Teper's work [19] the lowest 0^{--} mass in units of the string tension for SU(N) $(N \geq 3)$ gives in a large N expansion (see his table 34):

$$\frac{m^{--}}{\sqrt{\sigma}} = 5.91(21)(1 + 0.88(70)/N^2 + 1/N^4)\,. \tag{4.6}$$

For $N = 3$ one finds for this ratio 6.48(9).

[2] We use the same notation as for the vortex operator in (x, y, z) space as there is no risk for confusion.

4.1. The spatial Wilson loop

The spatial Wilson loop is of interest because it monitors the magnetic activity in the plasma. At zero temperature it obeys an area law identical to that of time-like loop, controlled by the zero temperature string tension.

The tension stays constant throughout the confined phase, and starts to rise about the critical temperature, indicating a new source of magnetic flux activity.

Let us begin with some basics: a representation built from k fundamental representations is said to have N-allity k. A center group transformation $\exp i\frac{2\pi}{N}$ is mapped into $\exp ik\frac{2\pi}{N}$ in such a representation. Write a Wilson loop formed with such a representation as $W_k(L)$. Its average will then give an area law with tension σ_k. At high T one can integrate out the hard modes, as they do not determine the string tension. One can also integrate out the electrostatic modes, and wind up with a path integral controlled by the magnetostatic action:

$$\exp -\sigma_k(T)A(L) = \frac{\int D\vec{A}W_k(L)\exp -S_{\mathrm{M}}(A)}{\int D\vec{A})\exp -S_{\mathrm{M}}(A)}\,. \tag{4.7}$$

The hard and electrostatic free energies f_{h} and f_{E} drop out in the ratio.

The only dimensionfull scale in the magnetostatic action is g^2_{M}. So the tension, having dimension $(\mathrm{mass})^2$, can be written as:

$$\sigma_k(T) = c_k g^4_{\mathrm{M}}\left(1 + O(g^3)\right). \tag{4.8}$$

So the dominant contribution to the tension is entirely from the magnetostatic sector. In Fig. 6 you see a fit of the tension data to this parametric expression for SU(3).

The authors took for the magnetic coupling $g^2_{\mathrm{M}} = g^2_{\mathrm{E}}$, so neglected renormalization effects of the scale gT, which are a few percent at $T = 2T_{\mathrm{c}}$. On the other hand they included two loop renormalization effects. Dropping those effects, and taking into account the uncertainty in the relation between $\Lambda_{\overline{MS}}$ and T_{c} there is still consistency between data and the one loop formula Eq. (3.3).

Notably the value of the tension at the critical temperature is within errors equal to the tension at zero temperature. So the tension of the spatial Wilson loop does not change within errors in the hadron phase.

The conclusion is quite clear: down to temperatures a few times T_{c}, the loop behaviour is determined by leading order magnetic sector effects! These effects are embodied in the dimensionless number $c_{k=1}$. The number $c_{k=1}$ is within errors equal to the purely 3D simulation of the loop.

The spatial Wilson loop measures in a sense to be specified later the magnetic flux in the system. The tension is flat from $T = 0$ to $T = T_{\mathrm{c}}$,

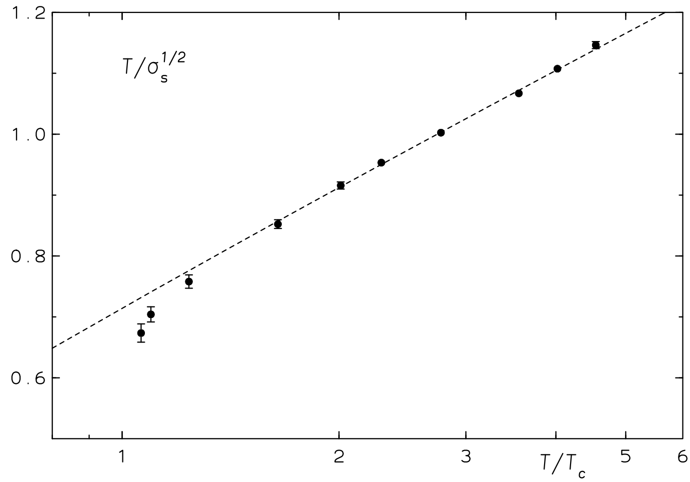

Fig. 6. The temperature over the square root of the spatial string tension *versus* T/T_{c} for SU(3). The dashed line shows a fit according to a two loop scaling formula for the coupling, see text below Eq. (4.8). From Ref. [14].

according to the data. In all of the confined phase the magnetic activity does not change.

Above T_{c} it starts to grow like $g_{\mathrm{M}}^4 T^2$. Apparently beyond the transition the activity goes up, and comes, as the data tell us, entirely from the magnetostatic sector.

5. A simple model

In close analogy with the 't Hooft loop one can do a quasi-particle calculation for the k-tension of the Wilson loop. But what are the magnetic quasi-particles?

We are making the simplest possible assumptions:

- The magnetic quasi-particles have a screening length $l_{\mathrm{M}} \sim g^2 T$ much smaller than their average distance.

- The magnetic quasi-particles are in the adjoint representation of $\mathrm{SU}(N)$.

Note here that the magnetic screening length defines a volume in which many elementary quanta are present, just like the Debye screening length. We assume here that the magnetic screening defines non-perturbative lumps, called magnetic quasi-particles.

We should have, like for the 't Hooft loop, a magnetic flux representation for the k-Wilson loop. One can argue that its average can be computed from:

$$\langle W_k(L)\rangle = \left\langle \exp ig \int d\vec{S}\cdot \mathrm{Tr}\vec{B}\frac{Y_k}{N}\right\rangle , \tag{5.1}$$

where, as before, $Y_k = \mathrm{diag}\,(k, k, ..., k, k-N, ...k-N)$ with $N-k$ entries k and k entries $k-N$, so that it generates the center group element:

$$\exp\left(i\frac{2\pi}{N}Y_k\right) = \exp ik\frac{2\pi}{N} = z_k. \tag{5.2}$$

The Y_k-charge of a magnetic quasi-particle is $\pm\frac{2\pi N}{g}$ with the same multiplicity $2k(N-k)$. It contributes -1 to the Wilson loop Eq. (5.1) because only one-half of its flux goes through the loop.

With our assumptions we can now, in precise analogy with the calculation of the 't Hooft loop, understand why the k-tension scales like:

$$\sigma_k = c(N)k(N-k)l_{\mathrm{M}}n_{\mathrm{M}}\,. \tag{5.3}$$

As in the gluon case n_{M} is the density of one quasi-particle species.

The coefficient $c(N)$ would be O(1) for all N with the simple Poisson distribution used in Eq. (3.19). And this would contradict the result from all orders in perturbation theory, that the Wilson loop tension is $O(1)$ for large N! This is easily repaired by insisting on a distribution with a width $w(N)$ of $O(N^{-1})$ and noting that $c(N)\sim w(N)$.

So for $k=1$ and large N we learn from Eq. (5.3) that :

$$l_{\mathrm{M}}^2\sigma_1 = c l_{\mathrm{M}}^3 n_{\mathrm{M}}, \tag{5.4}$$

and from Eq. (4.6) we find that $l_{\mathrm{M}}^2\sigma_1 = 0.028(2)$ for N large. This is an *a posteriori* justification of our assumption that the magnetic quasi-particle gas is dilute. Another justification comes from the lattice data for the ratios $\frac{\sigma_k}{\sigma_1}$ in which the $c(N)$ drops out.

The ratios found by simulation [20] are close — within a percent for the central value — :

$$\begin{aligned}\mathrm{SU}(4): \sigma_2/\sigma_1 &= 1.3548 \pm 0.0064\,,\\ \mathrm{SU}(6): \sigma_2/\sigma_1 &= 1.6160 \pm 0.0086; \sigma_3/\sigma_1 = 1.808 \pm 0.025\,.\end{aligned}$$

The results are that precise, that you see a two standard deviation, except for the second ratio of SU(6). As we said, magnetic quasi-particles are dilute but only approximately free.

There is a less precise determination of the ratio $\sigma_2/\sigma_1 = 1.52 \pm 0.15$ in SU(5) [21]. But the central value is within 1 to 2% of the predicted value 3/2.

6. Conclusions

In these lectures we concentrated on the order of the transition, and on the quasi-particle picture of the magnetic sector. The lattice data are consistent with the predictions within a few percent, the typical order of magnitude of their diluteness.

At large N the strong first order results suggest a quasi-particle picture down to the critical temperature and can be put into evidence by simulating at $T_{\rm c}$ spatial Wilson and 't Hooft loops. The simulation of the latter is now getting in a new stage, where we start to learn [25] the systematic errors from comparison with the known screening masses. From these results one could infer the corresponding $\mathrm{Z}(N)$ spin model.

How can one view the mechanism of the transition and the disappearance of the quasi-particles? First, as the densities of magnetic quasi-particles and gluons start to match as $g \to 1$, one might expect just above $T_{\rm c}$ a binding of the quasi-particles into dyons. This is because in our quasi-particle picture there is no correlation between electric and magnetic colour fields. From measurements of the topological susceptibility we know it should get restored in the critical region $T_{\rm c}^{+}$. A reasonable gues is that this happens through binding of electric and magnetic quasi-particles into dyons. Whatever the details of this binding, it would be witnessed by the equality of spatial Wilson and 't Hooft loops. At $T_{\rm c}$ the Wilson loop tension equals to a good approximation the zero temperature string tension. Hence the 't Hooft loop tension has to drop from this value to zero at $T_{\rm c}^{-}$ and signals a first order transition. This binding fails for SU(2), where we know from [9] that the 't Hooft tension tends to zero.

For the screening lengths the same is true.

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