SPONTANEOUS FISSION AND α-DECAY HALF-LIVES OF SUPERHEAVY NUCLEI IN DIFFERENT MACROSCOPIC ENERGY MODELS

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Dedicated to Adam Sobiczewski in honour of his 70th birthday

Spontaneous fission half-lives ($T_{\rm sf}$) of the heaviest nuclei are calculated in the macroscopic-microscopic approach based on the deformed Woods-Saxon potential. Four different models of the macroscopic energy are examined and their influence on the results is discussed. The calculations of ($T_{\rm sf}$) are performed within WKB approximation. Multi-dimensional dynamical-programming method (MDP) is applied to minimize the action integral in a 3-dimensional space of deformation parameters describing the nuclear shape $\{\beta_2, \beta_4, \beta_6\}$.

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1. Introduction

The region of superheavy nuclei is one of the most intensively studied in the last years (see for example [1, 2] and citations there). Nevertheless the experimental evidence is still far from being complete. The history of theoretical studies of spontaneous fission half-lives of the superheavy nuclei begun almost forty years ago. One of the first publications on this subject was related to Professor Adam Sobiczewski's name and it has been published in the early 60-th. Sobiczewski together with Gareev and Kalinnik [3] have foreseen the existence of the double-magic spherical superheavy nucleus with 114 protons and 184 neutrons.

In 1970–1982 a series of papers of Sobiczewski appeared in which he and his co-workers have introduced the so called "dynamical method" of calculation of the spontaneous fission half-lives. In this method the trajectory to fission was determined by minimization of the action integral in the multidimensional collective space of the deformation parameters [4–6]. The calculations were based on the single-particle potential of the Nilsson type [7–9] and the nuclear liquid drop [10] or droplet model [11] has been used to evaluate the macroscopic part of the nuclear energy. The spontaneous fission half-lives of nuclei in the region of $96 \leq Z \leq 104$ were reasonably reproduced when the liquid drop energy was used. The characteristic "parabolic" systematics of the $\log(T_{\rm sf})$ for investigated isotopes has been reconstructed pretty well.

The end of the eighties and the beginning of the nineties gave the cycle of relatively important papers [12–16], based on more realistic potential of the Woods–Saxon type [17] and a new form of the smooth part of nuclear energy, the so called folded-Yukawa plus exponential model [18]. In the midst of other things it was possible to predict the existence of "the deformed island of stability" in the neighborhood of the nucleus with $Z = 108$ and $N = 162$. The concept of "the double magic deformed nucleus" was introduced for the first time by Prof. Sobiczewski and is generally accepted in the literature.

All the calculations presented in the above mentioned papers have been carried out using the macroscopic-microscopic method in which the energy of fissioning nucleus was split into the smooth $E_{\rm smooth}$ (macroscopic) part and the shell $\delta E_{\rm shell}$ and pairing $\delta E_{\rm pair}$ energy corrections. The shell and pairing energy corrections depend on the form of the single-particle potential. There is a general believe that the deformed Woods–Saxon potential with the universal set of parameters gives the proper behavior of the fission barrier as a function of the deformation.

The expressions commonly used for the smooth part of nuclear energy are given by the liquid drop model [10], the droplet expansion [11], folded-Yukawa plus exponential approximation [18] and a very recently developed Lublin–Strasbourg drop (LSD) [19]. The LSD model represents the revised and improved version of the liquid drop formula, in which the parameters of the extended classical energy formula were adjusted to the known masses of isotopes. Due to the presence of the curvature term the LSD formula gives also the right fission barrier heights without any readjustment of the parameters.

It is well known that the smooth part of the energy considerably influences estimates of the spontaneous fission half-lives ($T_{\rm sf}$). Therefore, we would like to compare the fission probability given by the above models of the macroscopic energy. In particular, we would like to obtain the estimates of spontaneous fission ($T_{\rm sf}$) as well as the α-decay half-life times for the heavy and superheavy nuclei within the new LSD model.

In our analysis we have taken into account three deformation degrees of freedom $(\beta_2, \beta_4, \beta_6)$, describing well the shape of the heaviest nuclei during the penetration of the barrier. To minimize the action integrals related to the spontaneous fission probability we have used the multidimensional dynamic programming method (MDP) [20] and the WKB approximation.

The theoretical background of our model is shortly described in Section 2. The results are presented in Section 3. Conclusions and discussion are placed in Section 4.

2. Theoretical model

The potential energy surface of fissioning nuclei was obtained using the macroscopic–microscopic method and the single particle Woods–Saxon potential with the *universal* set of parameters [17]. According to the Strutinsky shell correction model, the collective potential energy V is split into a shell $\delta E_{\rm shell}$ and a pairing correction $\delta E_{\rm pair}$ parts as well as the smooth average background energy of the liquid drop [10], droplet [11], folded-Yukawa plus exponential approximation [18] and/or the LSD model [19] with the standard values of parameters. The residual pairing interaction is treated in the BCS approximation where the pairing strength constants are as in reference [24].

The collective mass parameters B_{kl}, describing the inertia of the fissioning nucleus are calculated in the adiabatic cranking model. The collective mass plays the role of a metric tensor in the multi-dimensional collective space.

The fission is treated as a tunneling through the collective potential energy barrier in the multidimensional deformation parameter space. The spontaneous-fission half-life is inversely proportional to the probability of the penetration of the barrier:

$$T_{\rm sf} = \frac{\ln 2}{n}\frac{1}{P}\,. \tag{1}$$

Here, n is the number of *assaults* of the nucleus on the fission barrier per unit of time: $n \approx 10^{20.38}{\rm s}^{-1}$. The penetration probability P in the one-dimensional WKB semi-classical approximation is given by the following formula:

$$P = (1 + {\rm e}^{2S})^{-1}\,, \tag{2}$$

where $S(L)$ is the action integral evaluated along the fission path $L(s)$ which minimizes the reduced action in the multidimensional collective space:

$$S(L) = \int\limits_{s_1}^{s_2} \left\{ \frac{2}{\hbar^2}\, B_{\rm eff}(s)[V(s) - E] \right\}^{1/2} ds\,. \tag{3}$$

An effective inertia associated with the fission motion along the path $L(s)$ is

$$B_{\rm eff}(s) = \sum_{k,l} B_{kl}\, \frac{dq_k}{ds}\frac{dq_l}{ds}\,, \tag{4}$$

where ds denotes the path-length element in the collective space. The integration limits s_1 and s_2 correspond to the classical turning points, determined by the equation $V(s) = E$, and E is the total energy of the nucleus.

The dynamic calculation of the $T_{\rm sf}$ means a quest for the trajectory $L_{\rm min}$ which fulfills a principle of stationary action:

$$\delta S(L) = 0\,. \tag{5}$$

To minimize the action integral we have used the multi-dimensional dynamic-programming method (MDP) [6,20].

Since the macroscopic–microscopic method is not analytical, it is necessary to evaluate the potential energy and all components of the inertia tensor on a grid in the multi-dimensional space of deformation parameters. This multi-dimensional collective space consists of three deformation parameters describing well the shape of fissioning heavy nuclei (β_2, β_4, β_6). We have used the following steps in the β_λ parameter grid: $\Delta\beta_2 = 0.05$, $\Delta\beta_4 = 0.04$ and $\Delta\beta_6 = 0.04$.

The spontaneous fission of the heaviest nuclei is accompanied by the α-decay. We have made the estimates of the energies of emitted α-particles (Q_α) and half-lives of the α-decay (T_α) using the Viola–Seaborg formulae [22] with the parameter set adjusted by Sobiczewski *et al.* [23].

3. Results

3.1. Spontaneous fission

The fission barrier of the Fermium isotopes with $N \in (142, 172)$ evaluated within all four discussed in the paper models are plotted in Fig. 1. The classic liquid drop fission barriers [10] are denoted by the dotted lines and the droplet ones [18] by dotted-dashed curves. The barriers obtained with the Yukawa plus exponential model [18] are marked by dashed lines while those of the LSD [19] by the solid ones.

One can see in Fig. 1 that for all the isotopes, the fission barriers in the liquid drop model are relatively high and wide. It is especially visible in the heavier isotopes with $N \geq 156$. This effect leads to the considerably longer spontaneous fission half-lives for heavier isotopes (see for example [4]).

One observes an interesting behavior of the fission barriers obtained with the droplet model. For lighter isotopes the barriers are in good agreement with the liquid drop ones while for heavier Fm nuclei the tendency to a large reduction of the height and thickness of the barrier can be noticed. In earlier papers dealing with the spontaneous fission half-lives this tendency was connected with an abrupt reduction of $T_{\rm sf}$ of heavier Fm isotopes [5].

The folded-Yukawa plus exponential model gives the macroscopic fission barriers similar to that of the drop model however the barrier heights are

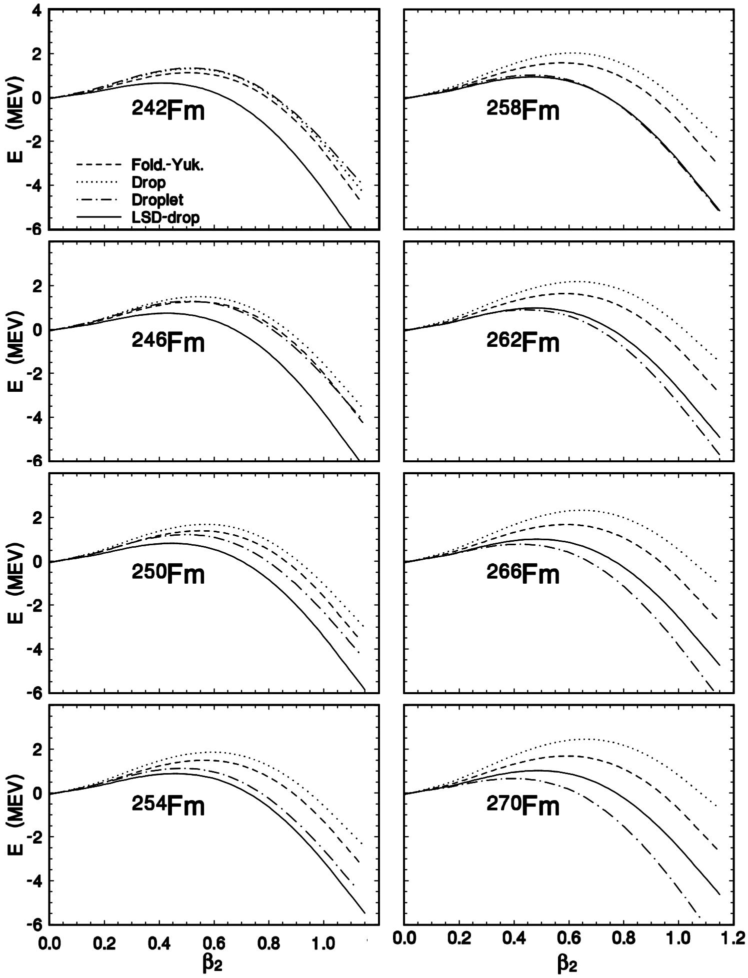

Fig. 1. The fission barriers for Fm isotopes with $N \in (142, 174)$ evaluated using four different expression for the macroscopic.

a little bit lower in case of the heavy isotopes. This decrease of the barrier heights influences the spontaneous fission half-lives T_{sf}: for the heavier isotopes the T_{sf} become considerably longer. Similar effect was already observed in the papers [20, 21].

The lowest barriers are observed for the new LSD model. The barriers change very weakly with increasing neutron number N and become only a little higher and wider. It is seen in Fig. 1 that the use of different formulae for the macroscopic energy can change the barrier heights by even 2–5 MeV. This effect should influence significantly the estimates of the spontaneous fission half-lives in different models.

Similar results but for the element $Z = 110$ are plotted in Fig. 2. The variance with growing neutron number of the fission-barrier heights and widths is close to those observed for the Fm isotopes in Fig. 1. In Fig. 3 the estimates of the spontaneous fission half-lives $T_{\rm sf}$ for the isotopes with atomic numbers $100 \leq Z \leq 110$ are compared with the experimental data (full diamonds). The theoretical results are obtained using the four discussed above models for the macroscopic energy. The data obtained in the liquid drop model are represented by full triangles and the results obtained with the droplet model by squares. The estimates made with the LSD are marked

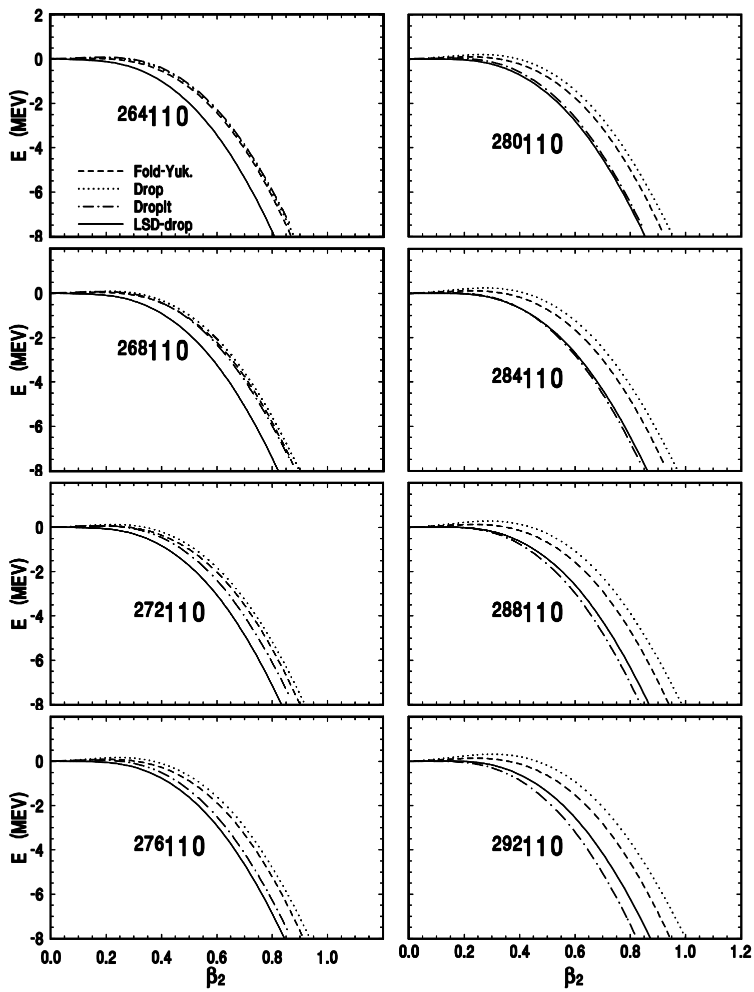

Fig. 2. The same as in Fig. 1 but for $Z = 110$ element.

by open triangles. It is seen that the spontaneous fission half-lives differ considerably depending on the model used. For the liquid drop and folded-Yukawa model the results are too large as compared to experiment, while these for droplet and LSD models are closer to the measured $T_{\rm sf}$.

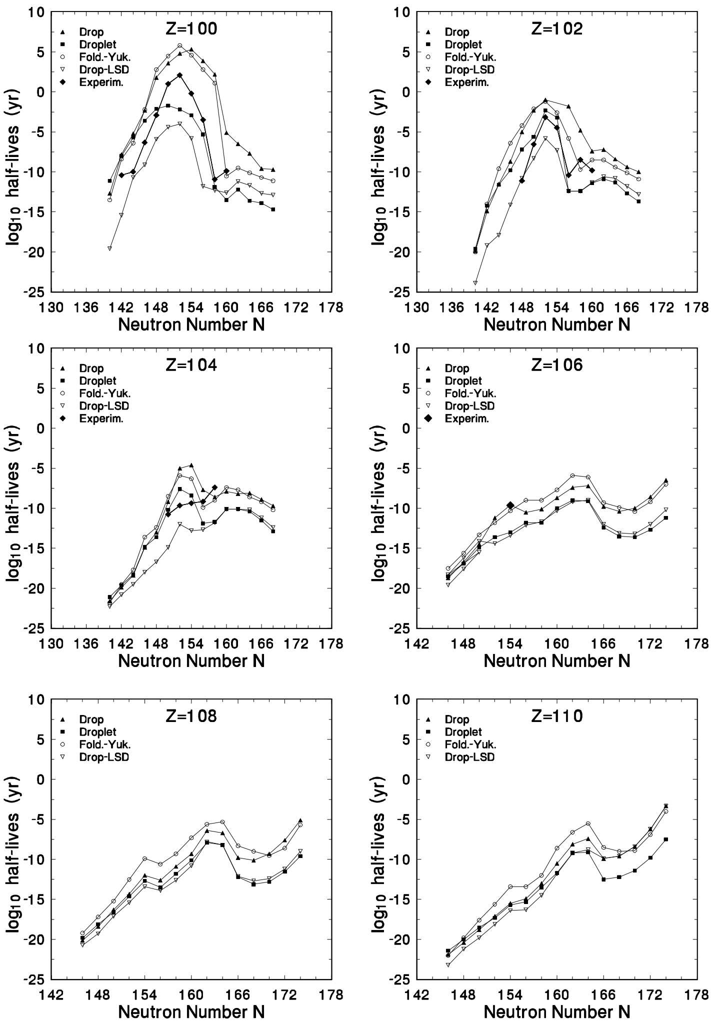

Fig. 3. Spontaneous fission half-lives ($T_{\rm sf}$) (in years) for the even–even isotopes with atomic with $Z \in (100, 110)$ plotted as a function of the neutron number N.

Traditionally, the estimations of the $T_{\rm sf}$ are made adding a zero-point energy (*e.g.*, E_0=0.5 MeV) to the Strutinsky ground-state energy of the nucleus. Some authors argue that such a procedure is inconsistent [25, 26]. It is easy to foresee the effect of discarding this zero-point energy. In Fig. 4 the

LSD estimates of $T_{\rm sf}$ of Fermium, Nobelium, Rutherdforium and Seaborgium isotopes are plotted with $E_0 = 0.5$ MeV and without the zero-point energy. One can see the increase of the fission life-times by about 3 orders of magnitude on the average. A rather good agreement of the theoretical estimates obtained with E_0=0 is observed for the Fermium and Nobelium isotopes only.

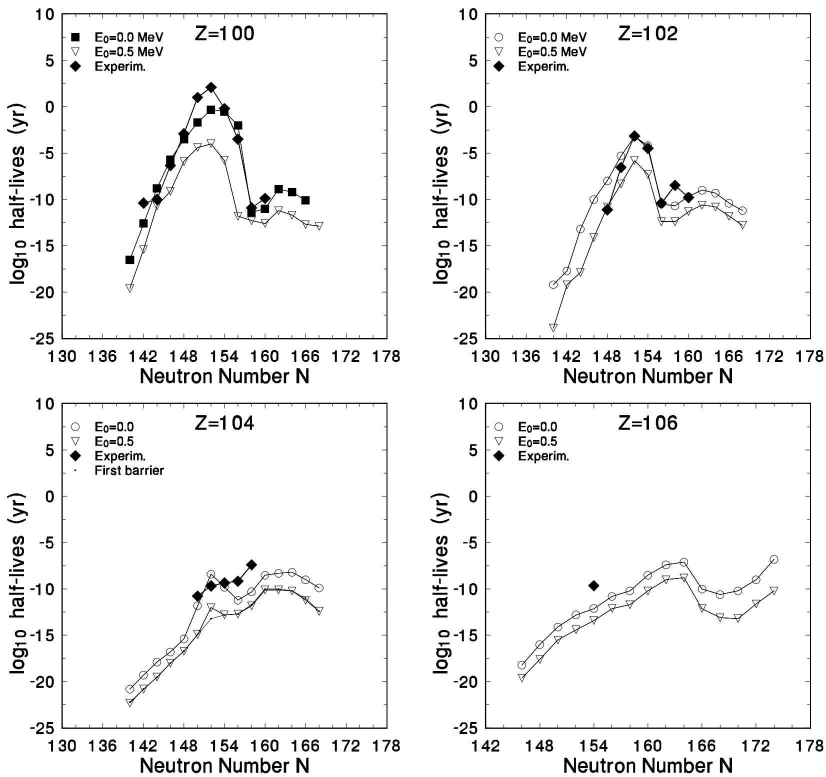

Fig. 4. Spontaneous fission half-lives ($T_{\rm sf}$) of the isotopes with atomic number $Z \in (100, 106)$ obtained with the LSD model and different zero-point energies (E_0).

For the Rutherfordium isotopes the agreement becomes rather poor. This is connected to the double fission barrier problem. For the Fermium and Nobelium isotopes the fission barriers are doubly humped and this has an impact on the behaviour of the spontaneous fission half-lives which increase very rapidly for the isotopes beyond $N = 152$ region and known "parabolic" systematics.

The situation fundamentally changes for the elements with $Z \geq 104$. The disappearance of the second barrier leads to practically linear systematics of

$T_{\rm sf}$ and the effect is very well visible in case of the experimental $T_{\rm sf}$ of the Rutherfordium isotopes. Unfortunately in our theoretical calculations the second barriers persistently exist for several isotopes and leads to unwanted increase of the fission lifetimes what is in disagreement with experimental data. The systematics of the fission life times astonishingly agrees with the experiment when one artificially neglects the penetration of the second barrier. This effect is seen in Fig. 5.

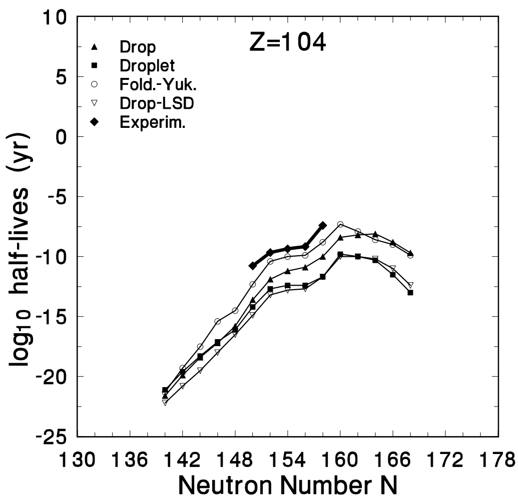

Fig. 5. Spontaneous fission half-lives ($T_{\rm sf}$) of the Rutherfordium isotopes calculated with the first barrier only (see text).

The calculations show that the theoretical estimations of the $T_{\rm sf}$ are the best if we adapt the macroscopic LSD model without zero-point energy.

3.2. α-decay

In addition to the calculations of the spontaneous fission half-lives it is useful to make the estimates of the Q_α values and the α-decay half-lives (T_α) for considered nuclei. In the heavy and superheavy region of nuclei both the decay modes compete.

The estimates of Q_α done for four discussed macroscopic models on the basis of Viola–Seaborg formulae [22] with parameters adjusted in Ref. [23] are presented in Fig. 6. The results obtained in all models are comparable. Similar results were reported for selfconsistent Hartree–Fock–Bogoliubov–Skyrme model with SLy4 force parameters by Ćwiok *et al.* in [27]. In Fig. 6 we show the Q_α values (in MeV) as a function of mass number A for different macroscopic models. Part (a) corresponds to the liquid drop of Myers and

Świątecki [10], part (b) to droplet model [11], part (c) to Yukawa plus exponential [18] and (d) to the LSD model [19]. The corresponding experimental values were retrieved from BNL nuclear data base [28]. In all cases shown in the figure one sees a good agreement between theoretically estimated values and the experimental ones.

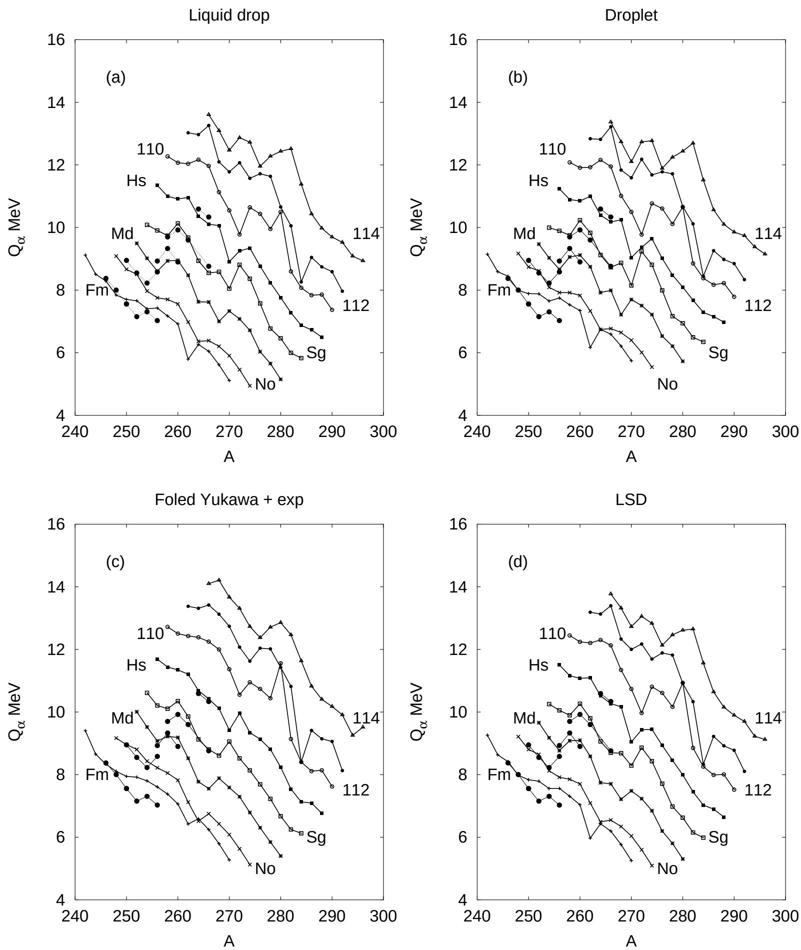

Fig. 6. Q_α values (in MeV) *vs* mass number A for different macroscopic models: (a) liquid drop of Myers and Świątecki [10], (b) droplet model [11], (c) Yukawa plus exponential [18] and (d) LSD [19]. The experimental values are retrieved from BNL nuclear data base [28].

In Fig. 7 there are shown the similar results for LSD (applied as a macroscopic part of the energy) and Thomas–Fermi (TF) [29] both for the case of a shell correction extracted from Myers–Swiatecki tables [29]. The root mean square deviation of the (rms-dev) is typed on each subfigure. Their values which are close to 0.23 MeV suggest that within the same accuracy, it is easier to use the LSD model instead of calculationally complicated TF model.

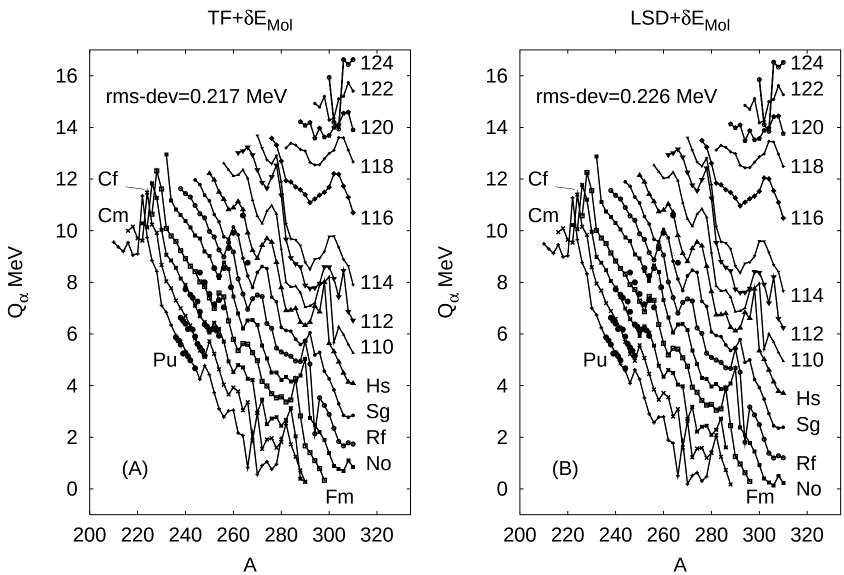

Fig. 7. Q_α values (in MeV) *vs* mass number A for (A) LSD [19] + Myers–Swiatecki shell correction. and (B) Thomas–Fermi (TF) + Myers–Swiatecki shell correction [29]. The experimental data are extracted from NL nuclear data base [28]. The rms deviation (rms-dev) is displayed for each case.

In Fig. 8 we show the contours of the logarithm of α-half-lives (sec) as calculated within the LSD and the Möller shell correction.

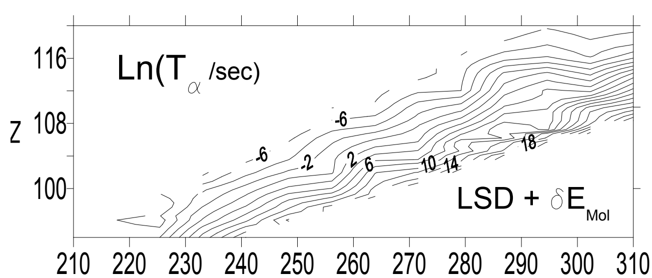

Fig. 8. The contours of the logarithm of α-half lives (sec) as calculated within the LSD and the Möller shell correction.

4. Conclusions and discussion

From our investigations one can draw the following conclusions:

- The macroscopic energy influences very considerably the calculated spontaneous fission half-lives T_{sf}.

- The Q_α values (or the α decay half-lives) calculated in different macroscopic models are nearly the same and model independent up to a small differences (see rms deviation on Fig. 8).

- The LSD model seems to be comparable in accuracy to the Thomas–Fermi macroscopic model and can be used as a fast and exact tool for calculation of the groundstate properties of the nucleus.

- One should still continue deep studies of the macroscopic part of the nuclear energy in order to choose the most appropriate model for barriers determination and the T_{sf} calculation.

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